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Design of Fractional order Recursive Digital Differentiator using Continued Fraction Expansion

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Abstract- In this paper, new discretized models of fractional order differentiator (FOD) based on different operators are proposed. Specifically in this work, one-third and one-fourth order differentiator models based on Simpson operator, Rectangular operator and Tustin operator have been derived. The implementation of fractional order digital differentiator requires two steps. In the first step, an integer order digital differentiator is designed then in second step continued fraction expansion (CFE) is used to obtain an efficient fractional order digital differentiator. The proposed one-third and one-fourth order differentiator approximates the ideal one-third and one-fourth order differentiator reasonably well over the entire Nyquist frequency range.
Keywords- Fractional calculus, One-third and one-fourth order differentiator, IIR filter, Continued fraction expansion.

I. INTRODUCTION

Fractional order systems are systems that are signified by differential equations that allow non-integer order. It is a generalization of the integer order integration and differentiation. Most of the real dynamic systems are fractional in nature and these systems can be more accurately evaluated by fractional calculus. This is the main reason behind the popularity of fraction order systems. They are widely used in various fields, such as signal processing, digital image processing and automatic control systems. Fractional order systems (integrators and differentiators) can be designed from integer order systems [1-2]. The design and improvement for the digital fractional order differentiators are becoming the key issue in the field of fractional order calculus. [3-4]. Considering the complexity factors when design the filter, the order of the FIR differential filters will be reserved and also the approximation effect the frequency response of the filter to the ideal frequency response is affected. So the recursive fractional order differentiator is considered to realize the fractional order operation in this paper. The common method used to expand the fractional-order includes the PSE (power series expansion) and CFE (continued fraction expansion) and CFE can make more impact on the functional approximation and has a faster convergence speed [5].

In this paper, discretized mathematical models of one-third and one-fourth order differentiators using Simpson operator, Rectangular operator and Tustin operator have been presented. The concept of linear interpolation is based on the assumption that combining two good integrators can result in a better integrator [6]. The result obtained show that the frequency response of the proposed fractional order differentiator model almost matches with the frequency response of ideal fractional order differentiator ($S^r; 0 < r < 1$) in continuous time domain.

The organization of this paper is as follows: Section II presents the typical IIR-type fractional order differentiators. Section III presents the new differentiators based on combining typical operators. MATLAB results of the various operators based on one-third and one-fourth order differentiators compared with ideal fractional order differentiator are presented in Section IV. Section V concludes the paper.

II. TYPICAL IIR-TYPE FRACTIONAL ORDER DIFFERENTIATOR

A. IIR-type fractional order digital differentiator based on Simpson operator

The Simpson differential operator is defined as:

$$H_S(z) = \frac{3}{T}\frac{1-z^{-2}}{1+4z^{-1}+z^{-2}} \quad (1)$$

So the transfer function of the Simpson fractional order differentiator can be defined as:

$$G_S^r(z) = \left(\frac{3}{T}\frac{1-z^{-1}}{1+4z^{-1}+z^{-2}}\right)^r \quad (2)$$

Where r denotes the differential order.

For any function $D(z)$, we can use the CFE (continued fraction expansion) to express it, that is:

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$$D(z) \cong a_0(z) + \cfrac{b_1(z)}{a_1(z)+\cfrac{b_2(z)}{a_2(z)+\cfrac{b_3(z)}{a_3(z)+\cdots}}} \quad (3)$$

Where the coefficients a_i, b_i are rational functions or constants for the variable z. We can acquire the finite order approximation function just by the truncated operation. When T= 0.001 sec, by using the CFE method to expand (2) we get the Simpson fractional order differentiator function $G_{Sn}^{r}(z)$, where r signifies the differential order and n denotes the filter order. From the point of view of the error and its computational complexity, the order of the fractional order differentiator is relatively suitable to choose five.

The discretization of one-third and one-fourth order differentiators (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (2) is calculated numerically and approximated transfer function is given as follows:

$$G_{S5}^{0.33}(z) = \frac{14.42\,z^5+89.56\,z^4+128.7\,z^3-73.49\,z^2-139.2\,z+31.39}{z^5+7.543\,z^4+16.1\,z^3+3.413\,z^2-12.54\,z-0.6247} \quad (4)$$

$$G_{S5}^{0.25}(z) = \frac{7.401\,z^5+47.21\,z^4+72.16\,z^3-32.23\,z^2-75.72\,z+14.07}{z^5+7.379\,z^4+15.13\,z^3+2.016z^2-12.45\,z-0.2246} \quad (5)$$

B. IIR-type fractional order digital differentiator based on Rectangular operator

The Rectangular differential operator is defined as:

$$H_R(z) = \frac{1}{T}\,\frac{1-z^{-1}}{1} \quad (6)$$

So the transfer function of the Rectangular fractional order differentiator can be defined as:

$$G_R^r(z) = \left(\frac{1}{T}\,\frac{1-z^{-1}}{1}\right)^r \quad (7)$$

The discretization of one-third and one-fourth order differentiators $G_{Rn}^{r}(z)$ (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (7) is calculated numerically and approximated transfer function is given as follows:

$$G_{R5}^{0.33}(z) = \frac{10\,z^5-26.67\,z^4+25.68\,z^3-10.7\,z^2+1.783\,z-0.07926}{z^5-2.333\,z^4+1.901\,z^3-0.6337\,z^2+0.07545\,z-0.001677} \quad (8)$$

$$G_{R5}^{0.25}(z) = \frac{5.623\,z^5-14.76\,z^4+13.94\,z^3-5.664z^2+0.9102\,z-0.03793}{z^5-2.375\,z^4+1.979\,z^3-0.6803\,z^2+0.08504z-0.002126} \quad (9)$$

C. IIR-type fractional order digital differentiator based on Tustin operator

The Tustin differential operator is defined as:

$$H_T(z) = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \quad (10)$$

So the transfer function of the Tustin fractional order differentiator can be defined as:

$$G_T^r(z) = \left(\frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\right)^r \quad (12)$$

The discretization of one-third and one-fourth order differentiators $G_{Tn}^{r}(z)$ (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (12) is calculated numerically and approximated transfer function is given as follows:

$$G_{T5}^{0.33}(z) = \frac{12.6\,z^5-4.2\,z^4-13.38\,z^3+3.215\,z^2+2.713\,z-0.2746}{z^5+0.333z^4-1.062\,z^3-0.2552\,z^2+0.2154\,z+0.02179} \quad (13)$$

$$G_{T5}^{0.25}(z) = \frac{6.687\,z^5-1.672\,z^4-7.245\,z^3+1.289\,z^2+1.506\,z-0.111}{z^5+0.25\,z^4-1.083\,z^3-0.1927\,z^2+0.2253\,z+0.0166} \quad (14)$$

III. NEW FRACTIONAL ORDER DIFFERENTIATOR CONSTRUCTED BY COMBINING TYPICAL OPERATORS

A. IIR-type fractional order digital differentiator based on Rectangular operator and Tustin operator

The Rectangular operator and Tustin operator have the best amplitude-frequency characteristics and phase-frequency characteristics

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respectively. So by combining these two operators using interpolation method we get nearly ideal integrator. The interpolation ratio is a user-specified weight that balances the effect of the selected integrator.

The new integrator, $I_A(z)$, can be mathematically specified as:

$$I_A(z) = \alpha I_R(z) + (1-\alpha) I_T(z) \tag{15}$$

The factor α, (0<α< 1) determines the fraction of involvement of each integrator in the new integrator. For integrator $I_A(z)$ value of α is $3/4$, $I_R(z)$ denotes Rectangular operator and $I_T(z)$ denotes Tustin operator.

By substitution of corresponding transfer function, we get

$$I_A(z) = \frac{3}{4}\left(\frac{T}{z-1}\right) + \frac{1}{4}\left(\frac{T}{2}\frac{(z+1)}{(z-1)}\right) \tag{16}$$

Simplifying we get

$$I_A(z) = \frac{T}{8}\frac{(z+7)}{(z-1)} \tag{17}$$

New differentiator is obtained by inverting the transfer function of the designed integrator $I_A(z)$ using the stabilization method described by Al-Alaoui [7]. The zero of (17) is not included in the unit circle, so the zero $z = -7$ is mapped to $z = -1/7$. Multiplying by 7 we obtain corresponding compensation for the amplitude and get the minimum phase integrator as follows:

$$I_A(z) = \frac{7T}{8}\frac{(z+\frac{1}{7})}{(z-1)} \tag{18}$$

By exchanging the numerator and denominator of (18) the obtained differentiator is given as follows:

$$D_A(z) = \frac{8}{7T}\frac{(z-1)}{(z+\frac{1}{7})} \tag{19}$$

So the corresponding fractional order differential operator $D_A^r(z)$ is:

$$D_A^r(z) = \left(\frac{8}{7T}\frac{z-1}{z+\frac{1}{7}}\right)^r \tag{20}$$

The discretization of one-third and one-fourth order differentiators $D_{An}^r(z)$ (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (20) is calculated numerically and approximated transfer function is given as follows:

$$D_{A5}^{0.33}(z) = \frac{10.21z^5-23.81\,z^4+18.52\,z^3-5.124\,z^2+0.1986\,z+0.0406}{z^5-1.954\,z^4+1.166\,z^3-0.1804\,z^2-0.01848\,z+0.002013} \tag{21}$$

$$D_{A5}^{0.25}(z) = \frac{5.814\,z^5-13.29\,z^4+10.05\,z^3-2.639\,z^2+0.0694\,z+0.0225}{z^5-2\,z^4+1.238\,z^3-0.2109\,z^2-0.01638\,z+0.00232} \tag{22}$$

B. IIR-type fractional order digital differentiator based on Tustin operator and Simpson operator

Tustin operator and Simpson operator both exists error in the high frequency region but their amplitude curves lie on the upper and lower bilateral respectively.

The new integrator, $I_B(z)$, can be mathematically specified as:

$$I_B(z) = \alpha I_T(z) + (1-\alpha) I_S(z) \tag{23}$$

The factor α, (0<α< 1) determines the fraction of involvement of each integrator in the new integrator. For integrator $I_B(z)$ value of α is $2/5$, $I_T(z)$ denotes Tustin operator and $I_s(z)$ denotes Simpson operator.

By substitution of corresponding transfer function, we get

$$I_B(z) = \frac{2T}{5}\frac{z^2+3z+1}{z^2-1} \tag{24}$$

The zeros of (24) are $r_1 = (-3+\sqrt{5})/2$ and $r_2 = (-3-\sqrt{5})/2$. To construct the minimum phase system, r_2 is mapped to r_1. At the same time, to keep the amplitude invariant we introduce the compensation factor $-r_2$ and get the integral operator as follows:

$$I_{B1}(z) = \frac{-2Tr_2}{5}\frac{(z-r_1)^2}{z^2-1} \tag{25}$$

By exchanging the numerator and denominator of (25) to get corresponding differentiator, we get

$$D_B(z) = \frac{-5r_1}{2T}\frac{(z^2-1)}{(z-r_1)^2} \tag{26}$$

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So the corresponding fractional order differential operator $D_B^r(z)$ is:

$$D_B^r(z) = \left(\frac{-5r_1}{2T}\frac{z^2-1}{(z-r_1)^2}\right)^r \quad (27)$$

The discretization of one-third and one-fourth order differentiators $D_{Bn}^r(z)$ (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (27) is calculated numerically and approximated transfer function is given as follows:

$$D_{B5}^{0.33}(z) = \frac{9.625\,z^5+0.6068\,z^4-13.71\,z^3+0.1281z^2+4.488\,z-0.513}{z^5+0.3151\,z^4-1.095\,z^3-0.2401\,z^2+0.2416\,z+0.01577} \quad (28)$$

$$D_{B5}^{0.25}(z) = \frac{5.559z^5+0.52\,z^4-7.718\,z^3-0.1317\,z^2+2.438\,z-0.2305}{z^5+0.2845\,z^4-1.139\,z^3-0.216\,z^2+0.2678\,z+0.01071} \quad (29)$$

C. IIR-type fractional order digital differentiator based on Rectangular operator and Simpson operator

The new integrator,$I_C(z)$, can be mathematically specified as:

$$I_C(z) = \alpha I_R(z) + (1-\alpha)I_S(z) \quad (30)$$

The factor α, (0<α< 1) determines the fraction of involvement of each integrator in the new integrator. For integrator $I_C(z)$ value of α is $5/8$, $I_R(z)$ denotes Rectangular operator and $I_s(z)$ denotes Simpson operator.

By substitution of corresponding transfer function, we get

$$I_C(z) = \frac{6T}{8}\frac{z^2+3/2z+1/6}{z^2-1} \quad (31)$$

The zeros of (31) are $r_1 = (-9+\sqrt{57})/12$ and $r_2 = (-9-\sqrt{57})/12$. To construct the minimum phase system,r_2 is mapped to r_1.At the same time, to keep the amplitude invariant we introduce the compensation factor $-r_2$ and get the integral operator as follows:

$$I_C(z) = \frac{-3Tr_2}{4}\frac{(z-r_1)(z-\frac{1}{r_2})}{z^2-1} \quad (32)$$

By exchanging the numerator and denominator of (32) to get corresponding differentiator, we get

$$D_C(z) = \frac{-4}{3Tr_2}\frac{z^2-1}{(z-r_1)(z-\frac{1}{r_2})} \quad (33)$$

So the corresponding fractional order differential operator $D_C^r(z)$ is:

$$D_C^r(z) = \left(\frac{-4}{3Tr_2}\frac{z^2-1}{(z-r_1)\left(z-\frac{1}{r_2}\right)}\right)^r \quad (34)$$

The discretization of one-third and one-fourth order differentiators $D_{Cn}^r(z)$ (r = 0.33, 0.25 and n=5) sampled at 0.001 sec for (34) is calculated numerically and approximated transfer function is given as follows:

$$D_{C5}^{0.33}(z) = \frac{9.507z^5+2.419\,z^4-15.45\,z^3-0.7743\,z^2+6.129\,z-0.9436}{z^5+0.5052\,z^4-1.282\,z^3-0.359\,z^2+0.3851\,z-0.0084} \quad (35)$$

$$D_{C5}^{0.25}(z) = \frac{5.507z^5+1.441\,z^4-8.754\,z^3-0.6776\,z^2+3.3852z-0.4677}{z^5+0.4732\,z^4-1.329z^3-0.3336\,z^2+0.41722\,z-0.01608} \quad (36)$$

IV. PERFORMANCE RESULTS AND DISCUSSION

The frequency response of the proposed one-third and one-fourth order models are compared with the response of the corresponding ideal equivalents in continuous time domain. MATLAB simulation results have been presented to validate the efficiency of the proposed design. Figure.1 is the frequency response curve of one-third order differentiator based on Rectangular operator, Tustin operator and Simpson operator. From figure, we can find that the amplitude curves of three filters are consistent to the ideal amplitude in the low frequency region, but with the frequency increasing the errors will increase sharply especially in the high frequency region. Amplitude characteristics based on Rectangular operator is the best, but the phase characteristic is poorer than two other operators. The advantage of the Tustin operator lies on its better phase characteristic and its phase characteristic is coincident to the phase curve of ideal frequency response in most regions. The Tustin operator and Simpson operator have better performance in the low frequency region.

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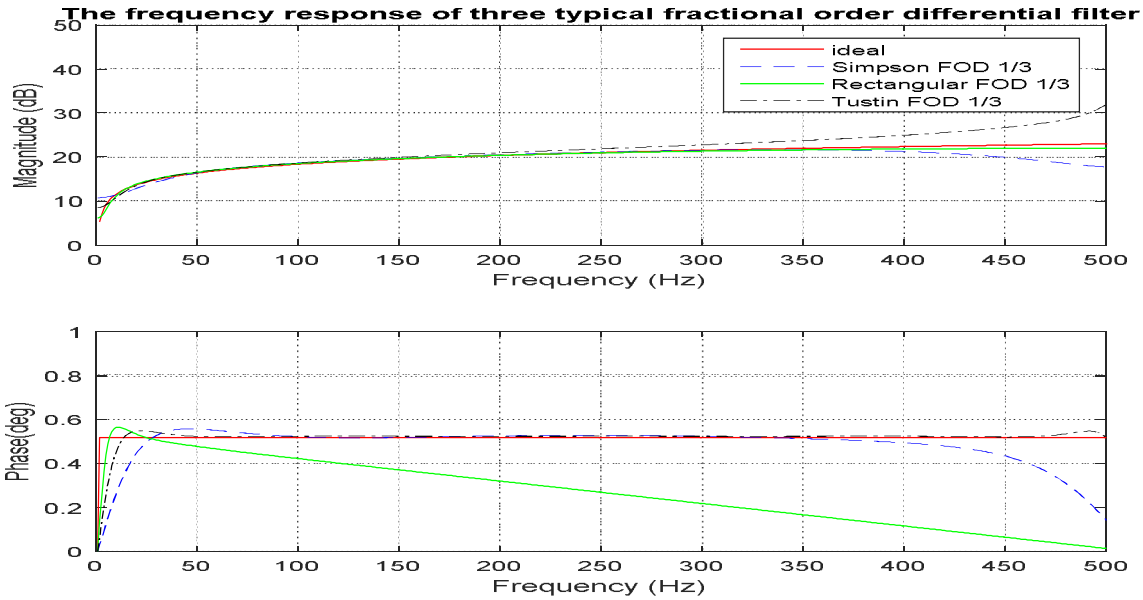

Fig.1 The frequency response of three typical one-third order differentiators

Figure.2 is the frequency response curve of one-third order differentiator based on three new operators. According to the curve the new operator D_{A5} is superior to the operator D_{B5} and D_{C5} in the frequency characteristics and its amplitude characteristic is basically close to the ideal frequency response curve from the low frequency region to high frequency region.

Figure.3 is the frequency response curve of one-fourth order differentiator based on three new operators. According to the curve the new operator D_{A5} is superior to the operator D_{B5} and D_{C5} in the frequency characteristics and its amplitude characteristic is basically close to the ideal frequency response curve from the low frequency region to high frequency region. Phase characteristic of new operator D_{A5} is approximate linearly increasing with the increasing frequency.

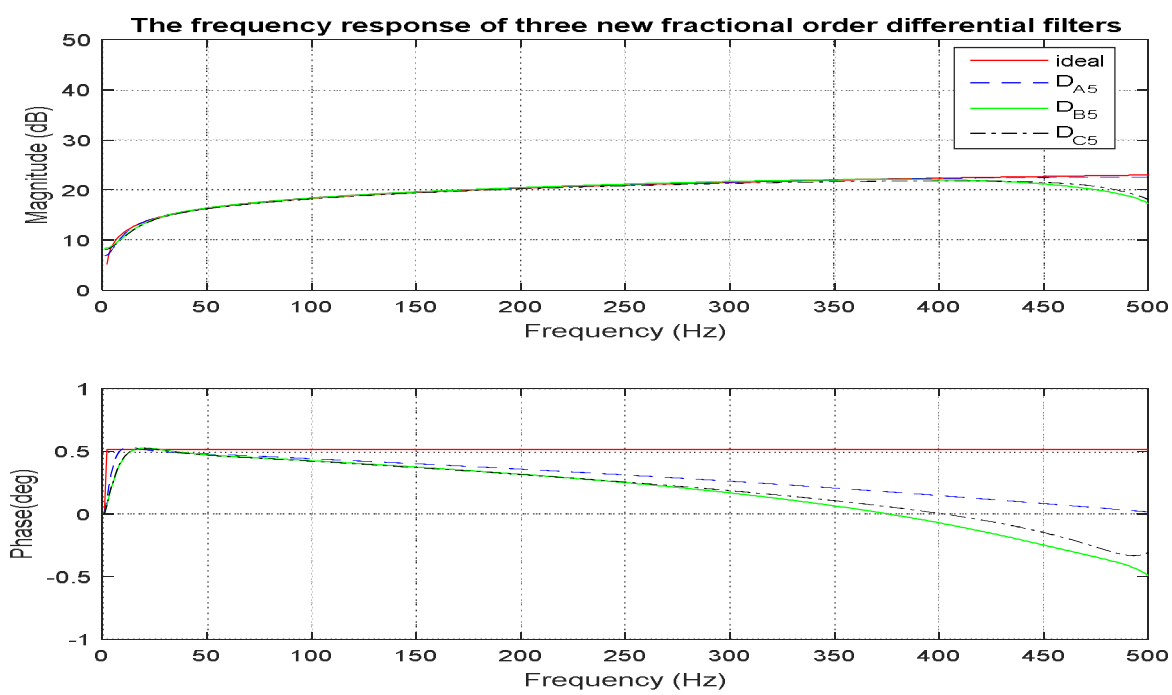

Fig.2 The frequency response of three new one-third order differentiators

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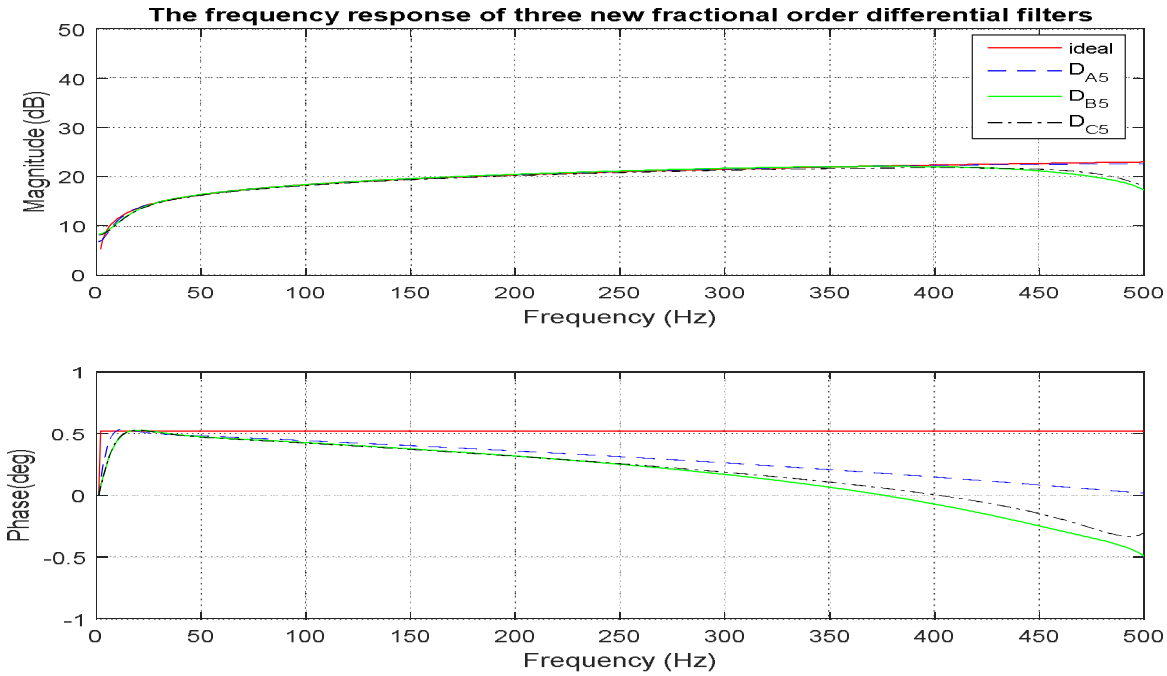


Fig.3 The frequency response of three new one-fourth order differentiators

V. CONCLUSION

This paper makes analysis on the design and implement for the IIR-type digital fractional order differentiator from the point of view of frequency region. In this paper, discretized mathematical models of one-third and one-fourth order differentiators using interpolation of Simpson operator, Rectangular operator and Tustin operator have been presented. CFE method used to transform the transfer function from the integer order form to fractional order form. By the analysis of frequency response of new operator, it is easy to know that the performance of fractional order differentiator can be improved obviously.

REFERENCES

[1] B.T.Krishna, K.V.V.S. Reddy, "Design of digital differentiators and integrators of order ½", World Journal of Modelling and Simulation, vol.4, no.3, pp. 182-187, May 2008

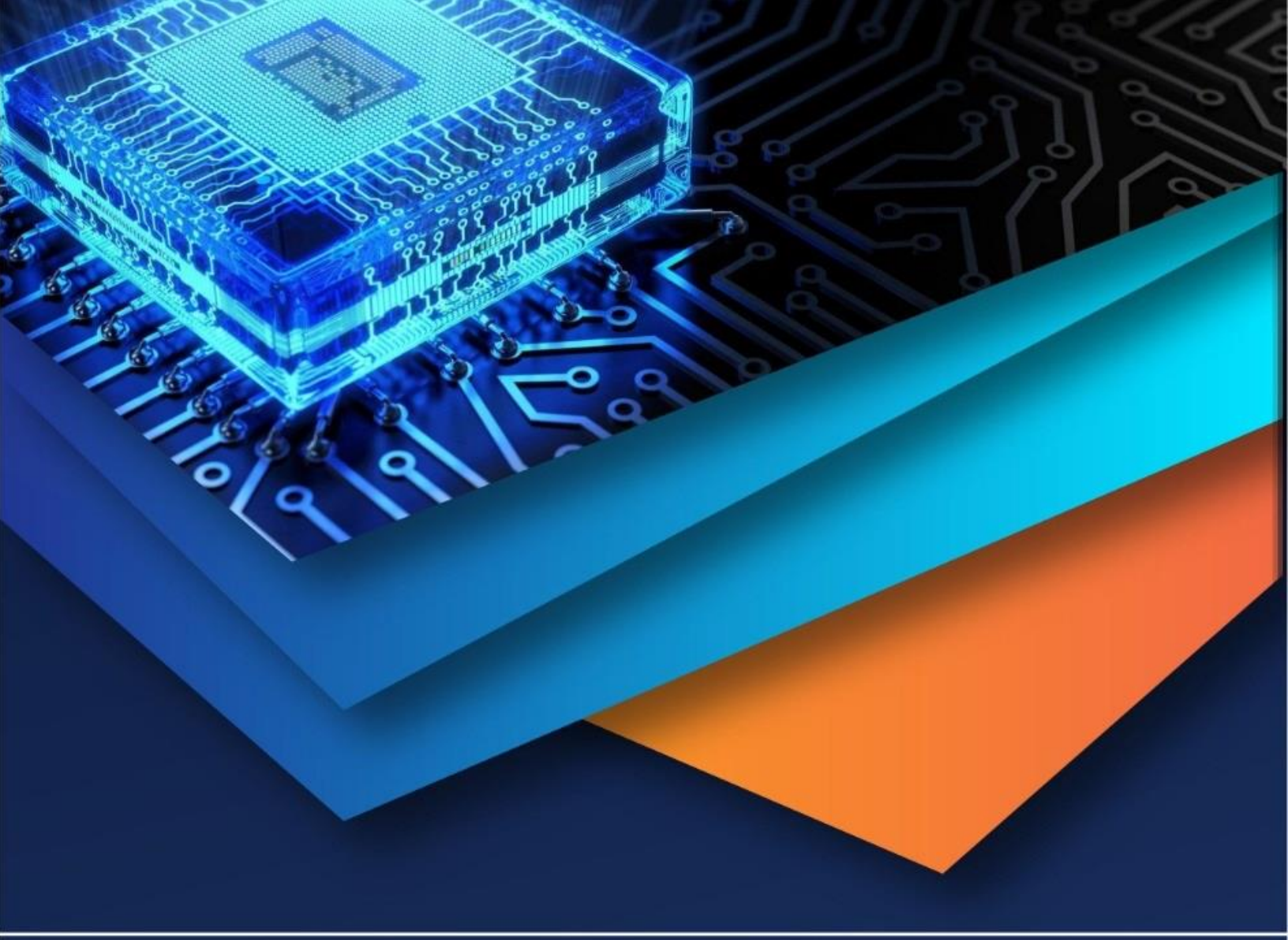
[2] M.A. Al-Alaoui, "Simulation and discretization of fractional order systems", Proceedings of the International Conference on Genetics and Evolutionary Methods (GEM 2009), pp. 249-255

[3] M.A. Ai-Alaoui, "Novel approach to designing digital differentiators", Electronics Letters, vol.28, no. 15, pp. 1376-1378, 1992

[4] Y.Q. Chen and B.M.Vinagre, "A new IIR-type digital fractional order differentiator", Elsevier J. Signal Processing: Special Issue on Fractional Signal Processing and Applications, vol. 83, no. 11, pp. 2359-2365, 2003

[5] Y.Q. Chen, B.M. Vinagre, Continued Fraction Expansion Approaches to Discretizing Fractional Order Derivatives-an Expository Review [J], Nonlinear Dynamics,2005,38:155-170

[6] Al-Alaoui, "Novel digital integrator and differentiator [J]", Electronics Letters, 1993, 29(4): 376-378



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