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Fixed Point Of Multivalued Mapping On Polish Space

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Abstract: *A general fixed point theorem, that incorporates several known random fixed point theorems, involving continuous random operators, is proved.*

Keywords: *Fixed point; Multivalued mapping; Polish space.*

1. INTRODUCTION

Random fixed point theorems are of fundamental importance in probabilistic functional analysis. They are stochastic generalization of classical fixed point theorems and are required for the theory of random equations. In a separable metric space random fixed point theorems for contractive mapping were proved by Spacek [1], Hans [2, 3], Mukherjee [4]. Itoh [5, 6] extended several fixed point theorems, i.e., for contraction, nonexpensive, mappings to the random case. Thereafter, various stochastic aspects of Schauder's fixed point theorem have been studied by Sehgal and Singh [7] and Lin [8]. Afterwards Beg and Shahzad [9], Badshah and Sayyad [10] studied the structure of common random fixed points and random coincidence points of a pair of compatible random operators and proved the random fixed points theorems for contraction random operators in polish spaces.

2. PRELIMINARIES

Let (X, d) be a Polish space; that is a separable, complete metric space and (Ω, a) be a measurable space. Let 2^X be a family of all subsets of X and $CB(X)$ denote the family of all non empty closed bounded subsets of X. A mapping $T:\Omega\to 2^X$, is called measurable, if for any open subset C of $X, T^{-1}(C)=\{\omega\in\Omega : T(\omega)\cap C\neq\phi\}\in a$. A mapping $\xi:\Omega\to X$, is said to be measurable selector of a measurable mapping $T:\Omega\to 2^X$, if ξ is measurable and for any $\omega\in\Omega$, $\xi(\omega)\in T(\omega)$. A mapping $f:\Omega\times X\to X$ is called a random operator if for any $x\in X$, $f(., X)$ is measurable. A mapping $T:\Omega\times X\to CB(X)$, is called random multivalued operator if for every $x\in X, T(., X)$ is measurable. A measurable mapping ¶ $:\Omega\to X$, is called the random fixed point of a random multivalued operator $T:\Omega\times X\to CB(X)$ ($f:\Omega\times X\to X$), if for every $\omega\in\Omega$, $\xi(\omega)\in T(\omega,\xi(\omega))$ ($f(\omega,\xi(\omega))=\xi(\omega)$). Let $T:\Omega\times X\to CB(X)$ be a random operator and $\{\xi_n\}$ a sequence of measurable mappings $\xi_n:\Omega\to X$. The sequence is said to be asymptotically T-regular, if $d(\xi_n(\omega), T(\omega,\xi_n(\omega)))\to 0.$

3. THEOREM

Let X be a Polish space. Let $T,S:\Omega\times X\to CB(X)$ be two continuous random multivalued operators. If there exist measurable mappings $\alpha,\beta,\gamma:\Omega\to(0,1)$ such that

$$
\begin{aligned}
(1)\ &\min\{H(S(\omega,x),T(\omega,y)), d(x,S(\omega,x))d(y,T(\omega,y)),\\
&d(y,T(\omega,y))\}+\alpha(\omega)\min\{d(x,T(\omega,y)), d(y,S(\omega,x))\}\\
&\le[\beta(\omega)d(x,S(\omega,x))+\gamma(\omega)d(x,y)]d(y,T(\omega,y))
\end{aligned}
$$

for each $x, y\in X$, where α,β,γ are real numbers such that $0<\beta(\omega)+\gamma(\omega)<1$ and there exists a

measurable sequence $\{\xi_n\}$ which is asymptotically regular with respect to S and T ,then there exists a common random fixed point of S and T. (H is Hausdorff metric on $CB(X)$ induced by metric d)

3.1 Proof. Let $\xi_0:\Omega\to X$ bne an arbitrary measurable mapping and choose a measurable mapping $\xi_1:\Omega\to X$ such that $\xi_1(\omega)\in S(\omega,\xi_0(\omega))$ for each $\omega\in\Omega$. Then for each $\omega\in\Omega$

$$
\begin{aligned}
&\min\left\{\begin{array}{l}H(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))),\\ d(\xi_0(\omega),S(\omega,\xi_0(\omega)))d(\xi_1(\omega),T(\omega,\xi_1(\omega))),\\ d(\xi_1(\omega),T(\omega,\xi_1(\omega)))\end{array}\right\}\\
&+\alpha(\omega)\min\{d(\xi_0(\omega),T(\omega,\xi_1(\omega))), d(\xi_1(\omega),S(\omega,\xi_0(\omega)))\}\\
&\le[\beta(\omega)d(\xi_0(\omega),S(\omega,\xi_0(\omega)))+\gamma(\omega)d(\xi_0(\omega),\xi_1(\omega))]\\
&\qquad d(\xi_1(\omega),T(\omega,\xi_1(\omega)))\cdot
\end{aligned}
$$

It further implies [lemma 2.3 of Beg (1993a)], that there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for any $\omega \in \Omega$, $\xi_2(\omega) \in T(\omega, \xi_1(\omega))$ and for $x = \xi_{n-1}(\omega)$ and $y = \xi_n(\omega)$ by condition (1), we have

$$\min\left\{\begin{array}{l} H(S(\omega,\xi_{n-1}(\omega)),T(\omega,\xi_n(\omega))), \\ d(\xi_{n-1}(\omega),S(\omega,\xi_{n-1}(\omega)))d(\xi_n(\omega),T(\omega,\xi_n(\omega))), \\ d(\xi_n(\omega),T(\omega,\xi_n(\omega))) \end{array}\right\}$$
$$+\alpha(\omega)\min\{d(\xi_{n-1}(\omega),T(\omega,\xi_n(\omega))),d(\xi_n(\omega),S(\omega,\xi_{n-1}(\omega)))\}$$
$$\le [\beta(\omega)d(\xi_{n-1}(\omega),S(\omega,\xi_{n-1}(\omega)))+\gamma(\omega)d(\xi_{n-1}(\omega),\xi_n(\omega))]\,d(\xi_n(\omega),T(\omega,\xi_n(\omega)))$$

Since $d(\xi_n(\omega),S(\omega,\xi_{n-1}(\omega)))=0$,
$\xi_n(\omega) \in S(\omega,\xi_{n-1}(\omega))$.

We have

$$\min\{d(\xi_n(\omega),\xi_{n+1}(\omega)),d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))\}$$
$$\le [\beta(\omega)d(\xi_{n-1}(\omega),\xi_n(\omega))+\gamma(\omega)d(\xi_{n-1}(\omega),\xi_n(\omega))]d(\xi_n(\omega),\xi_{n+1}(\omega))$$

and it follows that

$$\min\{d(\xi_n(\omega),\xi_{n+1}(\omega)),d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))\}$$
$$\le [\beta(\omega)+\gamma(\omega)]d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))$$

Since

$$d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))$$
$$\le [\beta(\omega)+\gamma(\omega)]d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))$$

is not possible (as $0<\beta(\omega)+\gamma(\omega)<1$).

We have

$$d(\xi_n(\omega),\xi_{n+1}(\omega))$$
$$\le [\beta(\omega)+\gamma(\omega)]d(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))$$

Or

$$d(\xi_n(\omega),\xi_{n+1}(\omega)) \le kd(\xi_{n-1}(\omega),\xi_n(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))$$

where

$$k=\beta(\omega)+\gamma(\omega), \qquad 0<k<1$$

Similarly proceeding in the same way, by induction we produce a sequence of measurable mapping $\xi_n : \Omega \to X$ such that for $n>0$ and any $\omega \in \Omega$

$$\xi_{2n+1}(\omega) \in S(\omega,\xi_{2n}(\omega)), \quad \xi_{2n+2}(\omega) \in T(\omega,\xi_{2n+1}(\omega))$$

and

$$d(\xi_n(\omega),\, \xi_{n+1}(\omega)) \le k\, d(\xi_{n-1}(\omega),\, \xi_n(\omega)) \le \ldots \le k^n d(\xi_o(\omega),\, \xi_1(\omega))$$

Further, for $m > n$,

$$d(\xi_n(\omega),\, \xi_m(\omega)) \le d(\xi_n(\omega),\, \xi_{n+1}(\omega)) + \ldots + d(\xi_{m-1}(\omega),\, \xi_m(\omega))$$
$$\le (k^n + k^{n+1} + \ldots + k^{m-1})\, d(\xi_o(\omega),\, \xi_1(\omega))$$
$$d(\xi_n(\omega),\, \xi_m(\omega))$$
$$\le \left(\frac{k^n}{1-k}\right) d(\xi_o(\omega),\, \xi_1(\omega))$$

which tends to zero as $n \to \infty$. It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$. It implies that $\xi_{2n+1}(\omega) \to \xi(\omega)$ and $\xi_{2n+2}(\omega) \to \xi(\omega)$. Thus, we have for any $\omega \in \Omega$,

$$d(\xi(\omega),\, S(\omega,\xi(\omega))) \le d(\xi(\omega),\, \xi_{2n+2}(\omega)) + d(\xi_{2n+2}(\omega),\, S(\omega,\xi(\omega)))$$
$$\le d(\xi(\omega),\, \xi_{2n+2}(\omega))$$
$$+ H(T(\omega,\, \xi_{2n+1}(\omega)),\, S(\omega,\, \xi(\omega))),$$

Or,

$$d(\xi(\omega),\, S(\omega,\xi(\omega)))$$
$$\le [\beta(\omega)d(\xi(\omega),\, S(\omega,\xi(\omega))) + \gamma(\omega)d(\xi(\omega),\xi_{2n+1}(\omega))]$$
$$\times\, d(\xi_{2n+1}(\omega),T(\omega,\xi_{2n+1}(\omega))),$$

which tends to zero.

So,

$\{\xi_{2n+1}\}$ is asymptotically T-regular.

Letting $n \to \infty$ we have

$$d(\xi(\omega),\, S(\omega,\xi(\omega))) \le 0.$$

Hence,

$\xi(\omega) \in S(\omega,\xi(\omega))$ for $\omega \in \Omega$.

Similarly, for

$$d(\xi(\omega),\, T(\omega,\xi(\omega)))$$
$$\le d(\xi(\omega),\xi_{2n+1}(\omega)) + H(S(\omega,\xi_{2n}(\omega)),T(\omega,\xi(\omega)))$$
$$d(\xi(\omega),\, T(\omega,\xi(\omega))) \le 0.$$

Hence,

$\xi(\omega) \in T(\omega,\xi(\omega))$ for each $\omega \in \Omega$.

3.2 Corollary. Let X be a Polish space. Let $T : \Omega \times X \to CB(X)$ be a continuous random multivalued operator. If there exist measurable mappings $\alpha,\beta,\gamma : \Omega \to (0,1)$ such that

$$\min\{H(T(\omega,x),T(\omega,y)),d(x,T(\omega,x))d(y,T(\omega,y)),d(y,T(\omega,y))\}$$
$$+\alpha(\omega)\min\{d(x,T(\omega,y)),d(y,T(\omega,x))\}$$
$$\le [\beta(\omega)d(x,T(\omega,x))+\gamma(\omega)d(x,y)]d(y,T(\omega,y)),$$

for each $x, y \in X$ and $\omega \in \Omega$ where α,β,γ are real numbers such that $0<\beta(\omega)+\gamma(\omega)<1$ then there exists a sequence $\{\xi_n\}$ of measurable mappings $\xi_n : \Omega \to X$ which asymptotically T-regular and converges to a random fixed point of *T*.

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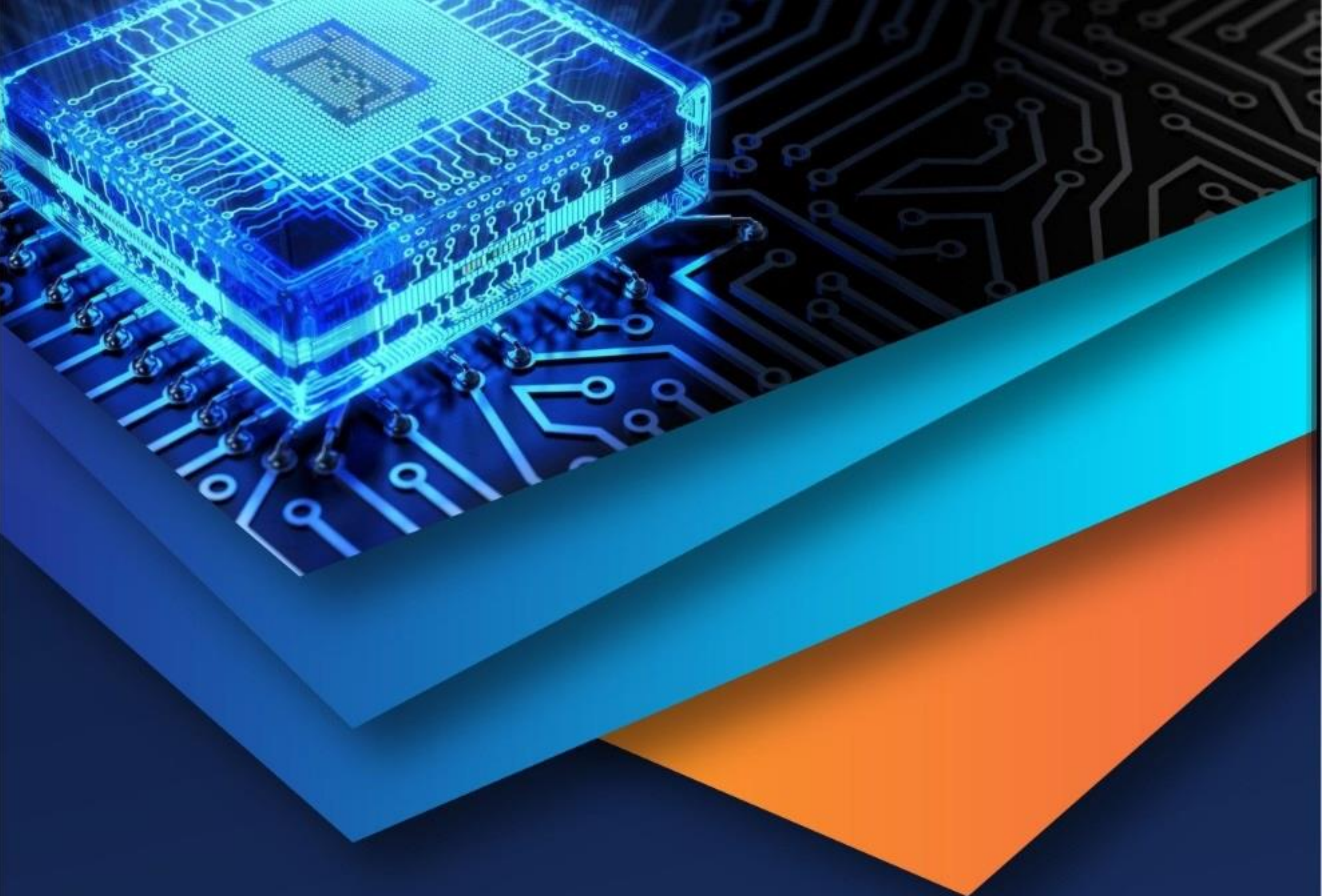

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