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# **Analysis of CT Liver Images Using Level Sets with Bayesian Analysis-A Hybrid Approach**

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**Abstract:** Computer tomography images are widely used in the diagnosis of liver tumor analysis because of its faster acquisition and compatibility with most life support devices. Accurate image segmentation is very sensitive in the field of medical image analysis. Active contours plays an important role in the area of medical image analysis. Active contour based tumor segmentation using classical level set methods easily suffer from deficiency in the presence of noise and other significant edges adjacent to the real boundary. This problem has not been effectively solved in the medical image analysis scenario. In this paper, we propose an improved energy function to tackle this problem by continuously rectifying the deviation of the level set function according to the signed distance function. This is achieved using an expectation-maximization algorithm. Experimental work shows the proposed framework outperforms the classical level set algorithms in accuracy and efficiency of image segmentation.

**Keywords:** Level set method, Level set scheme without re-initialization, Bayesian analysis, Energy Minimization

## **I. INTRODUCTION**

In this paper we focus on segmentation of liver tumor from different CT images by using level set methods with Bayesian analysis.. Liver and liver tumor segmentations are very important for a contemporary planning system of liver surgery. Over the past few years level set methods were used for image segmentation. The idea of the level set method [1] is to be implicitly represented a contour or interface as the zero level set. The level set method is used as numerical technique for analysis of medical images. With the level set representation the image segmentation problem can be formulated and solved in a principled way based on well established mathematical theories, including calculus of variations and partial differential equations. In classical level set methods, it is desirable to keep the evolving level set function close to a signed distance function [2]. Re-initialization has been commonly used in order to periodically re-initialize the level set function to a signed distance function during the evolution [3]. Many re-initializing strategies fail to push the zero level set towards its starting position, resulting in more intensive computation that “pulls” back the deviated level set. Possible solutions to the above problems have been intensively explored. For example, Caselles et al. [4] and Kichenassamy et al. [5] discovered the geodesic active contour schemes, which integrated both geometric and variational models in level set form. These models are expected to solve the initialization problem of active contour while handling complex topological changes automatically. Based on the Mumford–Shah minimal partition functional [6], Vese and Chan [7], Mansouri et al. [8] and Yezzi et al. [9] proposed different algorithms that did not depend on gradient computation for boundary detection. In spite of their promising performance, these methods lack robustness and efficiency in segmenting objects from complicated image backgrounds. Our new segmentation algorithm is the combination of a level sets method with Bayesian analysis, which is based on the approach of Li et al. [3]. This approach is intended to release the need of re-initialization during the evolution. It applies internal and external energy terms in order to reach a compromise between penalizing the deviation of the level set function and locating image features. We have to solve the classical partition problem by iteratively tuning the fronts according to the Bayes’ rule. This front evolution is undertaken in the context of an expectation–maximization (EM) algorithm so that the difference between the actual level set function and a signed distance function can be optimally reduced during the iteration. The main contributions of this paper are: (1) the combination of level set method and Bayesian analysis can be used to segment different CT liver images (2) Comparing the proposed strategy in contour detection with other classical algorithms.

## **II. ACTIVE CONTOUR MODEL**

### *A. Mumford and Shah Model*

The piecewise constant model is an alternative solution to the Mumford Shah model, proposed by Chan and Vese. They fit the original image  $I(x)$  by a piecewise constant function. For image  $I(x)$  on the image domain  $\Omega$ , PC model is formulated as the

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following energy functional.

$$E^{PC} = \lambda_1 \int_{\text{inside}(C)} |I(x) - c_1|^2 dx dy + \lambda_2 \int_{\text{outside}(C)} |I(x) - c_2|^2 dx dy + \mu |C|, \quad x \in \Omega \quad (1)$$

where  $\lambda_1$ ,  $\lambda_2$  and  $\mu$  are positive constants. Inside(C) and outside (C) denote the region inside and outside of curve C, respectively. The constants  $c_1$  and  $c_2$ , depending on C, are the average of  $I(x)$  inside C and outside C. Generally, we choose  $\lambda_1 = \lambda_2 = \lambda$  in practice. In order to solve this minimization problem, they replace the unknown curve C by the level set function  $\phi$ , and then rewrite the above function as follows

$$E^{PC}(\phi, c_1, c_2) = \lambda_1 \int_{\Omega} |I(x) - c_1|^2 H(\phi) dx dy + \lambda_2 \int_{\Omega} |I(x) - c_2|^2 (1 - H(\phi)) dx dy + \mu \int_{\Omega} \delta(\phi) |\nabla \phi| dx dy, \quad x \in \Omega \quad (2)$$

$H(\phi)$  is the Heaviside function and  $\delta(\phi)$  is the Dirac

### B. Level Set Formulation Without Re-Initialization

The idea of the Level set method is to be implicitly represented a contour or interface as the zero level set of a higher dimensional function, called the level set function, and formulate the evolution of the contour through the evolution of the level set function. In this paper it is desired to detect and delineate liver tumors in CT scans by using level set model. This technique is very suitable for medical organ segmentation since it can handle any of the cavities, concavities, convolutedness, splitting or merging. Another benefit of this technique is that this algorithm increases the capture range of the field flow. Level set is a deformable contour model where the user specifies a starting contour that is evolved to the image contour; the level set method is a geometric deformable model. The contour is described as a surface developed by partial differential equations[10, 11], where the contour is the zero level of the surface. The partial differential eqn. can then be written as

$$\frac{\partial \phi}{\partial t} = -|\nabla \phi| \cdot F \quad (3)$$

which is called the level set eqn. and where the symbol  $\phi$  denotes the level set function. In the above level set eqn. F is the velocity term that describes the level set evolution. By manipulating F, we can guide the level set to different areas or shapes, given a particular initialization of the level set function. F may also be dependent on an edge indicator function, which is defined as having a value zero on an edge, and non-zero otherwise. This causes F to slow the level set evolution when on an edge. The level set method proposed by Osher and Sethian is a versatile tool for tracing the interfaces that may separate an image  $\Omega$  into different parts. The main idea behind it is to characterize the interface function  $\Gamma(t)$  by a Lipchitz function  $\phi$ ,

$$\begin{cases} \phi(t, x, y) > 0(x, y) \text{ is inside } \Gamma(t) \\ \phi(t, x, y) = 0(x, y) \text{ is at } \Gamma(t) \\ \phi(t, x, y) < 0(x, y) \text{ is outside } \Gamma(t) \end{cases} \quad (4)$$

In this paper the evolution equation used is

$$\frac{\partial \phi}{\partial t} = \mu \left[ \nabla \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \right] + \lambda \delta(\phi) \text{div} \left( g \frac{\nabla \phi}{|\nabla \phi|} \right) + \gamma g \delta(\phi) \quad (5)$$

The second and the third term in the right hand side of eqn. correspond to the gradient flows of the energy functional

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$\lambda L_g(\Phi)$  and  $\gamma A_g(\Phi)$ , respectively, where

$$L_g(\Phi) = \int_{\Omega} g \delta(\Phi) |\nabla \Phi| dx dy \quad (6)$$

and

$$A_g(\Phi) = \int_{\Omega} g H(-\Phi) dx dy \quad (7)$$

where  $\delta$  is the univariate Dirac function, and  $H$  is the Heaviside function. These terms are responsible of driving the zero level curve towards the object boundaries. To explain the effect of the first term, which is associated to the internal energy  $\mu P(\phi)$ , we notice that the gradient flow

$$\nabla \phi - \text{div} \left( \frac{\nabla \Phi}{|\nabla \phi|} \right) = \text{div} \left[ \left( 1 - \frac{1}{|\nabla \Phi|} \right) \nabla \Phi \right] \quad (8)$$

has the factor  $\left( 1 - \frac{1}{|\nabla \Phi|} \right)$  as diffusion rate. If  $|\nabla \Phi| > 1$ , the diffusion rate is positive and the effect of this term is the usual diffusion. If  $|\nabla \Phi| < 1$ , the term has effect of reverse diffusion and therefore increase gradient.

### C. Bayesian Level Set Model

Active contour method proposed by Li et al. has achieved a great success, evidence shows that this algorithm requires further improvements. For example, it has been observed that in a noisy image with ambiguous boundaries this approach cannot ideally locate the object boundaries. This is mainly due to (1) the energy minimization by Eq. (5) is deviated due to the adjacent structures, and (2) these neighboring structures are difficult to be separated because of the mutual intersection, and hence no steep gradients can be used to propagate the front. One of the possible solutions to optimize the settlement of the contour is to iteratively tune the contour's position during the propagation. Given a minimized energy functional  $E$ , we shall have an appropriate contour location. In other words, we seek a maximized posterior probability  $p(C|\phi)$ , depending on the internal and external energy terms. Assuming the image points on the contour are conditionally independent, we have

$$p(C|\phi) = \prod_i p(C_i|\phi) \quad (9)$$

where  $i$  is the index of one of the image points on the contour, and  $C_i$  is the  $i^{\text{th}}$  point on  $C$ . Using Bayes' rule, we can have the following expression:

$$p(C_i|\phi) = \sum_j p(C_i|\delta_j, \phi) p(\delta_j) \quad (10)$$

where  $j$  is also an image index, and  $\delta_j$  denotes the energy on  $j$ . A maximum log-likelihood estimate is defined as follows:

$$L(\phi) = \log p(C_i|\phi) \quad (11)$$

This can be accomplished by using an EM algorithm. The entire EM algorithm starts from an initial estimate of the contour location. The total energy and the level set function are then deduced on the basis of the determined contour location. This is followed by the expectation of a new contour location. Such an iteration will not be terminated until a pre-determined threshold is satisfied. The likelihood is formulated as follows



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$$p(\delta_j | C_i, \phi_{t-1}) = \frac{p(\delta_j | \phi_{t-1}) p(C_i | \delta_j, \phi_{t-1})}{\sum_k [p(\delta_j | \phi_{t-1}) p(C_i | \delta_j, \phi_{t-1})]} \quad (12)$$

where  $p(\delta_j | \phi_{t-1})$  is the mean of the sum of  $p(C_i | \delta_j, \phi_{t-1})$  among the overall points of the contour. Next step is to find out a contour so that the conditional log-likelihood, the condition obtained is given by

$$Q(C_t | C_{t-1}) \geq Q(C_{t-1} | C_{t-1}) \quad (13)$$

We can evaluate the difference between  $(\delta_t(\phi) - \delta_{t-1}(\phi))$  to get a smooth solution of Eq. (13). The minimization condition obtained is given by

$$\frac{\partial \delta}{\partial t} = \frac{\partial \delta}{\partial \phi} \frac{\partial \phi}{\partial t} = 0 \quad (14)$$

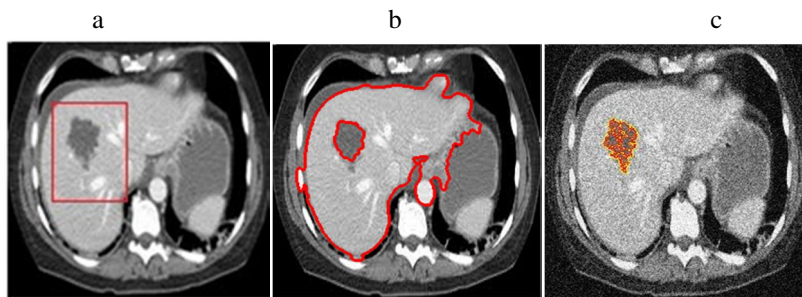
### III. ALGORITHM

- Step1: Initialize a contour;
- Step2: Computation of front evolution dynamics by using Eq. (5);
- Step3: Employment of Eqs. (12)–(14);
- Step4: Residual analysis;
- Step5: If the left side of Eq. (14) is less than 0.05
- Step6: Stop segmentation.
- Step7: Otherwise Continue steps (2–4).

### IV. RESULTS AND DISCUSSION

To demonstrate the performance of the proposed Bayesian level set scheme, compared to the classical level set techniques, we use CT liver images. Sometimes the images accompany background noise, which potentially has a negative impact on correct settlements of the contours. Due to the existence of other organs adjacent to liver with same intensity the contour searching may be strongly affected by the combination of noise and other edges. To achieve ideal image segmentation the method will have to make great efforts to get through the interference of image noise and neighboring structures. In this paper three methods are employed for performance and comparison, i.e. the original level set, the level set without re-initialization by Li et al. and our approach. The whole implementation is run on a PC with a 1.5GHz Intel(R) Pentium(R) CPU.

Fig. 1 demonstrates that the level set approach without re-initialization by Li et al. outperforms the original level set method by Osher and Sethian in tumor detection. Although the approach by Li et al. has achieved a great success, evidence shows that this algorithm requires further improvements. For example, it has been observed that in a noisy image with ambiguous boundaries this approach cannot ideally locate the tumor boundaries (Fig. 2). This is mainly due to (1) the energy minimization by Eq. (5) is deviated due to the same intensity in the adjacent organs, and (2) these neighboring pixels are difficult to be separated because of the mutual intersection, and hence no steep gradients can be used to propagate the front. One of the possible solutions to optimize the settlement of the contour is to iteratively tune the contour's position during the propagation.



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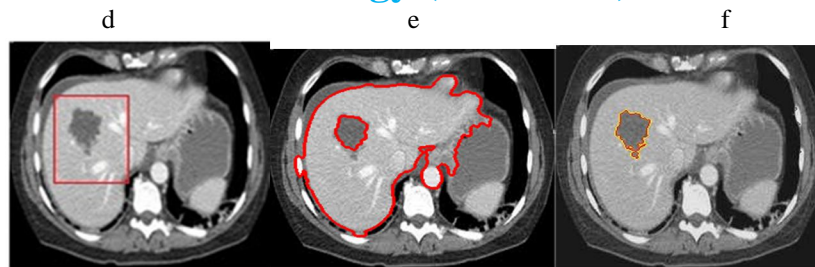


Fig. 1. Performance comparison between the original level set method (first row) and the level set scheme without re-initialization (second row).  $\lambda=2.5, \mu=0.4, v=1$  and time step = 3. (a) Initial, (b) iteration 200, (c) iteration 350, (d) initial, (e) iteration 200, (f) iteration 300.

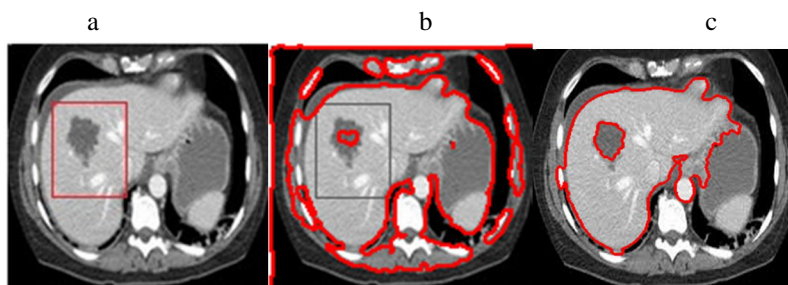


Fig. 2. Performance of the level set scheme without re-initialization.  $\lambda=2.5, \mu=0.4, v=1$ , and time step = 3. (a) Initial, (b) iteration 200, (c) iteration 300.

Fig. 3 illustrates individual performance of using the original level set algorithm and our approach, where the former is shown on the first row while the latter is demonstrated on the second row. The original level set method is distracted by the near edges of the liver tumors. Our method gets around this problem, and finally addresses on the ideal tumor boundary. This indicates that our approach is able to ignore the interference of neighboring organs edges, and fast approach to the tumor boundary.

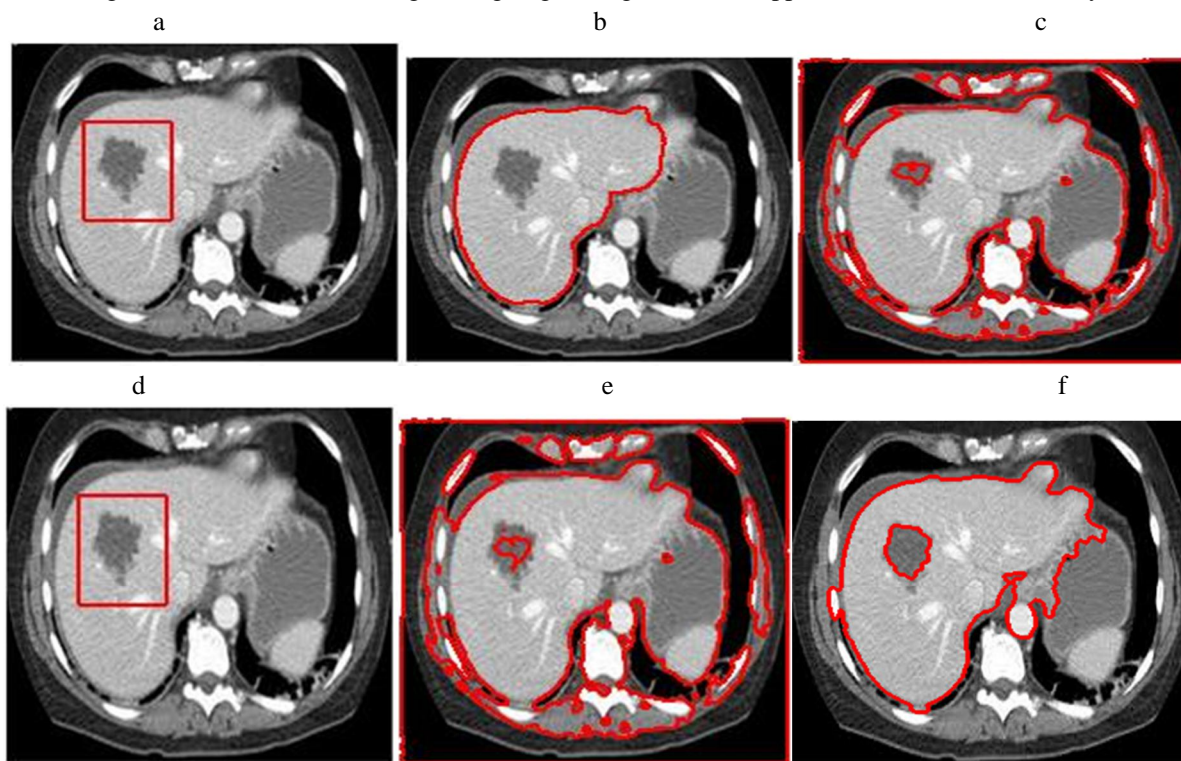


Fig. 3. Performance comparison of the original level set scheme and our approach.  $\lambda=3, \mu=0.3, v=2$ , and time step = 2.5. (a) Initial, (b) iteration 150, (c) iteration 300, (d) initial, (e) iteration 150, (f) iteration 250.



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Segmentation of multiple tumors is shown in Fig. 4. Multiple tumors exist in the image, and different tumors have different intensities. As seen in this figure, the original level set scheme cannot locate the edges of the tumor regions in the liver CT image. The edges have not been correctly outlined. These edges are vague somehow so that the energy minimization in the classical level set function cannot be ideally achieved. The model proposed by Li et al. has a better outcome of tumor detection. The accuracy of the edge detection still needs to be improved.. This may need more efforts to optimize the contour settlement. On the other hand, it is clear that our approach has a better performance than these two methods in terms of edge detection ( images on the third row).

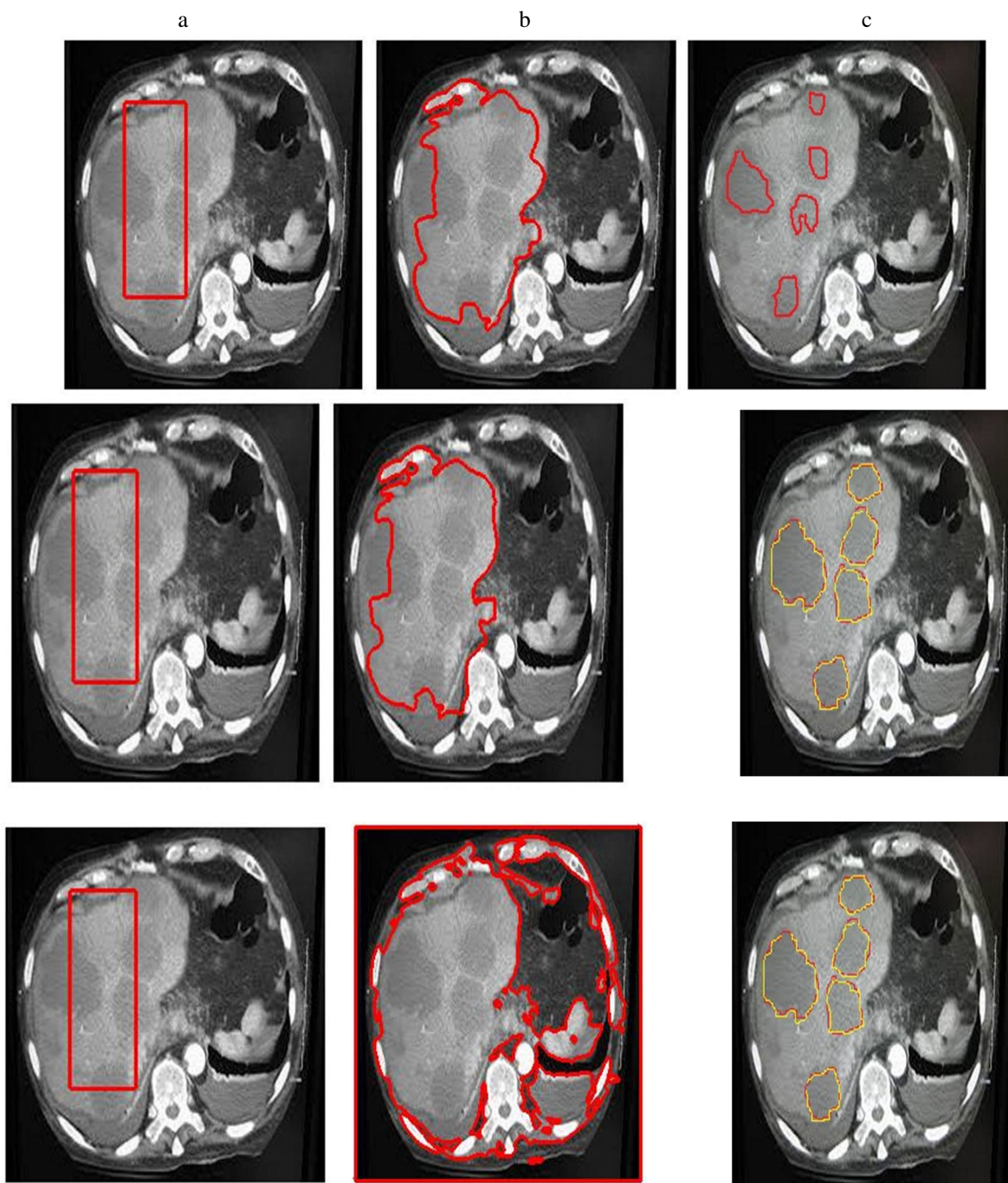


Fig. 4. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and our approach (third row).  $\lambda=2.5, \mu=0.4, v=2$ , and time step=2. 5  
(a) Initial, (b) iteration 150, (c) iteration 300.

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This experiment utilizes a CT liver tumor image where the tumor region is much brighter. Our target is to outline the correct tumor region using the available methods. Fig. 5 demonstrates that the proposed level set scheme has the best performance in tumor detection, where Li's method leads to errors in detecting the exact segmentation of the tumor region. Meanwhile, the original level set method cannot correctly locate the tumor boundary, resulting from the side-effects raised by the image noise.

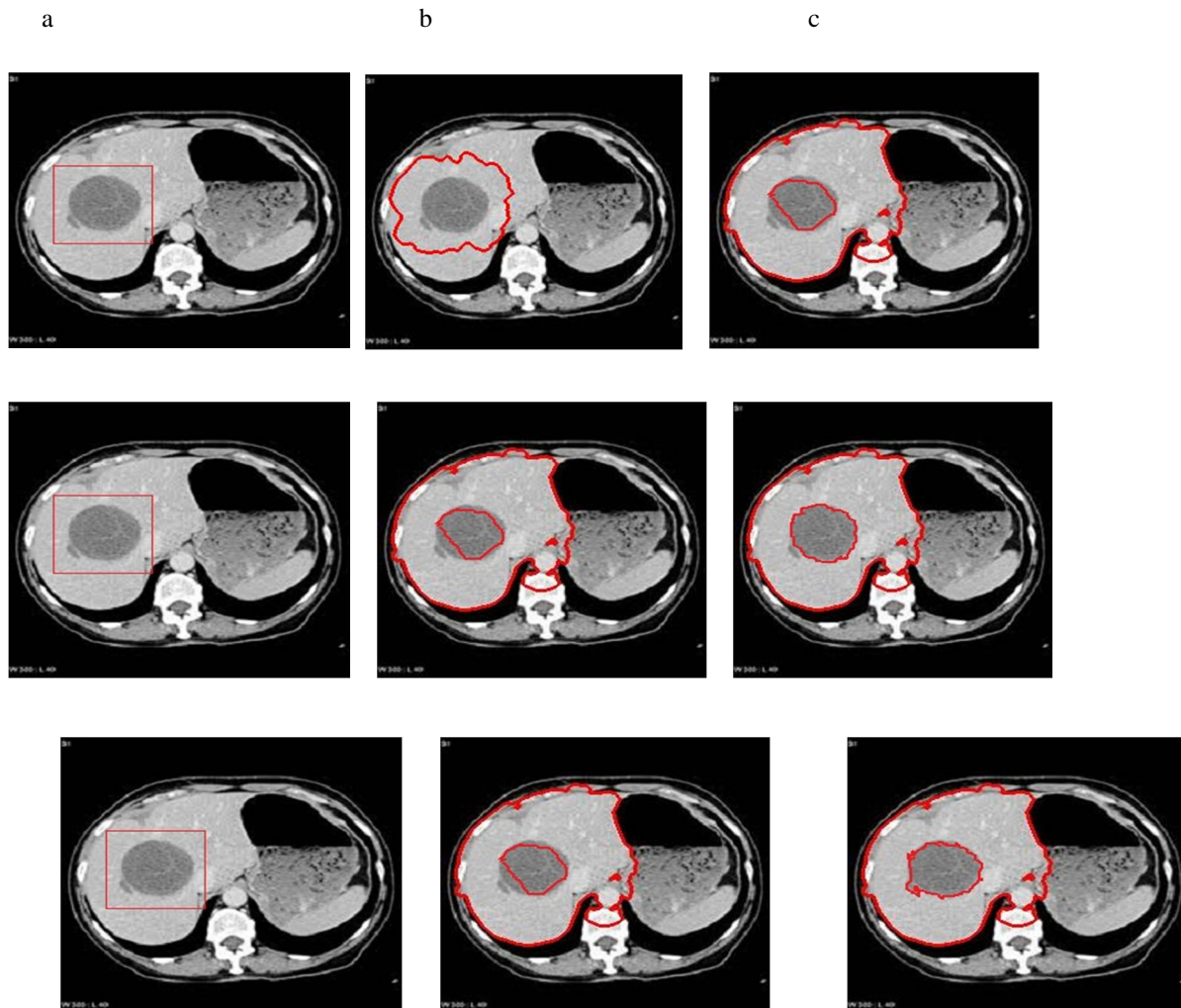
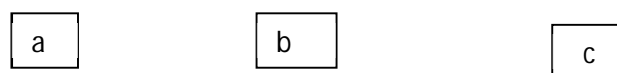


Fig. 5. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and our approach (third row).  $\lambda=2.5$ ,  $\mu=0.4$ ,  $v=2$ , and time step =2. 5. (a) Initial, (b) iteration 250, (c) iteration 350.

Fig.6 demonstrate how these methods cope with the noisy background. The original level set method failed to segment the tumor region. Meanwhile, Li's model and our method have been successfully segment the tumor region.. Interestingly, we observe that at iteration 150 the proposed scheme seems reluctant to pick up a concave area. However, it recovers very soon and successfully addresses on the exact tumor region at iteration 350. This may be due to the oscillation of the energy terms during the evolution (before iteration 350). Taking a closer look at the results from Li's model, we observe that this model has less detecting accuracy on the block corners of the tumor region than the proposed approach





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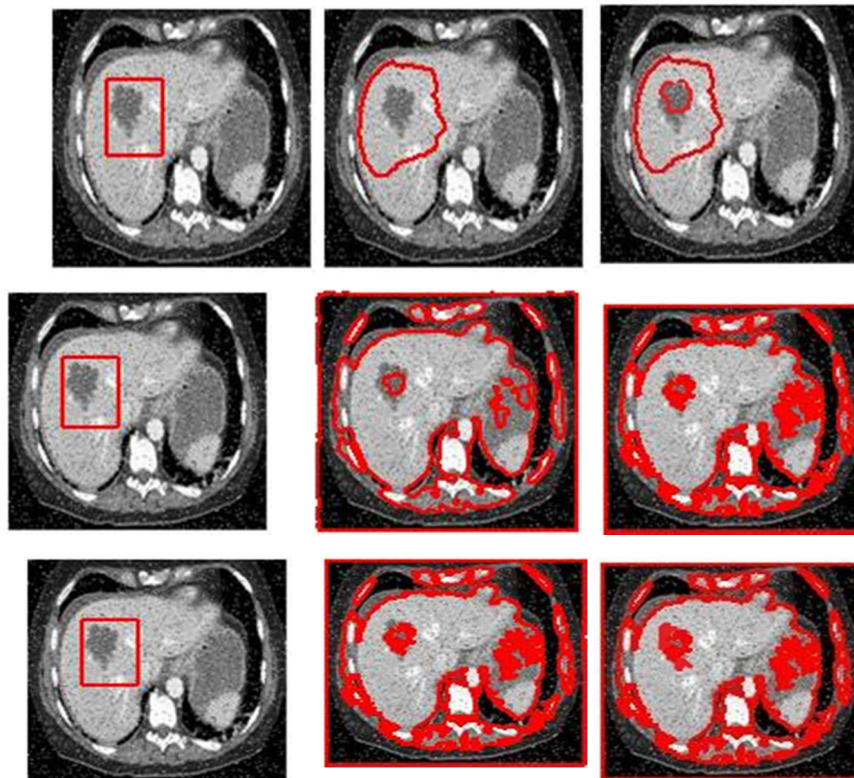


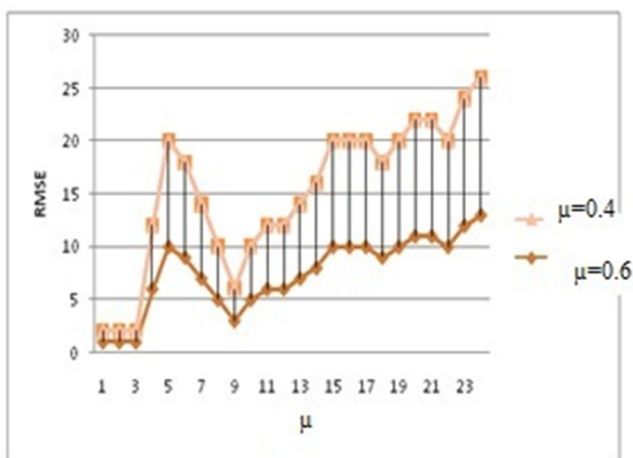
Fig. 6. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and our approach (third row).  $\lambda=2.5$ ,  $\mu=0.4$ ,  $v=2$ , and time step =2. 5. (a) Initial, (b) iteration 350, (c) iteration 400.

The root mean squared error (RMSE) is employed to evaluate the different methods and the equation is defined as

$$RMSE = \sqrt{\frac{\sum_{k=0}^{n-1} \left[ \left( x_k - \bar{x}_k \right)^2 + \left( y_k - \bar{y}_k \right)^2 \right]}{n}} \quad (15)$$

where  $(x_k, y_k)$ ,  $k=0, \dots, n-1$  denotes the coordinates of the points on the final contour, and  $(\bar{x}, \bar{y})$ ,  $k=0, 1, \dots, n-1$  is the corresponding point on the truth curve that has the closest distance from point  $(x_k, y_k)$ . To make RMSE comparable we utilized the CT liver images of figure 6, and these results come from the original level set method, level set scheme without re-initialization and the proposed model, respectively. The RMSE values are listed in Table 1. Fig.7 shows the RMSE curves with various values of  $\mu$ .

Table:1



Model	RMSE
Original level set	23.65
Level set scheme without re-initialization	16.43
proposed model	14.23

Fig.7 RMSE versus alpha for the liver CT image.

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## V. CONCLUSION

A hybrid model of traditional level set method and Bayesian analysis has been proposed in this paper to solve the segmentation problem in noisy and cluttered CT liver images. The energy function used for settling the contour is improved by iterating the energy minimization process. This procedure was performed in order to penalize the deviation of the level set function from the signed distance function. All the system parameters can be dynamically updated during the evolution. Results shows that the proposed algorithm has the best performance in tumor detection accuracy and computational efficiency, compared to two classical level set approaches.

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