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# Modeling and simulation of X-quadcopter control

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**Abstract-** Since the turn of the 19<sup>th</sup> century, air transport had taken a giant turn. It took decades to master the art of flying heavier than air objects for a variety of reasons. During those early days, air transport was considered a distant dream and one which attracted a lot of interest to dive into research. UAVs are currently in that same phase of development and lots of research is being conducted all around the world for a variety of applications. Evolution of quadcopters has hindered until very recently, because controlling four self-governing rotors has proven to be incredibly difficult and impossible without electronic aid. In order to achieve six degrees of freedom (three translational and three rotational) with only four independent inputs (rotor speeds), quadcopters are severely under-triggered. Hence rotational and translational motions are coupled. The resulting dynamic performance is highly nonlinear. This paper discusses on condensed model of quadcopter dynamics and design of a PID controller for the dynamics to follow a desired attitude and altitude for an unmanned aerial vehicle quadcopter. A simulation is performed to experiment and explore the control performances and responses of the quadcopter model. The attitude controller designs are implemented by Matlab Simulink.

**Keywords –** Quadcopter; x configuration; state equation.

## I.

## INTRODUCTION

Quadcopters are vehicles capable of flying in air with the help of four rotors. These four rotors provide the necessary thrust and lift for the vehicle to fly in the air. Easy to say, but the flight of quadcopter is not necessarily easy to control. There is a major factor of stability which needs to be perfected over the quadcopter due to the various harsh environments the vehicle may be subjected to and obstacles it may face en- route. Quadcopters are used in surveillance, search and rescue, and several other applications and hence it becomes imperative the performance of the vehicle need to be optimum. Two of the rotors will need rotating clock-wise and the other two counter-clockwise for proper stabilization.

The following section provides the mathematical model of a quadcopter. The third section provides simulation of Quad-copter using control inputs and using a PID Controller. The last section contains the conclusion of this paper.

## II. MATHEMATICAL MODEL OF QUADCOPTER

An important aspect of mathematical modelling of quadcopter is the coordinate system in use. The coordinate system will vary depending on the different configurations of quadcopter. The two configurations are the plus (“+”) or “X” configuration [1].

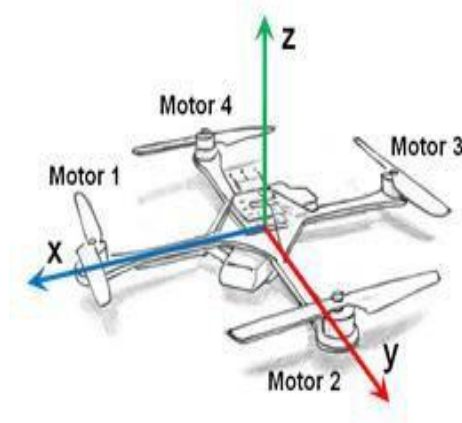


Fig.1. Plus (+) Configuration

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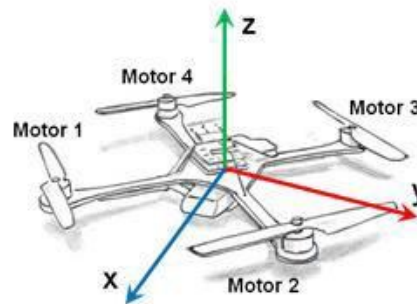


Fig.2. X Configuration

As seen in Fig. 1, the X axis lies along the Motor arm 1, Y axis along motor arm 2 and the Z axis pointing upwards. The Motor in arms 1 and 3 rotates in counter-clockwise direction whereas the motors in arm 2 and 4 rotate clockwise. When the Plus (“+”) configuration XY plane rotates about 45 degrees in positive upward direction [2], “X” configuration is achieved as in Fig. 2. The Euler angles over the three axes are provided below in Fig. 3.

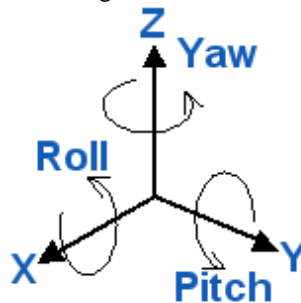


Fig. 3. Euler Angles

The following parameters are required to be calculated to model the quadcopter.

### A. Thrust Coefficient

The thrust produced the four motors on the quadcopter is the driving force behind all the maneuvers [1]. The thrust,  $T$ , provided by a single motor can be calculated as follows:

$$T = C_T \rho A_r r^2 \omega^2$$

Where  $C_T$  is the thrust coefficient for a specific rotor,  $\rho$  is the density of air,  $A_r$  is the cross-sectional area of the propeller's rotation,  $\omega$  is the angular velocity of the rotor and  $r$  is the radius of the rotor. For simple modelling of quadcopter, we propose a lumped parameter approach to simply the calculation:

$$T = c_T \omega^2$$

Where  $c_T$  is the lumped parameter thrust coefficient for an individual motor system.

### B. Torque Coefficient

The torque force is determined after performing lumped approach as shown below:

$$Q = c_Q \omega^2$$

Where  $Q$  is the torque created by the motor and  $c_Q$  is the torque coefficient for the motor.

### C. Gyroscopic Forces

The forces which result from gyroscopic precession are termed gyroscopic forces. The gyroscopic forces resulting on the body of the quadcopter are governed by the inertia of each motor's rotating components ( $J$ ), the speed of each motor ( $\omega_i$ ) and the rolling and pitching rates (P and Q) [3]. The gyroscopic torques created by motors for pitch and roll are given below:

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$$\begin{aligned}\tau_{\theta_{gyro}} &= JP\left(\frac{\pi}{30}\right)(-\omega_1 + \omega_2 - \omega_3 + \omega_4) \\ \tau_{\phi_{gyro}} &= JQ\left(\frac{\pi}{30}\right)(\omega_1 - \omega_2 + \omega_3 - \omega_4)\end{aligned}$$

### D. Throttle Command Relation

An important consideration for the controlling of quadcopter is that the coefficients of thrust and torque are based on a relationship with RPM of the motors and not directly determined by throttle command. Therefore, a regression is created that translates throttle command values to RPM values.

$$\omega_{ss} = (Throttle\%)c_R + b$$

Where  $\omega_{ss}$  is the steady-state motor RPM, is the throttle percentage to RPM conversion coefficient and  $b$  is the y- intercept of the linear regression relationship.

### E. Moments on the quadcopter

The moment is calculated resulting from the aerodynamics, thrusts and torques on the system as given below:

$$M = \begin{bmatrix} d_x c_T \omega_2^2 - d_x c_T \omega_4^2 + J_m Q\left(\frac{\pi}{30}\right)(\omega_1 - \omega_2 + \omega_3 - \omega_4) \\ -d_x c_T \omega_1^2 + d_x c_T \omega_3^2 + J_m P\left(\frac{\pi}{30}\right)(-\omega_1 + \omega_2 - \omega_3 + \omega_4) \\ -c_Q \omega_1^2 + c_Q \omega_2^2 - c_Q \omega_3^2 + c_Q \omega_4^2 \end{bmatrix}$$

Where  $d_x$  is the value for the distance between the motor/prop and the body's axes of rotation.

### F. Forces on the quadcopter

The force acting in the body frame on the quadcopter due to the aerodynamics and thrust as given below:

$$F = \begin{bmatrix} 0 \\ 0 \\ c_T(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix}$$

The lift force on the quadcopter is assumed to be oriented strictly in the positive z direction [4]. Additional effects such as blade flapping and aerodynamic drag are not considered since they are minimal.

### G. State Equations

The Angular Velocity State equation provides the change in roll ( $P$ ), pitch ( $Q$ ) and yaw ( $R$ ) rates of the quadcopter.

$$\dot{\omega} = (J^b)^{-1} [M - \Omega^b J^b \omega] = \begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix}$$

Where is the Inertia of the quadcopter relative to the body frame,  $M$  is the moment acting on the quadcopter body,  $\Omega^b$  is a cross-product matrix for rotational velocity and is the rotational velocity of the quadcopter body [1].

Rotational matrices provide rotation in Euclidean space.

According to the aerospace rotation sequence, the rotation of an aircraft is described as a rotation about the yaw axis (z) then a rotation about the pitch axis(y) followed by a rotation about the roll axis(x). Using these three rotations, a composite rotation matrix is given below for inertial to body frame rotation.

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$$C = \begin{bmatrix} c(\theta) c(\psi) & c(\theta) s(\psi) & s(\theta) \\ (-c(\phi) s(\psi) + s(\phi) s(\theta) c(\psi)) & (c(\phi) c(\psi) + s(\phi) s(\theta) s(\psi)) & s(\phi) c(\theta) \\ (s(\phi) s(\psi) + c(\phi) s(\theta) c(\psi)) & (-s(\phi) c(\psi) + c(\phi) s(\theta) s(\psi)) & c(\phi) c(\theta) \end{bmatrix}$$

Where  $c$  and  $s$  represent cosine and sine functions.

The acceleration of the center of mass of the rigid body quad model based on the forces and accelerations acting on the body is provided by the Velocity State equation as given below:

$$\dot{\mathbf{v}} = \left(\frac{1}{m}\right) \mathbf{F} + \mathbf{g}^b - \boldsymbol{\Omega}^b \boldsymbol{\omega}^b = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}$$

Where  $\mathbf{g}^b$  is the acceleration of gravity translated to act in the body frame by the rotation matrix  $C$ .

The velocity of the quadcopter in the body frame rotated into inertial frame is calculated using the Position State equation.

$$\dot{\mathbf{P}} = C \mathbf{v}^b = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}$$

$\mathbf{v}^b$  is the velocity of the quadcopter in the body frame.

### III. SIMULATION AND RESULTS

The quadcopter model is controlled using PID Controller. Controlling of quadcopter is performed using Attitude Control. In attitude control, the desired attitude of the quadcopter is provided in terms of the roll, pitch and yaw angles. The results of the simulation are published below:

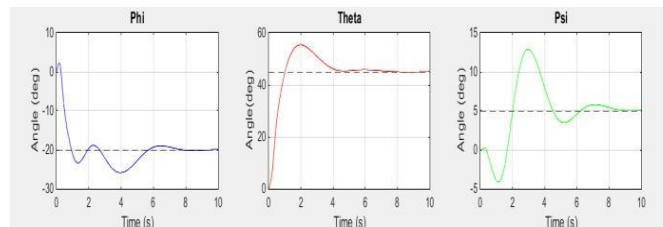


Fig.4. Desired Attitude Command

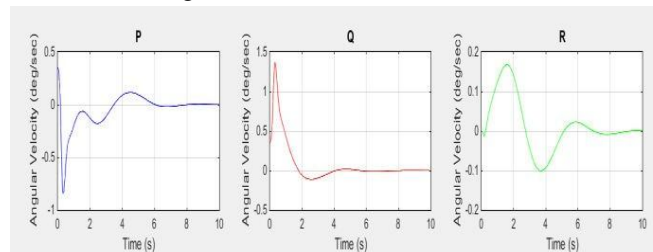


Fig.5. Roll, Pitch and yaw rate of the quadcopter

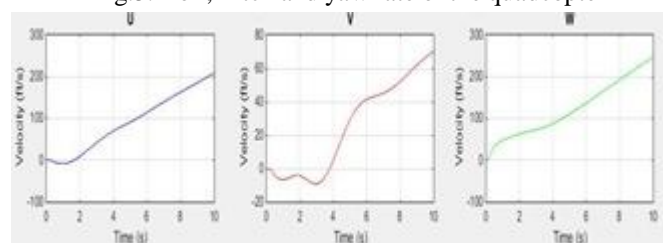


Fig.6. Velocity of quadcopter

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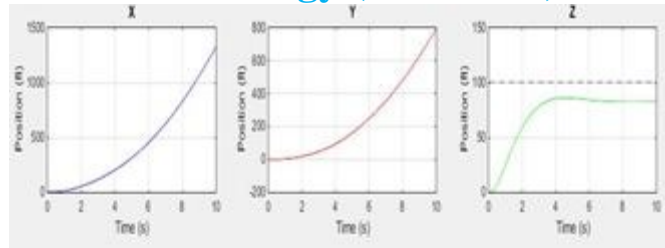


Fig.7. Position of quadcopter

The actual input provided to the UAV for its flight is given in Fig. 4. The Roll, Pitch and yaw angles are the input commands. On providing the input commands, the behavior of the UAV is simulated and shown in Fig.5, Fig.6 and Fig.7. Fig.5 shows the rate of roll, pitch and yaw the UAV attains over a period of 10 seconds when provided with the input desired commands. Fig.6 shows the velocity of the UAV over time and Fig.7 the actual projected position the UAV reaches in feet.

The graphs show an estimate of the actual UAV performance based on its mathematical model. The motors are the driving force behind the maneuvers and motion of the quadcopter. Based on the attitude commands to the quadcopter, a throttle command in percentage is arrived at which in turn determines the Motor speed command for the vehicle. The plot between the throttle command percentage and the Motor Speed Command in RPM for all the four motors are given in Fig. 8.

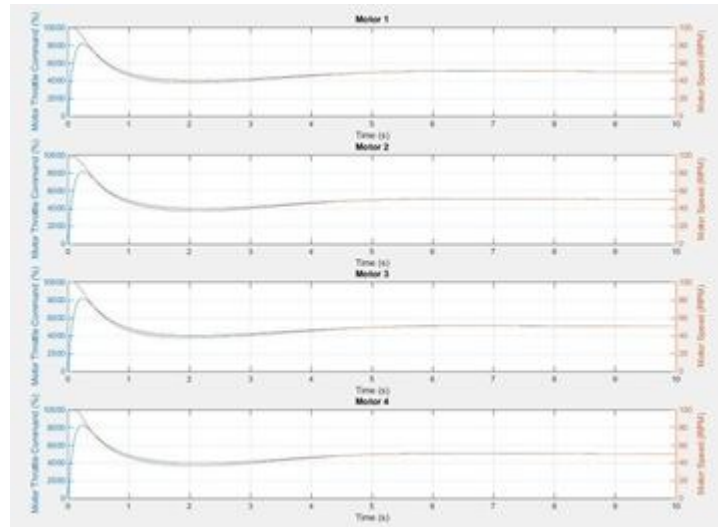


Fig. 8. Throttle % vs Motor Speed (RPM)

### IV. CONCLUSION

The modelling and control of X-quadcopter using a PID Controller has been presented in this paper. The Simulation is performed in Matlab Simulink and the model takes into account the assumptions that the quadcopter body and the propellers are rigid and the structure is supposedly symmetrical. Based on the attitude control commands in the form of desired roll, pitch and yaw angles, the quadcopter control is brought about by PID Controller. The Simulation results illustrate the velocity and position of the quadcopter along three different axes. This paper provides modelling of the UAV and also the projection of its performance based on the designed model. Earlier work on the same topic provided the modelling parameters alone and this paper takes the reference of it and extend it in measuring projections of its dynamic behavior.

Several extensions to this paper are of interest. First, this paper only discusses attitude control of quadcopter. This can be extended to position control too. Second, the vehicle is modeled taking into account only the forces and moments acting over it and doesn't include aerodynamic drag and blade flapping.

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