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γ - Splitting Graphs

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Abstract--Let $G(V,E)$ be a graph. A dominating set is a subset S of V such that every vertex not in S is adjacent to at least one vertex in S . The cardinality of a minimum dominating set is called the domination number, $\gamma(G)$. A dominating set with γ vertices is called a γ -set. Let η denote the number of γ -sets in G . For a graph G , the splitting graph $S(G)$, is obtained by adding a new vertex v' corresponding to each vertex v of G and joining v' to all vertices which are adjacent to v in G . Here we introduce a new type of graphs called minimum domination splitting graphs or simply γ -splitting graphs. Let G be a graph and let S_1, S_2, \dots, S_η be the γ -sets in G . The γ -splitting graph, $S_\gamma(G)$, of a graph G is the graph obtained from G by adding new vertices w_1, w_2, \dots, w_η and joining w_i to each vertex in S_i where $1 \leq i \leq \eta$. In this paper, we establish some results on γ -splitting graphs.

Keywords: Dominating set, domination number, splitting graph, γ -splitting graph.

AMS Subject Classification Code(2010): 05C(Primary)

I. INTRODUCTION

Throughout this paper, we consider only finite, simple, undirected graphs. For notations and terminology we follow [3]. Let $G(V,E)$ be a graph of order n . We denote the cycle on n vertices by C_n , the path of n vertices by P_n , and the complete graph on n vertices by K_n . The complete bipartite graph is denoted by $K_{m,n}$. In a graph G , degree of a vertex v is denoted by $d(v)$. If S is a subset of V , then $\langle S \rangle$ denotes the vertex induced subgraph of G induced by S . For any vertex $v \in V(G)$, the open neighbourhood $N(v)$ of $V(G)$ is the set of all vertices adjacent to v , that is, $N(v) = \{u \in V(G) / uv \in E(G)\}$, and the closed neighbourhood of v is defined by $N[v] = N(v) \cup \{v\}$. $N^c(v) = V - N(v)$ is called the neighbourhood complement. For any set S , $N(S) = \bigcup_{v \in S} N(v)$.

A full vertex of G is a vertex in G which is adjacent to all other vertices of G . A graph G is said to be r -regular if every vertex in G is of degree r . For any two integers k and d , $k \neq d$, a (k,d) - biregular graph is a graph in which every vertex is of degree either k or d . For any three integers x , a , and b , $x \neq a \neq b$, a (x,a,b) - triregular graph is a graph in which every vertex is of degree either x or a or b . For example, a $(2,3)$ -biregular and a $(1,2,6)$ - triregular graphs are shown in Figure 1.

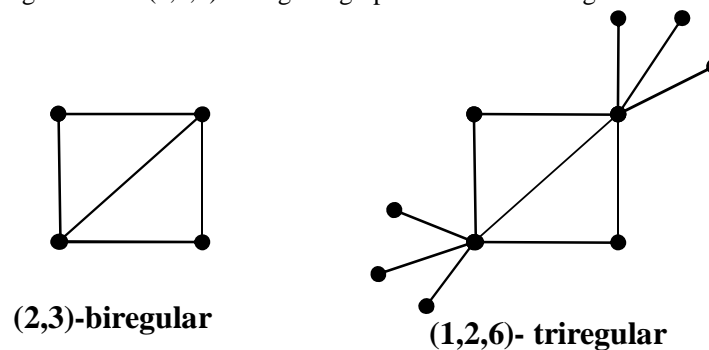


Figure 1

The distance $d(u,v)$ in G between two vertices u and v is the length of a shortest u - v path in G . The eccentricity $e(u)$, of a vertex u is the distance of a farthest vertex from u , and radius $\text{rad}(G)$ of G is the minimum eccentricity. The maximum distance between any two vertices in G is the diameter of G , denoted by $\text{diam}(G)$, that is, $\text{diam}(G) = \max_{u,v \in V(G)} \{d(u,v)\}$. A vertex u with $e(u) = \text{rad}(G)$ is called a central vertex.

A graph G for which $\text{rad}(G) = \text{diam}(G)$ is called a self-centered graph of radius $\text{rad}(G)$. Or equivalently, a graph is self-centered if all of its vertices are central vertices. For further basic definitions on distance in graphs one can refer [4].

Let $H_{n,n}$ denote the graph with vertex set $\{v_1, v_2, \dots, v_n ; u_1, u_2, \dots, u_n\}$ and edge set $\{v_i u_j / 1 \leq i \leq n, n-i+1 \leq j \leq n\}$. The graph $B_{m,n}$ is the bistar obtained from the stars $K_{1,m}$ and $K_{1,n}$ by joining their central vertices by means of an edge. For example, the graph $H_{4,4}$

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and the bistar $B_{4,5}$ are shown in Figure 2.

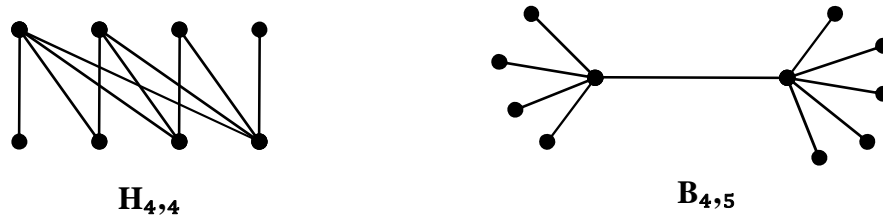


Figure 2

The *join* $G \vee H$ of the graph G and H is the graph obtained from $G \cup H$ by joining every vertex of G to each vertex of H by means of an edge. The graph $W_n = C_{n-1} \vee K_1$ is called the *wheel* graph on n vertices. The *corona* $G \circ H$ of two graphs G and H is obtained by taking one copy of G and $|V(G)|$ copies of H , and by joining each vertex in the i^{th} copy of H to the i^{th} vertex of G , where $1 \leq i \leq |V(G)|$. The corona graph $C_5 \circ K_2$ is depicted in Figure 3, for reference,

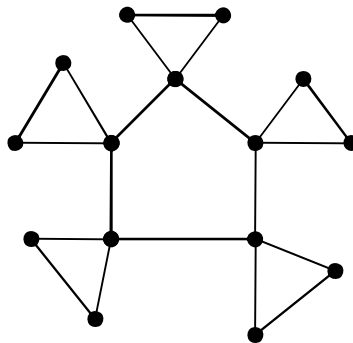


Figure 3

In a graph G , the process of deleting an edge uv and introducing a new vertex w and the edges uw and vw is called the *subdivision of the edge* uv . A *spider* is a tree on $2n + 1$ vertices obtained by subdividing each edge of a star $K_{1,n}$. In other words, spider is nothing but $K_{1,n} \circ K_1$. A *wounded spider* is a graph obtained from subdividing at most $n - 1$ edges of a star $K_{1,n}$. The wounded spider includes K_1 , the star $K_{1,n-1}$. For example, a wounded spider G the graph shown in Figure 4. The *cartesian product* of two graphs G_1 and G_2 is denoted by $G_1 \times G_2$. The graph $K_{1,m} \times P_2$ is called the *m-book* graph and it is denoted by B_m . For example, the book graph B_4 is shown in Figure 5.

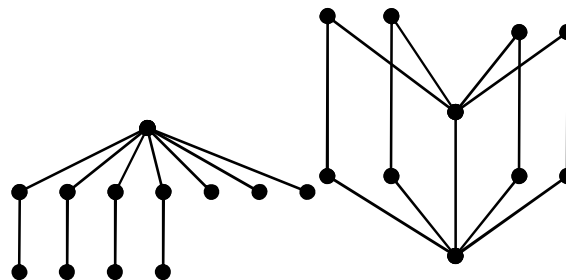
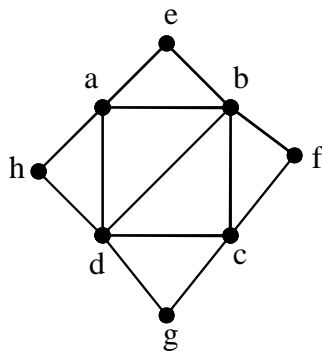


Figure 4

Figure 5

A *dominating set* is a subset S of the vertex set V such that every vertex is either in S or adjacent to a vertex in S , that is, such that every vertex in $V-S$ is adjacent to at least one vertex in S . The *domination number* is the number of vertices in a smallest dominating set of G , it is denoted by $\gamma(G)$. A dominating set with γ elements is called a γ -set. For example, $S_1 = \{b,d\}$ and $S_2 = \{a,c\}$ are the minimum dominating sets of the graph G can be verified in Figure 6. For further results on domination in graphs, one can refer [5].

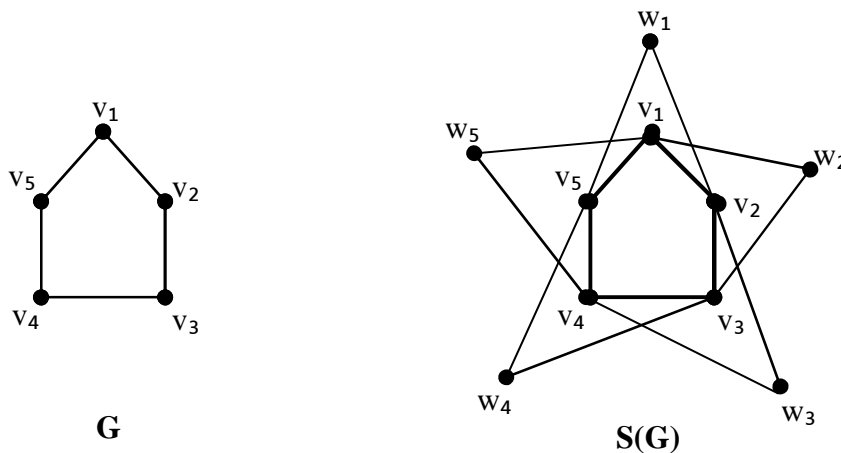
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G

Figure 6

Note that $S_3 = \{a, b, c, d, e, f, g, h\}$ and $S_4 = \{a, b, c, d\}$, etc., are also dominating sets in G . The concept of splitting graph was introduced by Sampath Kumar and Walikar [6]. The *splitting graph* $S(G)$, is the graph obtained from G , by adding a new vertex w for every vertex $v \in V(G)$, and joining w to all vertices of G adjacent to v . For example, a graph G and its splitting graph $S(G)$ are shown in Figure 7.

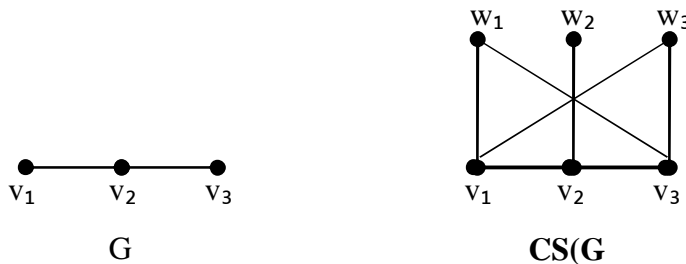


G

S(G)

Figure 7

The concept of cosplitting graphs has been recently introduced by Selvam Avadayappan and M. Bhuvaneshwari [1]. Let G be a graph with vertex set $\{v_1, v_2, \dots, v_n\}$. The *cosplitting graph* $CS(G)$ is the graph obtained from G , by adding a new vertex w_i for each vertex v_i and joining w_i to all vertices which are not adjacent to v_i in G . As an illustration, a graph G and its cosplitting graph $CS(G)$ are shown in Figure 8.



v₁

v₂

v₃

G

w₁

w₂

w₃

v₁

v₂

v₃

CS(G)

Figure 8

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The concept of β -splitting graph has been introduced by Selvam Avadayappan, M. Bhuvaneshwari and B. Vijaya Lakshmi [2]. Let S_1, S_2, \dots, S_ρ be the maximum independent sets of G . The β -splitting graph $S_\beta(G)$ of a graph G is a graph obtained from G by adding new vertices w_1, w_2, \dots, w_ρ such that each w_i is adjacent to each vertex in S_i , for $1 \leq i \leq \rho$. For example, a graph G and its β -splitting graph $S_\beta(G)$ are shown in Figure 9.

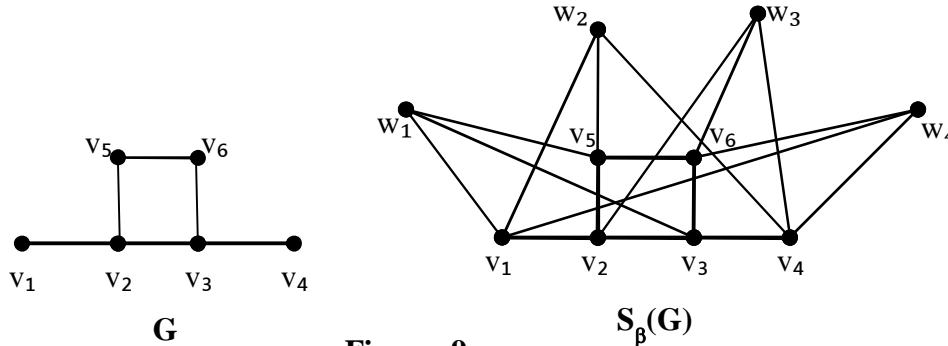


Figure 9

In this paper, we introduce a new type of splitting graphs called γ -splitting graphs. Let G be a graph and let η be the number of γ -sets in G . Let S_1, S_2, \dots, S_η be the minimum dominating sets in G . The γ -splitting graph, $S_\gamma(G)$, of a graph G is the graph obtained from G by adding new vertices w_1, w_2, \dots, w_η and joining w_i to each vertex in S_i where $1 \leq i \leq \eta$. For example, the γ -splitting graph of P_4 is shown in Figure 10.

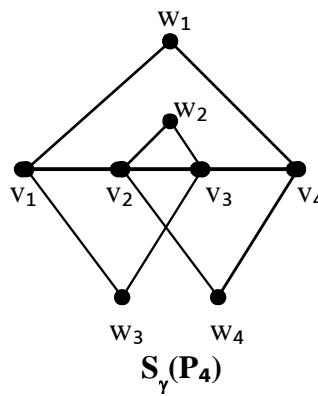


Figure 10

Clearly, $S_1 = \{v_1, v_4\}$, $S_2 = \{v_2, v_3\}$, $S_3 = \{v_1, v_3\}$, $S_4 = \{v_2, v_4\}$ are the γ -sets in P_4 , also w_1, w_2, w_3, w_4 are newly added vertices in $S_\gamma(P_4)$. Here, we discuss a few results on γ -splitting graphs. In this paper, we independently characterise graphs for which $S_\gamma(G)$ is a regular, biregular, tree, unicyclic graph. We attain bounds for the maximum and minimum degree of a vertex in $S_\gamma(G)$. Finally we study the distance properties of γ -splitting graphs.

II. CHARACTERISATION OF γ -SPLITTING GRAPHS

The following facts can be easily verified for γ -splitting graphs. For a vertex v in $S_\gamma(G)$, let $d^*(v)$ denote the degree of v in $S_\gamma(G)$.

Fact 2.1 The newly added vertices $\{w_1, w_2, \dots, w_\eta\}$ are independent in $S_\gamma(G)$, that is, $d(w_i, w_j) \geq 2$, for any $i, j, 1 \leq i, j \leq \eta$.

Fact 2.2 $d^*(w_i) = \gamma(G)$, for $i, 1 \leq i \leq \eta$.

Fact 2.3 For any vertex $v \in V(G)$, $d(v) \leq d^*(v)$.

Fact 2.4 Every graph G is an induced subgraph of $S_\gamma(G)$. Even more G is a proper subgraph of $S_\gamma(G)$, since every graph contains at least one γ -set.

Fact 2.5 The graph having only one full vertex, bistar graph, the graph $H_{n,n}$, the path P_{3k} , $k \geq 1$ and the book graph B_m are some

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graphs whose γ -splitting graphs contain exactly one newly added vertex.

Fact 2.6 $S_\gamma(K_n) \cong K_n \circ K_1$ for any $n \geq 1$.

Fact 2.7 $S_\gamma(K_{1,n}) \cong K_{1,n+1}$ for any $n \geq 2$.

Fact 2.8 $S_\gamma(K_n^c) \cong K_{1,n}$ for any $n \geq 1$.

The following theorems establish some properties of γ -splitting graphs.

$$\text{Proposition 2.9 For any } m \geq 1 \text{ and } n \geq 1, \eta(K_{m,n}) = \begin{cases} 1 & \text{if } m = 1, n \geq 2 \\ 2 & \text{if } m = n = 1 \\ 6 & \text{if } m = n = 2 \\ mn + 1 & \text{if } m = 2, n > 2 \\ mn & \text{if } m \geq 3, n \geq 3. \end{cases}$$

Proof Let $V = \{u_1, u_2, \dots, u_m; v_1, v_2, \dots, v_n\}$ be the vertex set of $K_{m,n}$.

Case (i) Suppose $m = n = 1$, then clearly $\{u_1\}$ and $\{v_1\}$ are only the γ -sets and hence $\eta(K_{m,n}) = 2$.

Case (ii) If $m = n = 2$, then clearly $\{u_1, v_1\}, \{u_2, v_2\}, \{u_1, v_2\}, \{u_2, v_1\}, \{u_1, u_2\}$ and $\{v_1, v_2\}$ are the only γ -sets in $K_{2,2}$ and hence $\eta(K_{m,n}) = 6$.

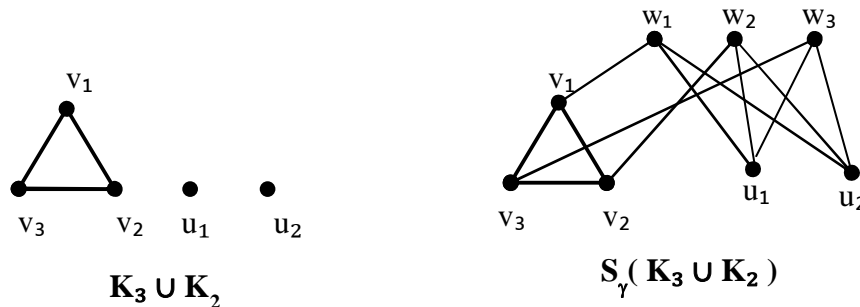
Case (iii) If $m = 1$ and $n \geq 2$, then $G \cong K_{1,n}$, and therefore $\{u_1\}$ is the only γ -set. That is, $\eta(G) = 1$.

Case (iv) Suppose $m = 2$ and $n > 2$. Then $\{u_1, u_2\}$ and $\{u_j, v_k\} \ 1 \leq j \leq 2, 1 \leq k \leq n$ are the γ -sets of G . Thus $\eta(K_{m,n}) = mn + 1$.

Case (v) If $m \geq 3$ and $n \geq 3$, then clearly $\{u_i, v_k\} \ 1 \leq i \leq m, 1 \leq k \leq n$. Thus $\eta(K_{m,n}) = mn$. ■

Theorem 2.10 For any $n \geq 1$, there exists a graph G of order n , such that $S_\gamma(G)$ is n -regular.

Proof When $n = 1, G \cong K_1$, for which $S_\gamma(G) \cong K_2$ is the required graph. Therefore assume that $n \geq 2$, consider the graph $G \cong K_n \cup K_{n-1}^c$ with vertex set $\{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_{n-1}\}$ with edge set $\{v_i v_j \mid 1 \leq i, j \leq n\}$. For any $i, 1 \leq i \leq n$, clearly $\{v_i, u_1, u_2, \dots, u_{n-1}\}$ is a γ -set of G , that is, $\gamma(G) = n$. Hence there are n such γ -sets in G . Let w_1, w_2, \dots, w_n be the newly added vertices in $S_\gamma(G)$. Now for any $i, j, 1 \leq i \leq n, 1 \leq j \leq n-1$. Thus $d^*(v_i) = d^*(w_i) = d^*(u_j) = n$. Hence $S_\gamma(G)$ is n -regular. Thus G is the required graph. For example, the graph $K_3 \cup K_2^c$ and $S_\gamma(K_3 \cup K_2^c)$ which is a 3-regular graph are shown in Figure 11.



Figure

Now, consider the star graph $K_{1,n-1}, n \geq 3$, which is biregular. In addition $S_\gamma(K_{1,n-1})$ is also biregular. This shows that there are biregular graphs G whose $S_\gamma(G)$ are also biregular. Some examples are listed below:

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Graph G	Degree set of G	S _γ (G)	Degree set of S _γ (G)
K _{1,n-1} , n ≥ 3	{1, n-2}	K _{1,n}	{1, Δ(G)+1}
P ₅	{1, 2}	S _γ (P ₅)	{2, Δ(G)+2}
B _m	{2, m+1}	S _γ (B _m)	{2, Δ(G)+1}

Theorem 2.11 The graph S_γ(K_{m,n}) is biregular if m = n and S_γ(K_{m,n}) is triregular if m ≠ n for m ≥ 2.

Proof Let V = {v₁, v₂, ..., v_m; u₁, u₂, ..., u_n} be the vertex set of K_{m,n}.

Case (i) Suppose m = n, and m ≥ 3. The graph S_γ(K_{m,m}), then d*(w_i) = 2. Also, by Proposition 1, η = m². Each u_i or v_i belongs to exactly m γ-sets. Hence d*(u_i) = d*(v_i) = 2m. Then S_γ(K_{m,m}) is a (2m, 2)-biregular graph when m = n.

Case (ii) Let m ≠ n. The graph S_γ(K_{m,n}), then d*(w_i) = 2, and η = mn. Each u_i belongs to n γ-sets and each v_i belongs to m γ-sets. Then d*(u_i) = 2n and d*(v_i) = 2m. Hence S_γ(K_{m,n}) is a (2m, 2n, 2)-triregular graph when m ≠ n. Hence the proof. For example, the graph K_{2,2} and S_γ(K_{2,2}) are shown in Figure 12.

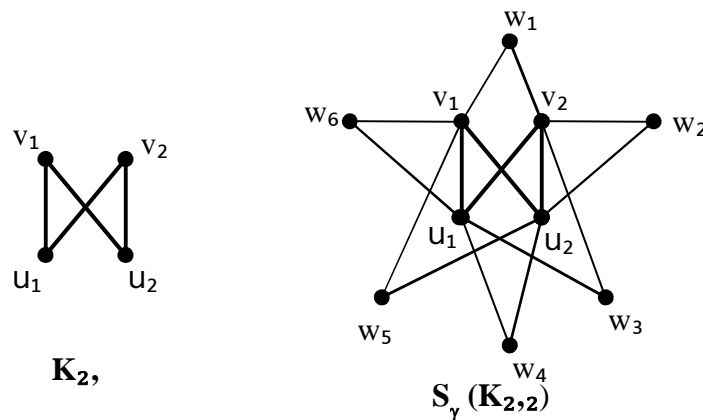


Figure 12

Theorem 2.12 The graph S_γ(G) is a tree if and only if G is one among the following graphs K_n^c, P₂, (∪_{i=1}^k K_{1,n_i}) ∪ K_m^c, k ≥ 1, n_i

≥ 2, m ≥ 1, or ∪_{i=1}^k K_{1,n_i}, k ≥ 1, n_i ≥ 2.

Proof Consider a graph G for which S_γ(G) is a tree. Since G is an induced subgraph of S_γ(G), G is acyclic. If G contains only two vertices, then obviously G ≅ K₂ or K₂^c for which S_γ(G) ≅ P₄ or P₃ respectively. So we assume that G contains at least three vertices.

Case (i) Suppose G is a tree. Then G contains at most one full vertex. If G contains only one full vertex, then G ≅ K_{1,n} for which S_γ(G) ≅ K_{1,n+1}. If G contains no full vertex, then γ(G) > 1 and thus G contains at least two vertices u and v in any γ-set S of G. Let w be the newly added vertex in S_γ(G), corresponding to S. Now the u-v path together with the edges uw and vw forms a cycle in S_γ(G), which is a contradiction to our assumption that S_γ(G) is a tree. Therefore, this case does not arise.

Case (ii) Let G be a forest. If a γ-set contains at least two vertices in the same component, then S_γ(G) contains a cycle, which is a contradiction. Therefore every component must contain exactly one vertex of each γ-set of G, which is possible when each

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component is a star or a trivial graph and hence $G \cong \left(\bigcup_{i=1}^k K_{1,n_i} \right) \cup K_m^c$, $k \geq 1$, $n_i \geq 2$ and $m \geq 1$ or $G \cong \bigcup_{i=1}^k K_{1,n_i}$, $k \geq 1$, $n_i \geq 2$.

And the converse is obvious. ■

For example, the graph $S_7 \left(\bigcup_{i=1}^3 K_{1,3} \right)$ and $S_7 \left(\left(\bigcup_{i=1}^2 K_{1,3} \right) \cup K_3^c \right)$ are shown in Figure 13.

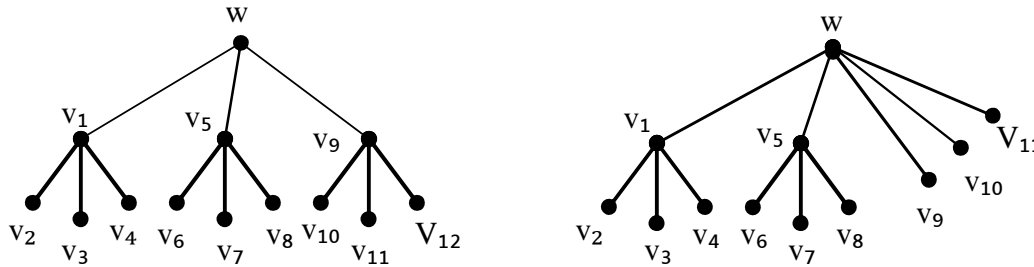


Figure 13

Let $P_k(m,n)$, where $k \geq 2$ and $m,n \geq 1$, be the graph obtained by identifying the centre vertices of the stars $K_{1,m}$ and $K_{1,n}$ at the ends of P_k respectively. The graph $C_3(m_1, m_2, m_3)$, where $m_i \geq 0$, is obtained from the cycle $C_3 = v_1 v_2 v_3 v_1$ by identifying the centre of the star K_{1,m_i} , at v_i of C_3 , for $1 \leq i \leq 3$. For example, the graph $P_5(3, 4)$ and $C_3(3, 0, 0)$ are shown in Figure 14.

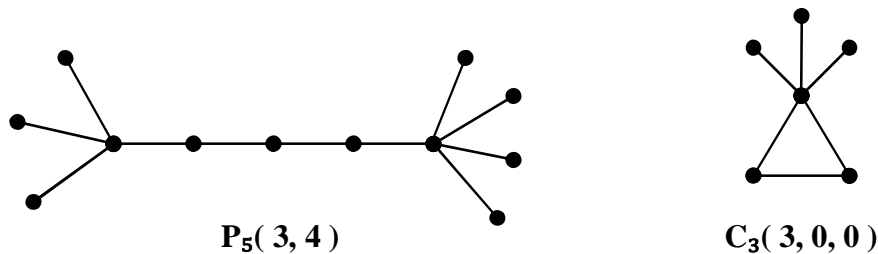


Figure 14

Theorem 2.13 The graph $S_\gamma(G)$ is unicyclic if and only if G is isomorphic to any one of the following graphs: (i) $P_2 \cup K_1$, (ii) K_3 , (iii) $B_{m,n}$, $m > 1$, $n > 1$, (iv) $P_k(m,n)$, $k = 3, 4$ and $m,n \geq 1$, (v) $B_{m,n} \cup K_t^c$, $m > 1$, $n > 1$, $t \geq 1$, (vi) $P_k(m,n) \cup K_t$, $k = 3, 4$ and $m, n \geq 1$, $t \geq 1$, (vii) $C_3(m_1, 0, 0) \bigcup_{p=0}^r pK_{1,n} \bigcup_{q=0}^s qK_n^c$ where $m_1 \geq 1$.

Proof Consider the graph G for which $S_\gamma(G)$ is unicyclic. Then there arise two cases.

Case (i) Suppose G is acyclic. Then clearly the cycle contains a newly added vertex w in $S_\gamma(G)$. Therefore, $\gamma(G) \neq 1$. Let G be a connected graph. Then $\eta = 1$, that is, G contains exactly one γ -set, since every newly added vertex forms a new cycle. In particular, $\gamma(G) = 2$ with the γ -set $\{u,v\}$. Let w be the newly added vertex in $S_\gamma(G)$. Then the (u,v) -path in G together with the newly added edges wu and wv forms the unique cycle in $S_\gamma(G)$, this is possible only when $G \cong B_{m,n}$, $m > 1$, $n > 1$, $P_k(m,n)$, $k = 3, 4$ and $m, n \geq 1$.

Let G be disconnected. If G has more than one component, with at least one edge, then $S_\gamma(G)$ has more cycles, which is a contradiction to our assumption that $S_\gamma(G)$ is unicyclic. Hence only one component G_1 of G can contain edges and the others are isolated vertices. If G_1 contains only one edge, then G must be $P_2 \cup K_1$. If G_1 contains more than one edge, then G_1 is isomorphic to $B_{m,n}$, $m > 1$, $n > 1$, $P_k(m,n)$, $k = 3, 4$ and $m,n \geq 1$ and hence $G \cong B_{m,n} \cup K_t^c$, $m > 1$, $n > 1$, $t \geq 1$, $P_k(m,n) \cup K_t$, $k = 3, 4$ and $m, n \geq 1$, $t \geq 1$.

Case (ii) Suppose G is unicyclic. Let G be a connected graph. Then newly added edges cannot be in a cycle. This is possible only

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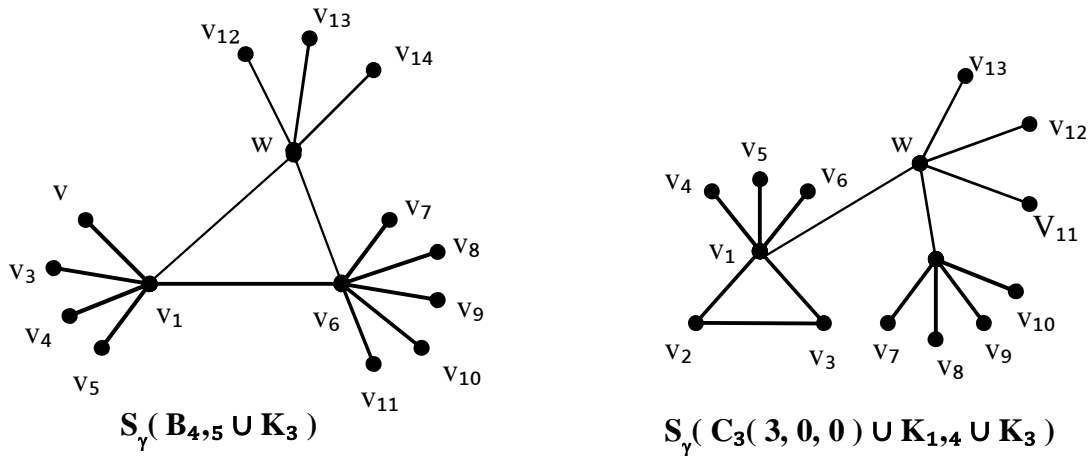
when $\gamma(G) = 1$. This forces that $G \cong K_3$ or $C_3(m_1, 0, 0)$ where $m_1 \geq 1$.

Let G be disconnected graph. Then $\omega(G) \geq 2$. Clearly, one of the component of G is unicyclic and the remaining are trees. Since every component is connected, by the above argument exactly one vertex of each component belongs to γ -set of G . Also, the γ -set

must be unique to avoid cycles formed by newly added vertices. Such a graph is isomorphic to $C_3(m_1, 0, 0) \bigcup_{p=0}^r pK_{1,n} \bigcup_{q=0}^s qK_n^c$

where $m_1 \geq 1$. And the converse is obvious. ■

For example, the graphs $S_\gamma(B_{4,5} \cup K_3^c)$ and $S_\gamma(C_3(3, 0, 0) \cup K_{1,4} \cup K_3^c)$ are shown in Figure 15.



Figure

Theorem 2.14 Let G be a graph. Then $S_\gamma(G)$ has a full vertex if and only if $G \cong K_n^c$ or $H \vee K_1$ where H is a graph without a full vertex.

Proof Let w_i be the newly added vertices in $S_\gamma(G)$ for $1 \leq i \leq \eta$. Let v be a full vertex in $S_\gamma(G)$.

Case (i) Suppose v is a newly added vertex. Since w_i 's are all independent in $S_\gamma(G)$, v is the only newly added vertex. And hence $V(G)$ is the only dominating set of G . This is possible only when $G \cong K_n^c$.

Case (ii) Let $v \in V(G)$. Then v is a full vertex of G . If G has a full vertex u other than v , then there are w_1 and w_2 corresponding to the γ -sets $\{u\}$ and $\{v\}$. But w_1 and w_2 are not adjacent. In addition uw_2 and vw_1 are not the edges in $S_\gamma(G)$. Thus $S_\gamma(G)$ contains no full vertices, a contradiction. Therefore, G has exactly one full vertex. In other words, $G \cong H \vee K_1$ where H has no full vertex.

Conversely, assume that $G \cong H \vee K_1$. The graph $S_\gamma(G)$ is nothing but a graph obtained from $H \vee K_1$ by adding a new vertex and join it to the vertex of K_1 . Also $S_\gamma(K_n^c) \cong K_{1,n}$. In both the cases, $S_\gamma(G)$ has a full vertex. Hence the proof.

Proposition 2.15 For any connected graph G , $\Delta(G) \leq \Delta(S_\gamma(G)) \leq \max\{\Delta(G) + \eta, \gamma\}$.

Proof Let v be a vertex of maximum degree in $S_\gamma(G)$. If v is a newly added vertex, then $\Delta(S_\gamma(G)) = \gamma$. Otherwise, if $v \in V(G)$, then there arise two cases. When $v \notin \bigcup S_i, 1 \leq i \leq \eta$, then $\Delta(S_\gamma(G)) = \Delta(G)$. When $v \in \bigcap S_i, 1 \leq i \leq \eta$, $\Delta(S_\gamma(G)) = \Delta(G) + \eta$. Hence the maximum degree of the graph $S_\gamma(G)$ varies as, $\Delta(G) \leq \Delta(S_\gamma(G)) \leq \max\{\Delta(G) + \eta, \gamma\}$. Hence the proof. ■

For any $n \geq 6$, there exists a graph of order n with $\Delta(S_\gamma(G)) = \gamma(G)$, $P_{3k}, k \geq 2$ is one such a graph. Also the spider graph proves the existence of graphs with $\Delta(S_\gamma(G)) = \Delta(G)$. The wounded spider graph stands as an example of graphs with $\Delta(S_\gamma(G)) = \Delta(G) + \eta$. For example the graphs G_1, G_2, G_3 with $\Delta(S_\gamma(G_1)) = \Delta(G_1)$, $\Delta(S_\gamma(G_2)) = \Delta(G_2) + \eta$ and $\Delta(S_\gamma(G_3)) = \gamma(G)$ respectively are shown in Figure 16. Here G_1 is the spider graph on 9 vertices, G_2 is the wounded spider graph on 5 vertices and G_3 is the path graph on 12 vertices.

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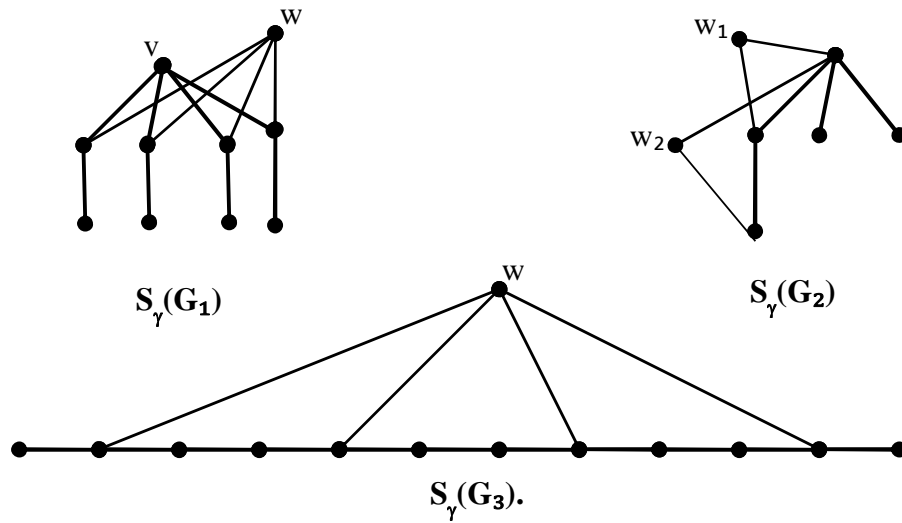


Figure 16

III. DISTANCE PROPERTIES OF γ -SPLITTING GRAPHS

Here we are interested in studying about the distance properties in S_γ -graphs. Also normally we expect $\text{diam}(S_\gamma(G)) < \text{diam}(G)$. But there are graphs with $\text{diam}(S_\gamma(G)) \geq \text{diam}(G)$. This behaviour gives rise to following three definitions S_γ^+ -graphs, S_γ^- -graphs, and S_γ^* -graphs as given below:

A graph G is called a S_γ^+ -graph if $\text{diam}(G) < \text{diam}(S_\gamma(G))$.

It is called a S_γ^- -graph if $\text{diam}(G) > \text{diam}(S_\gamma(G))$.

Finally, it is said to be a S_γ^* -graph if $\text{diam}(G) = \text{diam}(S_\gamma(G))$. For example, S_γ^+ , S_γ^- , and S_γ^* -graphs are shown in Figure 17.

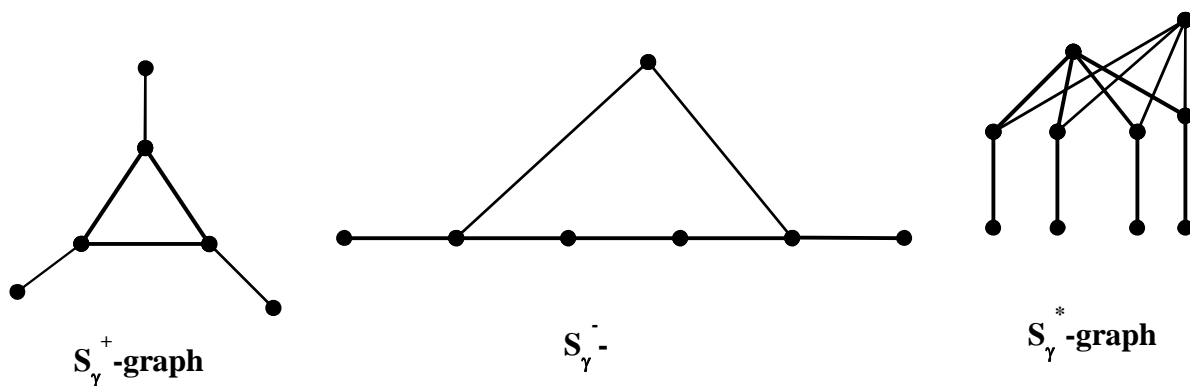


Figure 17

Some standard graphs with their diameters and corresponding families are listed below:

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Graph G	diam(G)	diam($S_\gamma(G)$)	Type
K_n	1	3	S_γ^+ -graph
$K_{m,n}$	2	3	S_γ^+ -graph
W_3	1	2	S_γ^+ -graph
Graph with exactly one full vertex	2	2	S_γ^* -graph
$H_{n,n}$	3	3	S_γ^* -graph
B_m	3	3	S_γ^* -graph
$B_{m,n}$	3	3	S_γ^* -graph
Spider	4	4	S_γ^* -graph

Theorem 3.1 For any graph G, the distance between newly added vertices in $S_\gamma(G)$ is 2 or 3.

Proof Let G be any graph of order n, and w_1 and w_2 be any two newly added vertices in $S_\gamma(G)$. We know that $d^*(w_i) = \gamma(G)$, $1 \leq i \leq \eta$ and $d(w_1, w_2) \geq 2$ (Fact 2.1).

Case (i) Suppose $N(w_1) \cap N(w_2) \neq \emptyset$. Let $x \in N(w_1) \cap N(w_2)$. Then x is the common neighbour of w_1 and w_2 , and so $d(w_1, w_2) = 2$.

Case (ii) Suppose $N(w_1) \cap N(w_2) = \emptyset$. Then let $x \in N(w_1)$. Since $N(w_1)$ is a γ -set, every vertex in $N^c(w_1)$ is adjacent to at least one vertex in $N(w_1)$. But $N(w_2) \subseteq N^c(w_1)$. Therefore, there exists a vertex $y \in N(w_2)$ such that y is adjacent to a vertex x in $N(w_1)$. Then $d(w_1, w_2) = 3$. ■

Theorem 3.2 For any graph G, $\text{diam}(S_\gamma(G)) \leq 4$.

Proof Let G be any graph and $S_\gamma(G)$ be its corresponding γ -splitting graph. Let u and v be any two vertices in $S_\gamma(G)$. We claim that $d(u, v) \leq 4$ for every $u, v \in V(G)$.

Case (i) If u and v are newly added vertices in $S_\gamma(G)$. By Theorem 3.1, $d(u, v) \leq 3$.

Case (ii) If u is a newly added vertex and $v \in V(G)$. Then $N(u)$ is a dominating set, and therefore $v \in N(u)$ or v is adjacent to a vertex in $N(u)$ in $S_\gamma(G)$. This forces that $d(u, v) \leq 2$.

Case (iii) Suppose $u, v \in V(G)$. Then there arise two subcases.

Subcase (i) Let u belong to a γ -set S. Then there is a newly added vertex w corresponding to S. If $v \in S$, then uwv is a u-v path of length 2 in $S_\gamma(G)$. Therefore $d(u, v) \leq 2$. If $v \notin S$, then there is a vertex v_1 in S, adjacent to v. Therefore uwv_1v is a u-v path of length 3, and so $d(u, v) \leq 3$. If v belongs to any other γ -set, then in a similar way we can show that $d(u, v) \leq 3$.

Subcase (ii) Neither u nor v belongs to any γ -set. Fix a newly added vertex w. Clearly, $N(w)$ is a γ -set. So $V(G) \subseteq N(N(w))$ in $S_\gamma(G)$. Therefore, $d(u, v) \leq 4$. Hence $\text{diam}(S_\gamma(G)) \leq 4$. ■

The inequality stated above is strict. For example, $\text{diam}(S_\gamma(P_{3k})) = 4$, for any $k \geq 2$. For example, $\text{diam}(S_\gamma(P_6)) = 4$ can be verified in Figure 18.

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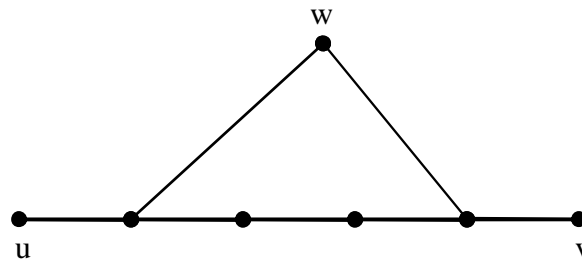


Figure 18

The following corollary gives a characterisation of S_γ^- -graphs.

Corollary 3.3 Any connected graph G with $\text{diam}(G) > 4$, is a S_γ^- -graph.

Proof Suppose G is a connected graph and $\text{diam}(G) > 4$. Let $S_\gamma(G)$ be its corresponding $\text{diam}(S_\gamma(G)) \leq 4$ and the result follows. ■

γ -splitting graph. By Theorem 3.2,

It has been prove in [7], that $\eta(P_n) = \begin{cases} 1 & \text{if } n = 3k, k \geq 1 \\ \frac{k^2 + 5k + 2}{2} & \text{if } n = 3k+1, k \geq 0 \\ k+2 & \text{if } n = 3k+2, k \geq 0 \end{cases}$ and

$$\eta(C_n) = \begin{cases} 3 & \text{if } n = 3k, k \geq 1 \\ \frac{(3k+1)(k+2)}{2} & \text{if } n = 3k+1, k \geq 1 \\ 3k+2 & \text{if } n = 3k+2, k \geq 1 \end{cases}$$

Proposition 3.4 The path graph P_n is S_γ^+ -graph if $n \leq 2$, S_γ^* -graph if $n = 3, 4$, and S_γ^- -graph if $n \geq 5$.

Proposition 3.5 The cycle graph C_n is S_γ^+ -graph if $n \leq 5$, S_γ^* -graph if $n = 6, 7$, and S_γ^- -graph if $n \geq 8$.

REFERENCES

- [1] Selvam Avadayappan and M. Bhuvaneshwari, Cosplitting and coregular graphs, International Journal of Mathematics and Soft Computing Vol.5, (2015), 57-64.
- [2] Selvam Avadayappan, M. Bhuvaneshwari and B. Vijaya Lakshmi, β -splitting graphs, Preprint.
- [3] R. Balakrishnan and K. Ranganathan, A Text Book of graph Theory, Springer-Verlag, New York, Inc(1999).
- [4] F. Bunkley and F.Harary, Distance in Graph, Addison-Wesley Reading, 1990.
- [5] T.W. Haynes, S.T. Hedetneimi and P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker Inc., New York, (1998).
- [6] Sampath Kumar. E, Walikar. H.B, On the Splitting graph of a graph, (1980), J. Karnatak Uni. Sci 25 : 13.
- [7] H. B. Walikar, H. S. Ramane, B. D. Acharya, H.S. Shekhareppa, S. Arumugum, Partially balanced incomplete block design arising from minimum dominating sets of path and cycles, AKCE J. Graph. combin., 4(2) (2007), 223-232.



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