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Perfect Degree Support Product Graphs

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Abstract--For a graph $G(V,E)$, the support $s(v)$ of a vertex v is defined as the sum of degrees of its neighbours. A graph G is said to be *balanced* (*highly unbalanced*), if the support of all the vertices are same (distinct). Let k be any positive integer. A graph G is said to be a k – *perfect degree support graph* (k – *pds graph*) if for any vertex v in G , the ratio of its support and its degree is the constant k . A graph G is called a k – *linear degree support graph* (k – *lds graph*) if, for any two vertices in V with distinct degrees, the ratio of difference between their supports and the difference between their degrees is the constant k . The properties of k – *lds graphs* and k – *pds graphs* in various product graphs have been studied in this paper.

Keywords--Support, balanced graphs, highly unbalanced graphs, k – perfect degree support graphs, k – linear degree support graphs.

AMS Subject Classification code (2000): 05C (Primary)

I. INTRODUCTION

Throughout this paper, we consider only finite, simple, undirected graphs. For notations and terminology we follow [3]. A graph G is said to be r – *regular*, if every vertex of G has degree r . Let G_1 and G_2 be any two graphs. The graph $G_1 \circ G_2$ obtained from one copy of G_1 and $|V(G_1)|$ copies of G_2 by joining each vertex in the i^{th} copy of G_2 to the i^{th} vertex of G_1 is called the *corona* of G_1 and G_2 . The *cartesian product* of G and H is denoted by $G \times H$ and their *join* is denoted by $G \vee H$. The *composition graph* of G_1 to G_2 is denoted by $G_1[G_2]$. The concepts of support, balanced graphs, highly unbalanced graphs have been introduced and studied by Selvam Avadayappan and G. Mahadevan [1]. The *support* $s(v)$ of a vertex v is the sum of degrees of its neighbours. That is, $s(v) = \sum_{u \in N(v)} d(u)$. Note that the support of any vertex in an r – regular graph is r^2 .

A graph G is said to be a *balanced graph*, if the support of every vertex in G is equal. It is easy to observe that the complete bipartite graphs $K_{m,n}$ and the regular graphs are balanced graphs. A graph G is said to be *highly unbalanced*, if distinct vertices of G have distinct supports. For example, a highly unbalanced graph is shown in Figure 1.

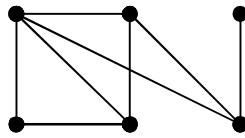


Figure 1

The following results have been proved in [1]:

Result 1 $\sum_{v \in V} s(v) = \sum_{v \in V} d(v)^2$.

Result 2 $s(v) = (n - 1)^2$ for every $v \in G$ if and only if $G \cong K_n$.

Result 3 For any balanced graph G , $\delta(G) = 1$ if and only if $G \cong K_2$ or $K_{1,n}$.

Result 4 For any $n \geq 6$, there is a highly unbalanced graph of order n .

Consequently the concepts of k – perfect degree support graph and k – linear degree support graph have been defined in [2].

A graph G is said to be a k – *perfect degree support graph* (or simply a k – *pds graph*), if for any vertex v in G , $\frac{s(v)}{d(v)} = k$. For example, the graph shown in Figure 2 is a 3 – pds graph. In general, $C_n \circ K_2$ is a 3 – pds graph for any $n > 2$.

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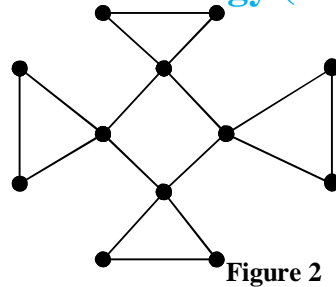


Figure 2

A graph G is said to be a k – linear degree support graph (or simply a k – lds graph), if for any two vertices u and v in G with $d(u) \neq d(v)$ in G , $\frac{s(u) - s(v)}{d(u) - d(v)} = k$, for some integer k . Or equivalently, a graph in which for any vertex v , $s(v) = k d(v) + c$ for some constant c is called a k – lds graph. Note that in a k – lds graph, $s(u) - k d(u) = s(v) - k d(v)$. For example, the graph shown in Figure 3 is a 3 – lds graph.

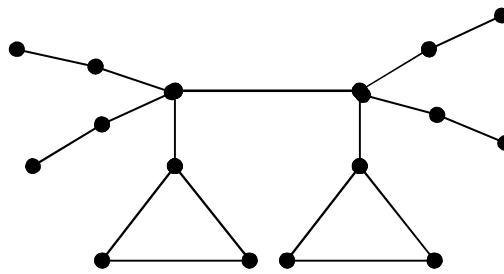


Figure 3

In general any k – pds graph is a k – lds graph but not the converse. For example, the graph shown in Figure 3 is a 3 – lds graph but it is not a 3 – pds graph. In fact k – pds graphs are k – lds graphs with $c = 0$. All balanced graphs are 0 – lds graphs and any r – regular graphs are r – pds graphs and hence r – lds graphs.

Note that in a k – lds graph, two vertices of same degree have the same support. That is, $d(u) = d(v)$ implies that $s(u) = s(v)$. A few families of k – pds graphs and k – lds graphs with some constraints have been constructed in [2]. Also k – pds trees have been characterized in [2].

In this paper, many new k – lds and k – pds graphs have been generated using various graph products.

k – pds and k – lds product graphs

Regarding the properties of k – lds and k – pds graphs in product graphs, the following facts can be easily verified:

Fact 2.1 A disconnected graph is a k – lds graph if and only if each of its components is a k – lds graph.

Fact 2.2 Let G_1 be a k_1 – lds graph and G_2 be a k_2 – lds graph. Then $G_1 \cup G_2$ is a k – lds graph if and only if $k_1 = k_2$.

By a (p, q) – graph, we mean a graph with p vertices and q edges.

Fact 2.3 Let G_1 be a (n_1, m_1) – graph and G_2 be a (n_2, m_2) – graph. Then we can easily verify the following:

(i) For any $v \in V(G_1 \cup G_2)$, we have

$$d_{G_1 \cup G_2}(v) = d_{G_1}(v) \text{ and } S_{G_1 \cup G_2}(v) = S_{G_1}(v), \text{ if } v \in V(G_1) \text{ and}$$

$$d_{G_1 \cup G_2}(v) = d_{G_2}(v) \text{ and } S_{G_1 \cup G_2}(v) = S_{G_2}(v), \text{ if } v \in V(G_2)$$

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(ii) For any $v \in V(G_1 \vee G_2)$, we have

$$d_{G_1 \vee G_2}(v) = d_{G_1}(v) + n_2; S_{G_1 \vee G_2}(v) = S_{G_1}(v) + n_2 d_{G_1}(v) + 2m_2, \text{ if } v \in V(G_1);$$

$$d_{G_1 \vee G_2}(v) = d_{G_2}(v) + n_1; S_{G_1 \vee G_2}(v) = S_{G_2}(v) + n_1 d_{G_2}(v) + 2m_1, \text{ if } v \in V(G_2)$$

(iii) For any $(u, v) \in V(G_1 \times G_2)$, we have

$$d_{G_1 \times G_2}((u, v)) = d_{G_1}(u) + d_{G_2}(v) \text{ and}$$

$$S_{G_1 \times G_2}((u, v)) = 2d_{G_1}(u) d_{G_2}(v) + S_{G_2}(v) + S_{G_1}(u),$$

(iv) For any $(u, v) \in V(G_1[G_2])$, we have

$$d_{G_1[G_2]}((u, v)) = n_2 d_{G_1}(u) + d_{G_2}(v).$$

(v) Let $V(G_1) = \{v_1, v_2, \dots, v_{n_1}\}$ and $V(G_2) = \{u_1, u_2, \dots, u_{n_2}\}$. In $V(G_1 \circ G_2)$, let u_{ij} denote the vertex corresponding to u_j in the i^{th} copy of G_2 . Then we have,

$$d_{G_1 \circ G_2}(v_i) = d_{G_1}(v_i) + n_2 \text{ and } S_{G_1 \circ G_2}(v_i) = S_{G_1}(v_i) + n_2 d_{G_1}(v_i) + 2m_2 + n_2,$$

$$d_{G_1 \circ G_2}(u_{ij}) = d_{G_2}(u_{ij}) + 1 \text{ and } S_{G_1 \circ G_2}(u_{ij}) = S_{G_2}(u_{ij}) + d_{G_2}(u_{ij}) + d_{G_1}(v_i) + n_2.$$

Now let us prove some results on k – lds and k – pds product graphs.

Theorem 2.4 If G is a k – lds graph of order n , then $G \vee G$ is a $(k + n)$ – lds graph.

Proof Let G be any k – dsl graph of order n . Then for any two vertices v_i and v_j of distinct degrees, $\frac{s_G(v_i) - s_G(v_j)}{d_G(v_i) - d_G(v_j)} = k$. Now consider $G \vee G$. Then the degree of every vertex gets increased by n and the support of any vertex gets increased by its degree times n . For any two vertices of distinct degrees, $\frac{s_{G \vee G}(v_i) - s_{G \vee G}(v_j)}{d_{G \vee G}(v_i) - d_{G \vee G}(v_j)} = \frac{s_G(v_i) + n d_G(v_i) - s_G(v_j) - n d_G(v_j)}{d_G(v_i) - d_G(v_j)} = k + n$. Hence, $G \vee G$ is a $(k + n)$ – lds graph. For example, a 1 – lds graph P_4 and the corresponding 5 – lds graph $P_4 \vee P_4$ are shown in Figure 3.

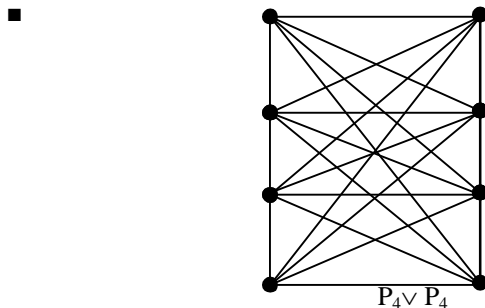


Figure 3

Theorem 2.5 Let G be any (n, m) – graph. If $G \vee K_1$ is a k – pds graph, then $\frac{2m}{n} = k - 1$.

Proof Let G be a graph with n vertices and m edges such that $G \vee K_1$ is a k – pds graph. Let v be the vertex of K_1 . Then clearly v is a full vertex with support $S_{G \vee K_1}(v) = \frac{\sum_{u \in V(G)} d_{G \vee K_1}(u)}{n} = \frac{\sum_{u \in V(G)} d_G(u) + n}{n} = \frac{2m + n}{n} = k$. Hence the proof.

Theorem 2.6 There does not exist a k – pds graph G such that $G \vee K_1$ is also k – pds.

Proof Suppose G is a k – pds graph such that $G \vee K_1$ is a k – pds graph. Then for any vertex v in G , $S_G(v) / d_G(v) = S_{G \vee K_1}(v) /$

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$d_{G \vee K_1}(v) = k$. Now by using Fact 5.8 (ii), $\frac{s_G(v) + d_G(v) + n}{d_G(v) + 1} = k$. That is, $\frac{kd_G(v) + d_G(v) + n}{d_G(v) + 1} = k$, which implies $d_G(v) = k - n$. Since $G \vee K_1$ is connected, we have $k > n$. In other words, $s_G(v) > n d_G(v)$ which is impossible. Therefore such a graph G does not exist ■

Theorem 2.7 For an r -regular graph G of order n_1 and a k -regular graph H of order n_2 , $G \circ H$ is a $(r + k)$ -pds graph.

Proof Let G and H be r -regular and k -regular graphs respectively. Then by Fact 2.3 (v), $G \circ H$ is a biregular graph. In particular, $s_{G \circ H}(v) = r + n_2$; $s_{G \circ H}(v) = r^2 + n_2(r + k + 1)$, for any $v \in V(G)$ and $d_{G \circ H}(u) = k + 1$; $s_{G \circ H}(u) = k^2 + k + r + n_2$, for all $u \in V(H)$. Therefore we get $\frac{s_{G \circ H}(v) - s_{G \circ H}(u)}{d_{G \circ H}(v) - d_{G \circ H}(u)} = r + k$. Hence $G \circ H$ is a $(r + k)$ -pds graph. For example, $K_{3,3} \circ K_2$ which is a 5-pds graph is shown in Figure 4. ■

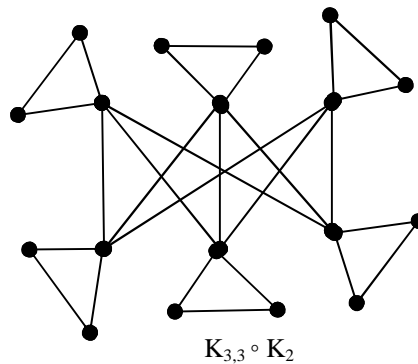


Figure 4

Theorem 2.8 For any two regular graphs G and H , $G[H]$ is a k -pds graph.

Proof Suppose G is an r -regular graph of order n_1 and H is a k -regular graph of order n_2 . Then it is easy to note that $G[H]$ is a $(n_2r + k)$ -regular graph and hence a $(n_2r + k)$ -pds graph. For example, $C_4[K_2]$ which is a 5-pds graph is shown in Figure 5. ■

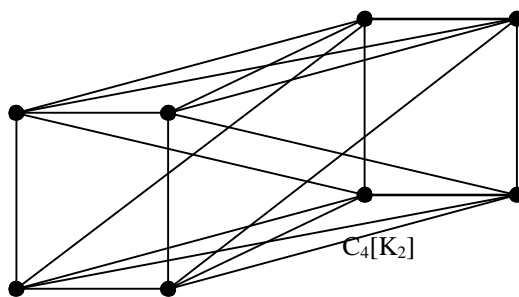


Figure 5

Theorem 2.9 If G is a k -lds graph, then $G \times H$ is a $(k + 2r)$ -lds graph, for any r -regular graph H .

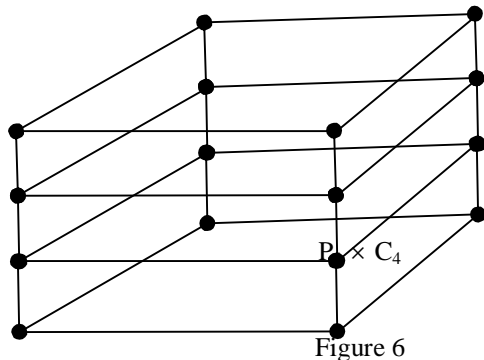
Proof Let G be any k -lds graph and H be any r -regular graph. Then $\frac{s_G(v_i) - s_G(v_j)}{d_G(v_i) - d_G(v_j)} = k$, for any two vertices v_i and v_j of distinct degrees in G and $s_H(w) = r^2$ for any vertex w in H . Using Fact 2.3 (iii), for any two vertices in $G \times H$, we have $s_{G \times H}(v_i, w_k) - s_{G \times H}$

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$(v_j, w_l) = (k + 2r) \{d_{G \times G}(v_i, w_k) - d_{G \times G}(v_j, w_l)\}$, which implies that $G \times H$ is a $(k + 2r)$ – graph.

For example, a 1 – lds graph P_4 and the corresponding 5 – lds graph $P_4 \times C_4$ are shown in Figure 6.

■



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