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# (1, 2) – Double Domination in Graphs

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**Abstract** - In this paper , we introduced the ( 1, 2 ) – double domination number and also we discussed about its properties .

**Keywords** - ( 1, 2 ) – double domination number , independent double dominating set , independent triple dominating set .

**2000 Mathematics Subject Classification** - 05C69

## I. INTRODUCTION

Dominating queens is the origin of the study of dominating set in graphs . Berge [1] and Ore [7] were the first to define dominating sets. A new type of dominating set, ( 1, 2 ) – dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [8] . In this paper , we introduced the ( 1, 2 ) – double domination number and also we discussed about its properties .

## II. PRELIMINARIES

*Definition 2.1* : A graph is said to be complete if each of its vertices is adjacent to every other vertex.

*Definition 2.2* : A graph is said to be regular if each of its vertices has the same degree.

*Definition 2.3* : A graph is said to be cubic graph if each of its vertices is of degree three.

*Definition 2.4* : A bipartite graph is a graph in which vertices can be divided into two disjoint sets A and B such that every edge connects a vertex in A to one in B.

*Definition 2.5* : A ( 1, 2 ) – dominating set in a graph  $G = (V,E)$  is a set S having the property that for every vertex  $v$  in  $V - S$  there is atleast one vertex in S at distance 1 from  $v$  and a second vertex in S at distance atleast 2 from  $v$ .

*Definition 2.6* : The order of the smallest (1,2)- dominating set of G is called the (1,2) – domination number of G and we denote it by  $\gamma(1,2)$  .

*Remark 2.1* : From the definition of 2.1 , we see that a (1,2) – dominating set contains atleast 2 vertices, (1,2) – domination number of a graph will be always  $\geq 2$  and (1,2) – dominating sets occur in graphs of order atleast 3.

*Definition 2.7* : For each vertex  $x$  in a graph G, we introduce a new vertex  $x'$  and join  $x$  and  $x'$  by an edge. The resulting graph is called the *corona* of G.

## III. ( 1, 2 ) – DOUBLE DOMINATING SET

*Definition 3.1* : A ( 1, 2 ) – double dominating set in a graph  $G = (V, E)$  is a set S having the property that for every vertex  $v$  in  $V - S$  there is atleast two vertex in S at distance 1 from  $v$  and a second vertex in S at distance atleast 2 from  $v$ .

*Definition 3.2* : The order of the smallest ( 1, 2 ) - double dominating set of G is called the (1,2) – double domination number of G and we denote it by  $\gamma_d(1,2)$

From the definition of (1,2) – double dominating sets, we see that a (1,2) – double dominating set contains atleast 2 vertices, (1,2) – double domination number of a graph will be always  $\geq 3$  and (1,2) – double dominating sets occur in graphs of order atleast 3.

*Example 3.1* : Consider the graph

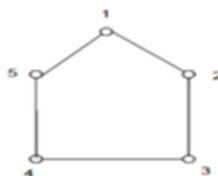


Fig. 1

In Fig. 1, { 1, 4, 3 } , { 1, 4, 2 } is a (1,2) – double dominating set.

*Definition 3.3* : A dominating set S is an independent double dominating set if no two vertices in S are adjacent, that is, S is an independent set. The independent double domination number  $i_2(G)$  of a graph G is the minimum cardinality of an independent double dominating set. Thus,  $i_2(G) = \min\{ |S| \text{ dominates and } \Delta(\{S\}) \}$  .

*Example 3.2* : In example 3.1 [9] ,  $i_2(G) = 3$  .

*Definition 3.4* : A double dominating set S is called a perfect double dominating set if for every vertex , The perfect double

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domination number is denoted as  $u \in V$ ,  $|N[u] \cap S| = 1$ . The perfect double domination number is denoted as  $\gamma_{pd}(G)$ .

**Definition 3.5 :** A double dominating set  $S$  is called an efficient double dominating set if for every vertex  $u \in V - S$ ,  $|N(u) \cap S| = 1$ . Equivalently, a dominating set is efficient if the distance between any two vertices in  $S$  is at least three, that is,  $S$  is a packing.

We note that, if a graph has an efficient double dominating set, then all efficient double dominating sets in  $G$  have the same cardinality namely  $\gamma(G)$ .

**Theorem 3.1 :** All (1,2) – double dominating sets are dominating sets.

**Proof :** The result is trivial from the definition of (1,2) – double dominating sets.

But the converse need not be true.

**Example 3.2 :** In example 3.1,  $\{1,4\}$  is a dominating set.

But it is not a (1,2) – dominating set.

$\{2,3,4\}$  is a (1,2) – dominating set.

$\{1,4,3\}$  is a (1,2) – double dominating set and it is a dominating set also.

### IV. (1,2) – DOUBLE DOMINATION IN COMPLETE GRAPHS

**Theorem 3.2.1 :** (1,2) – double domination is not possible in complete graphs.

**Proof:** In a complete graph, each vertex is adjacent to every other vertices. So we cannot find a (1,2) – double dominating set. No vertex can be found at a distance atmost 2 from any other vertex. Let  $G$  be a complete graph with  $n$  vertices. Then it will have  $nC2$  edges and each vertex is of degree  $n - 1$ . The minimum number of edges to be deleted so as to become the resulting graph (1,2) – double dominating is  $n - 2$ . If we delete  $n - 2$  edges from a complete graph, then in the resulting graph, we can find a (1,2) – double dominating set.

**Lemma 3.2.1 :** If a graph  $G$  with  $n$  vertices, has a vertex of degree  $n - 1$ , we cannot find a (1,2) – dominating set.

**Example 3.2.1 :** In this graph, we cannot find a (1,2) – double dominating set since each vertex is adjacent to all other vertices.

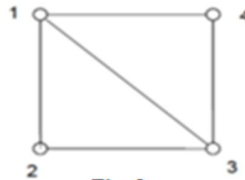


Fig. 2

In graph Fig. 2, we cannot find a (1,2) – double dominating set since each vertex is adjacent to all other vertices.

### V. RELATION BETWEEN DOMINATION NUMBER AND (1,2) – DOUBLE DOMINATION NUMBER

In this section we consider different types of graphs and find out their domination number, (1,2) - domination number and (1,2) – double domination number and check the relation between them.

**Example 3.3.1 :**



Fig. 3

In Fig. 3,

$\{1,3\}$ ,  $\{1,4\}$ ,  $\{2,4\}$ ,  $\{2,3\}$  are all dominating sets.  $\gamma(G) = 2$ .

$\{1,4\}$  is a (1,2) – dominating set.

$\gamma(1,2) = 2$ .

$\{1,3,4\}$  is a (1,2) – double dominating set and double dominating set.

$\gamma_{d(1,2)} = \gamma_d = 3$ .

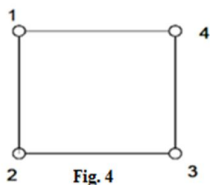
$\therefore \gamma(1,2) < \gamma_{d(1,2)}$ .

$\gamma < \gamma_{d(1,2)}$ .

$\gamma_{d(1,2)} = \gamma_d$ .

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Example 3.3.2 :



In Fig. 4,

$\{1,3\}, \{1,4\}, \{2,4\}, \{2,3\}$  are all dominating sets.  $\gamma(G) = 2$ .

$\{2,3\}$  is a  $(1, 2)$  – dominating set.

$\gamma(1,2) = 2$ .

$\{2, 4\}$  is a double dominating set.

$\{2, 3, 4\}$  is a  $(1, 2)$  – double dominating set.

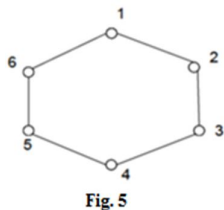
$\gamma_d(1,2) = 3$ .

$$\therefore \gamma(1,2) < \gamma_d(1,2) .$$

$$\gamma < \gamma_d(1,2) .$$

$$\gamma_d < \gamma_d(1,2) .$$

Example 3.3.3 :



In Fig. 5 ,

$\{1, 3, 5\}, \{2, 4, 6\}$  are dominating sets.

$\gamma(G) = 3$ .

$\{1, 4, 6\}$  is a  $(1, 2)$  – dominating set.

$\gamma(1,2) = 3$ .

$\{1, 5, 3\}$  is a double dominating set.

$\{1, 3, 4, 6\}$  is a  $(1, 2)$  – double dominating set.

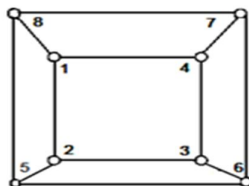
$\gamma_d(1,2) = 4$

$$\therefore \gamma(1,2) < \gamma_d(1,2) .$$

$$\gamma < \gamma_d(1,2) .$$

$$\gamma_d < \gamma_d(1,2) .$$

Example 3.3.4 :



In Fig. 6 ,

$\{1,2,3,4\}, \{5,6,7,8\}$  are dominating.

$\gamma(G) = 4$ .

$\{1,2,3,4\}$  is a  $(1,2)$  – dominating set.

$\gamma(1,2) = 3$ .

$\{4, 8, 2, 6\}$  is a double dominating set.

$\{1, 3, 5, 6, 7, 8\}$  is a  $(1, 2)$  – double dominating set.

$\gamma_d(1,2) = 6$

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$$\therefore \gamma(1,2) < \gamma_{d(1,2)} .$$

$$\gamma < \gamma_{d(1,2)} .$$

$$\gamma_d < \gamma_{d(1,2)} .$$

Example 3.3.5 : Consider the bipartite graph G



Fig. 7

In Fig. 7 ,

{1,2} is a dominating set.

$$\gamma(G) = 2.$$

{1,4,5} is a (1,2) – dominating set.

$$\gamma(1,2) = 3.$$

{3, 4, 5 } is a double dominating set.

{ 2, 3, 4, 5 } is a ( 1, 2 ) – double dominating set.

$$\gamma_{d(1,2)} = 4$$

$$\therefore \gamma(1,2) < \gamma_{d(1,2)} .$$

$$\gamma < \gamma_{d(1,2)} .$$

$$\gamma_d < \gamma_{d(1,2)} .$$

Example 3.3.6 : Consider the cubic bipartite graphs G ,

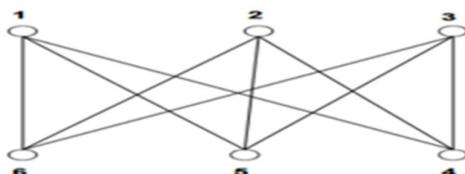


Fig. 8

In Fig. 8 ,

{ 1, 5 }, { 2, 6 } is a dominating set.

$$\gamma(G) = 2.$$

{ 1, 5 } is a (1,2) – dominating set.

$$\gamma(1,2) = 2.$$

{ 2, 4, 6, 5 } is a ( 1, 2 ) – double dominating set.

$$\gamma_{d(1,2)} = 4$$

$$\therefore \gamma(1,2) < \gamma_{d(1,2)} .$$

$$\gamma < \gamma_{d(1,2)} .$$

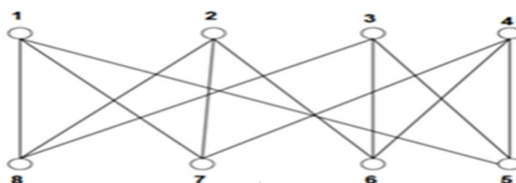


Fig. 9

In Fig. 9,

{ 1, 6 } is a dominating set.

$$\gamma(G) = 2.$$

{ 1, 6 } is a (1,2) – dominating set.

$$\gamma(1,2) = 2.$$

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$\{ 1, 3, 6, 7, 8 \}$  is a  $(1, 2)$  – double dominating set.

$$\gamma_{d(1,2)} = 5 .$$

$$\therefore \gamma(1,2) < \gamma_{d(1,2)} .$$

$$\gamma < \gamma_{d(1,2)} .$$

*Remark 3.3.1* : In all the above examples , we conclude the following

- i) domination number is less than  $(1, 2)$  – double domination number .
- ii) double domination number is less than  $(1, 2)$  – double domination number .
- iii)  $(1, 2)$  – domination number is less than  $(1, 2)$  – double domination number .

From the above examples we have the following theorem.

*Theorem 3.3.1* : In a graph  $G$ , domination number is less than or equal to  $(1, 2)$  – double domination number.

*Proof* : Let  $G$  be a graph and  $D$  be its double dominating set. Then every vertex in  $V - D$  is adjacent to a vertex in  $D$ . That is, in  $D$ , for every vertex  $u$ , there is a 2 vertex which is at distance 1 from  $u$ . But it is not necessary that there is a second vertex at distance atmost 2 from  $u$ . So if we find a  $(1, 2)$  – dominating set, it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to  $(1,2)$  – domination number.

*Theorem 3.3.2* : In a graph  $G$ ,  $(1, 2)$  – domination number is less than or equal to  $(1, 2)$  – double domination number.

*Proof* : Similar to theorem 3.3.1.

*Theorem 3.3.3* : In a graph  $G$ , double domination number is less than or equal to  $(1, 2)$  – double domination number.

*Proof* : Similar to theorem 3.3.1.

*Theorem 3.3.4* : If  $G$  is a 2-regular graph, then the  $(1, 2)$  – double domination number of the corona of  $G$  is equal to the number of vertices of  $G$ .

*Proof* : Let  $G$  be a 2- regular graph. Then each of its vertices will be of degree 2. In the corona of  $G$ , for each vertex  $x$ , we introduce a new vertex and join them. Consequently, an edge is added to each of its vertices. By the definition of  $(1,2)$  – double dominating set each vertex  $v$  in  $V - S$  has atleast two vertex in  $S$  at distance 1 from  $v$  and a second vertex in  $S$  at distance atmost 2 from  $v$ . Hence  $(1,2)$  – double dominating set of the corona of  $G$  will consist of all the vertices of  $G$ .

*Theorem 3.3.5* : If in a graph  $G$ , an edge  $e$  is added,  $\gamma_{d(1,2)}(G + e) \geq \gamma_{d(1,2)}(G)$  .

*Proof*: Let  $G$  be a graph. Let  $S$  be the  $(1, 2)$  – double dominating set of  $G$ . If we add an edge to a vertex in  $S$ , that will not affect the cardinality of  $S$ . If we add an edge to a vertex in  $V - S$ , the cardinality of  $(1, 2)$  – double dominating set will increase. Therefore,  $\gamma_{d(1,2)}(G + e) \geq \gamma_{d(1,2)}(G)$  .

*Theorem 3.3.6* : If  $G$  is a complete bipartite graph, then the  $(1, 2)$  – double domination number  $\gamma_{d(1,2)}$  is 3.

*Proof*: Let  $G$  be a complete bipartite graph. Then  $V(G)$  can be partitioned in to 2 disjoint sets  $X$  and  $Y$  and each edge has one end in  $X$  and other end in  $Y$ . Since  $G$  is complete bipartite, each vertex of  $X$  is joined to every vertex in  $Y$ . A set of 2 vertices, one from  $X$  and another from  $Y$  will constitute a  $(1, 2)$  – double dominating set. Therefore,  $\gamma_{d(1,2)} = 3$ .

## VI. CONCLUSIONS

We considered the problem of finding a  $(1, 2)$  – double dominating set in graphs and compared them with the domination number. Also some preliminary theorems on  $(1, 2)$  - dominating sets are proved.

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