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# Perfect Degree Support Product Graphs

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**Abstract**--For a graph  $G(V,E)$ , the support  $s(v)$  of a vertex  $v$  is defined as the sum of degrees of its neighbours. A graph  $G$  is said to be *balanced* (*highly unbalanced*), if the support of all the vertices are same (distinct). Let  $k$  be any positive integer. A graph  $G$  is said to be a  $k$  – *perfect degree support graph* ( $k$  – *pds graph*) if for any vertex  $v$  in  $G$ , the ratio of its support and its degree is the constant  $k$ . A graph  $G$  is called a  $k$  – *linear degree support graph* ( $k$  – *lds graph*) if, for any two vertices in  $V$  with distinct degrees, the ratio of difference between their supports and the difference between their degrees is the constant  $k$ . The properties of  $k$  – *lds graphs* and  $k$  – *pds graphs* in various product graphs have been studied in this paper.

**Keywords**--Support, balanced graphs, highly unbalanced graphs,  $k$  – perfect degree support graphs,  $k$  – linear degree support graphs.

**AMS Subject Classification code (2000):** 05C (Primary)

## I. INTRODUCTION

Throughout this paper, we consider only finite, simple, undirected graphs. For notations and terminology we follow [3]. A graph  $G$  is said to be  $r$  – *regular*, if every vertex of  $G$  has degree  $r$ . Let  $G_1$  and  $G_2$  be any two graphs. The graph  $G_1 \circ G_2$  obtained from one copy of  $G_1$  and  $|V(G_1)|$  copies of  $G_2$  by joining each vertex in the  $i^{\text{th}}$  copy of  $G_2$  to the  $i^{\text{th}}$  vertex of  $G_1$  is called the *corona* of  $G_1$  and  $G_2$ . The *cartesian product* of  $G$  and  $H$  is denoted by  $G \times H$  and their join is denoted by  $G \vee H$ . The *composition graph* of  $G_1$  to  $G_2$  is denoted by  $G_1[G_2]$ . The concepts of support, balanced graphs, highly unbalanced graphs have been introduced and studied by Selvam Avadayappan and G. Mahadevan [1]. The support  $s(v)$  of a vertex  $v$  is the sum of degrees of its neighbours. That is,  $s(v) = \sum_{u \in N(v)} d(u)$ . Note that the support of any vertex in an  $r$  – regular graph is  $r^2$ .

A graph  $G$  is said to be a *balanced graph*, if the support of every vertex in  $G$  is equal. It is easy to observe that the complete bipartite graphs  $K_{m,n}$  and the regular graphs are balanced graphs. A graph  $G$  is said to be *highly unbalanced*, if distinct vertices of  $G$  have distinct supports. For example, a highly unbalanced graph is shown in Figure 1.

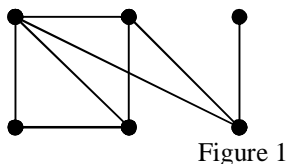


Figure 1

The following results have been proved in [1]:

Result 1  $\sum_{v \in V} s(v) = \sum_{v \in V} d(v)^2$ .

Result 2  $s(v) = (n - 1)^2$  for every  $v \in G$  if and only if  $G \cong K_n$ .

Result 3 For any balanced graph  $G$ ,  $\delta(G) = 1$  if and only if  $G \cong K_2$  or  $K_{1,n}$ .

Result 4 For any  $n \geq 6$ , there is a highly unbalanced graph of order  $n$ .

Consequently the concepts of  $k$  – perfect degree support graph and  $k$  – linear degree support graph have been defined in [2].

A graph  $G$  is said to be a  $k$  – *perfect degree support graph* (or simply a  $k$  – *pds graph*), if for any vertex  $v$  in  $G$ ,  $\frac{s(v)}{d(v)} = k$ . For example, the graph shown in Figure 2 is a 3 – pds graph. In general,  $C_n \circ K_2$  is a 3 – pds graph for any  $n > 2$ .

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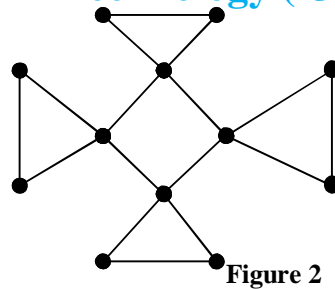


Figure 2

A graph  $G$  is said to be a  $k$  – linear degree support graph (or simply a  $k$  – lds graph), if for any two vertices  $u$  and  $v$  in  $G$  with  $d(u) \neq d(v)$  in  $G$ ,  $\frac{s(u) - s(v)}{d(u) - d(v)} = k$ , for some integer  $k$ . Or equivalently, a graph in which for any vertex  $v$ ,  $s(v) = k d(v) + c$  for some constant  $c$  is called a  $k$  – lds graph. Note that in a  $k$  – lds graph,  $s(u) - k d(u) = s(v) - k d(v)$ . For example, the graph shown in Figure 3 is a 3 – lds graph.

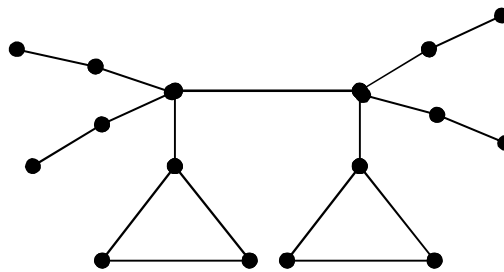


Figure 3

In general any  $k$  – pds graph is a  $k$  – lds graph but not the converse. For example, the graph shown in Figure 3 is a 3 – lds graph but it is not a 3 – pds graph. In fact  $k$  – pds graphs are  $k$  – lds graphs with  $c = 0$ . All balanced graphs are 0 – lds graphs and any  $r$  – regular graphs are  $r$  – pds graphs and hence  $r$  – lds graphs.

Note that in a  $k$  – lds graph, two vertices of same degree have the same support. That is,  $d(u) = d(v)$  implies that  $s(u) = s(v)$ . A few families of  $k$  – pds graphs and  $k$  – lds graphs with some constraints have been constructed in [2]. Also  $k$  – pds trees have been characterized in [2].

In this paper, many new  $k$  – lds and  $k$  – pds graphs have been generated using various graph products.

$k$  – pds and  $k$  – lds product graphs

Regarding the properties of  $k$  – lds and  $k$  – pds graphs in product graphs, the following facts can be easily verified:

Fact 2.1 A disconnected graph is a  $k$  – lds graph if and only if each of its components is a  $k$  – lds graph.

Fact 2.2 Let  $G_1$  be a  $k_1$  – lds graph and  $G_2$  be a  $k_2$  – lds graph. Then  $G_1 \cup G_2$  is a  $k$  – lds graph if and only if  $k_1 = k_2$ .

By a  $(p, q)$  – graph, we mean a graph with  $p$  vertices and  $q$  edges.

Fact 2.3 Let  $G_1$  be a  $(n_1, m_1)$  – graph and  $G_2$  be a  $(n_2, m_2)$  – graph. Then we can easily verify the following:

(i) For any  $v \in V(G_1 \cup G_2)$ , we have

$$d_{G_1 \cup G_2}(v) = d_{G_1}(v) \text{ and } s_{G_1 \cup G_2}(v) = s_{G_1}(v), \text{ if } v \in V(G_1) \text{ and}$$

$$d_{G_1 \cup G_2}(v) = d_{G_2}(v) \text{ and } s_{G_1 \cup G_2}(v) = s_{G_2}(v), \text{ if } v \in V(G_2)$$

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(ii) For any  $v \in V(G_1 \vee G_2)$ , we have

$$d_{G_1 \vee G_2}(v) = d_{G_1}(v) + n_2; S_{G_1 \vee G_2}(v) = S_{G_1}(v) + n_2 d_{G_1}(v) + 2m_2, \text{ if } v \in V(G_1); \quad d_{G_1 \vee G_2}(v) = d_{G_2}(v) + n_1; S_{G_1 \vee G_2}(v) = S_{G_2}(v) + n_1 d_{G_2}(v) + 2m_1, \text{ if } v \in V(G_2)$$

(iii) For any  $(u, v) \in V(G_1 \times G_2)$ , we have

$$d_{G_1 \times G_2}((u, v)) = d_{G_1}(u) + d_{G_2}(v) \text{ and}$$

$$S_{G_1 \times G_2}((u, v)) = 2d_{G_1}(u) d_{G_2}(v) + S_{G_2}(v) + S_{G_1}(u),$$

(iv) For any  $(u, v) \in V(G_1[G_2])$ , we have

$$d_{G_1[G_2]}((u, v)) = n_2 d_{G_1}(u) + d_{G_2}(v).$$

(v) Let  $V(G_1) = \{v_1, v_2, \dots, v_{n_1}\}$  and  $V(G_2) = \{u_1, u_2, \dots, u_{n_2}\}$ . In  $V(G_1 \circ G_2)$ , let  $u_{ij}$  denote the vertex corresponding to  $u_j$  in the  $i^{\text{th}}$  copy of  $G_2$ . Then we have,

$$d_{G_1 \circ G_2}(v_i) = d_{G_1}(v_i) + n_2 \text{ and } S_{G_1 \circ G_2}(v_i) = S_{G_1}(v_i) + n_2 d_{G_1}(v_i) + 2m_2 + n_2,$$

$$d_{G_1 \circ G_2}(u_{ij}) = d_{G_2}(u_{ij}) + 1 \text{ and } S_{G_1 \circ G_2}(u_{ij}) = S_{G_2}(u_{ij}) + d_{G_2}(u_{ij}) + d_{G_1}(v_i) + n_2.$$

Now let us prove some results on  $k$  – lds and  $k$  – pds product graphs.

**Theorem 2.4** If  $G$  is a  $k$  – lds graph of order  $n$ , then  $G \vee G$  is a  $(k + n)$  – lds graph.

**Proof** Let  $G$  be any  $k$  – dsl graph of order  $n$ . Then for any two vertices  $v_i$  and  $v_j$  of distinct degrees,  $\frac{s_G(v_i) - s_G(v_j)}{d_G(v_i) - d_G(v_j)} = k$ . Now consider  $G \vee G$ . Then the degree of every vertex gets increased by  $n$  and the support of any vertex gets increased by its degree times  $n$ . For any two vertices of distinct degrees,  $\frac{s_{G \vee G}(v_i) - s_{G \vee G}(v_j)}{d_{G \vee G}(v_i) - d_{G \vee G}(v_j)} = \frac{s_G(v_i) + n d_G(v_i) - s_G(v_j) - n d_G(v_j)}{d_G(v_i) - d_G(v_j)} = k + n$ . Hence,  $G \vee G$  is a  $(k + n)$  – lds graph. For example, a 1 – lds graph  $P_4$  and the corresponding 5 – lds graph  $P_4 \vee P_4$  are shown in Figure 3.

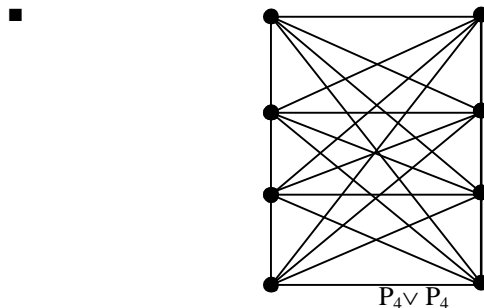


Figure 3

**Theorem 2.5** Let  $G$  be any  $(n, m)$  – graph. If  $G \vee K_1$  is a  $k$  – pds graph, then  $\frac{2m}{n} = k - 1$ .

**Proof** Let  $G$  be a graph with  $n$  vertices and  $m$  edges such that  $G \vee K_1$  is a  $k$  – pds graph. Let  $v$  be the vertex of  $K_1$ . Then clearly  $v$  is a full vertex with support  $S_{G \vee K_1}(v) = \frac{\sum_{u \in V(G)} d_{G \vee K_1}(u)}{n} = \frac{\sum_{u \in V(G)} d_G(u) + n}{n} = \frac{2m + n}{n} = k$ . Hence the proof.

**Theorem 2.6** There does not exist a  $k$  – pds graph  $G$  such that  $G \vee K_1$  is also  $k$  – pds.

**Proof** Suppose  $G$  is a  $k$  – pds graph such that  $G \vee K_1$  is a  $k$  – pds graph. Then for any vertex  $v$  in  $G$ ,  $S_G(v) / d_G(v) = S_{G \vee K_1}(v) /$

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$d_{G \vee K_1}(v) = k$ . Now by using Fact 5.8 (ii),  $\frac{s_G(v) + d_G(v) + n}{d_G(v) + 1} = k$ . That is,  $\frac{kd_G(v) + d_G(v) + n}{d_G(v) + 1} = k$ , which implies  $d_G(v) = k - n$ . Since  $G \vee K_1$  is connected, we have  $k > n$ . In other words,  $s_G(v) > n d_G(v)$  which is impossible. Therefore such a graph  $G$  does not exist ■

**Theorem 2.7** For an  $r$  – regular graph  $G$  of order  $n_1$  and a  $k$  – regular graph  $H$  of order  $n_2$ ,  $G \circ H$  is a  $(r + k)$  – pds graph.

**Proof** Let  $G$  and  $H$  be  $r$  – regular and  $k$  – regular graphs respectively. Then by Fact 2.3 (v),  $G \circ H$  is a biregular graph. In particular,  $s_{G \circ H}(v) = r + n_2$ ;  $s_{G \circ H}(v) = r^2 + n_2(r + k + 1)$ , for any  $v \in V(G)$  and  $d_{G \circ H}(u) = k + 1$ ;  $s_{G \circ H}(u) = k^2 + k + r + n_2$ , for all  $u \in V(H)$ . Therefore we get  $\frac{s_{G \circ H}(v) - s_{G \circ H}(u)}{d_{G \circ H}(v) - d_{G \circ H}(u)} = r + k$ . Hence  $G \circ H$  is a  $(r + k)$  – pds graph. For example,  $K_{3,3} \circ K_2$  which is a 5 – pds graph is shown in Figure 4. ■

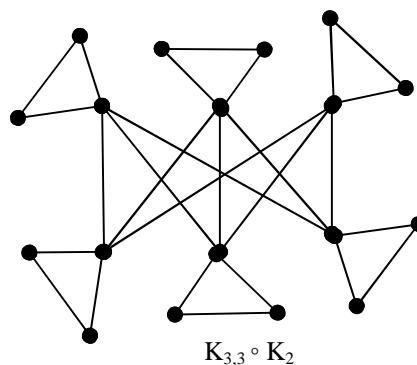


Figure 4

**Theorem 2.8** For any two regular graphs  $G$  and  $H$ ,  $G[H]$  is a  $k$  – pds graph.

**Proof** Suppose  $G$  is an  $r$  – regular graph of order  $n_1$  and  $H$  is a  $k$  – regular graph of order  $n_2$ . Then it is easy to note that  $G[H]$  is a  $(n_2r + k)$  – regular graph and hence a  $(n_2r + k)$  – pds graph. For example,  $C_4[K_2]$  which is a 5 – pds graph is shown in Figure 5. ■

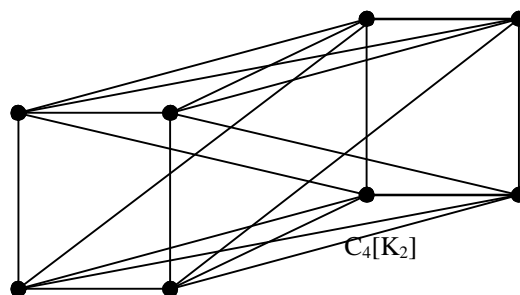


Figure 5

**Theorem 2.9** If  $G$  is a  $k$  – lds graph, then  $G \times H$  is a  $(k + 2r)$  – lds graph, for any  $r$  – regular graph  $H$ .

**Proof** Let  $G$  be any  $k$  – lds graph and  $H$  be any  $r$  – regular graph. Then  $\frac{s_G(v_i) - s_G(v_j)}{d_G(v_i) - d_G(v_j)} = k$ , for any two vertices  $v_i$  and  $v_j$  of distinct degrees in  $G$  and  $s_H(w) = r^2$  for any vertex  $w$  in  $H$ . Using Fact 2.3 (iii), for any two vertices in  $G \times H$ , we have  $s_{G \times H}(v_i, w_k) - s_{G \times H}$

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$(v_j, w_l) = (k + 2r) \{d_{G \times G}(v_i, w_k) - d_{G \times G}(v_j, w_l)\}$ , which implies that  $G \times H$  is a  $(k + 2r) -$  graph.

For example, a 1 – lds graph  $P_4$  and the corresponding 5 – lds graph  $P_4 \times C_4$  are shown in Figure 6.

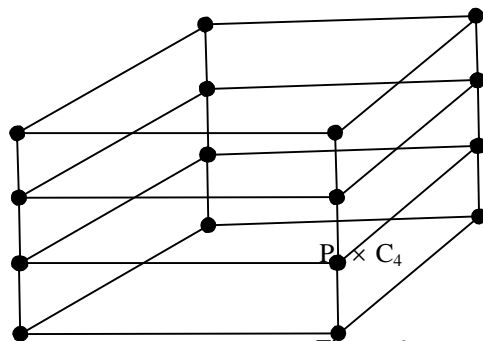


Figure 6

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