iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 4 Issue: VI Month of publication: June 2016

DOI:

www.ijraset.com
IC Value: 13.98

Volume 4 Issue VI, June 2016
ISSN: 2321-9653

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Fuzzy HX Subring of a HX Ring

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***Abstract*— In this paper, we define the concept of a fuzzy HX ring and define a new algebraic structure of a fuzzy HX subring of a HX ring. We also discuss some related properties of it.**

***Keywords*— HX ring, fuzzy HX ring, fuzzy set, fuzzy subring, fuzzy HX subring,.**

I. INTRODUCTION

In 1965, Zadeh [10] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval 0 and 1 and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic , set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [9] defined the idea of fuzzy subgroups and gave some of its properties. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1,2] gave the structures of HX ring on a class of ring. In this paper we define a new algebraic structure of a fuzzy HX subring of a HX ring and investigate some related properties.

II. PRELININARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R ,+, \cdot)$ is a Ring, e is the additive identity element of R and xy, we mean x.y.

III. PROPERTIES OF FUZZY HX RING

In this section we define the concept of fuzzy HX ring and discuss some related results.

A. *Definition*

Let R be a ring. Let μ be a fuzzy set defined on R . Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\mathfrak{R}$ is called a fuzzy HX ring on $\mathfrak{R}$ or a fuzzy ring induced by μ if the following conditions are satisfied. For all $A , B \in \mathfrak{R}$,

i. $\lambda^\mu (A – B) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$,

ii. $\lambda^\mu (AB) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$,

where $\lambda^\mu (A) = \max \{ \mu(x) \ / \ \text{for all } x \in A \subseteq R \}$.

B. *Remark*

For a fuzzy HX subring λ^μ of a HX ring $\mathfrak{R}$, the following result is obvious.

i. $\lambda^\mu (A) \leq \lambda^\mu (0)$ and $\lambda^\mu (A) = \lambda^\mu (-A)$, for all $A \in \mathfrak{R}$.

ii. $\lambda^\mu (A – B) = 0$ implies that $\lambda^\mu (A) = \lambda^\mu (B)$.

C. *Example*

Let $C^0 = C – \{0\}$, where C is the set of all complex numbers.

For all $a, b \in C^0$, define the operators $\oplus$ and $\otimes$ on C^0 as $a \oplus b = ab$ and $a \otimes b = |a|^{\ln|b|}$.

Clearly, $(C^0, \oplus, \otimes)$ is a **ring**.

Define, a fuzzy set μ on C^0 as,

$$\mu (x) = \mu (a + ib) = \begin{cases} 0.8 & \text{if} \quad a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if} \quad a < 0 \text{ and } b = 0 \\ 0.4 & \text{if} \quad b \neq 0 \end{cases}$$

where, a is the real part of x lies in X-axis and b is the imaginary part of x lies in the Y-axis.

Then, Clearly $\mu (x \Theta y) = \mu (x \oplus (-y)) \geq \min \{ \mu (x) , \mu (y) \}$,

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ISSN: 2321-9653
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$\mu (x \otimes y) \geq \min \{ \mu (x) , \mu (y) \}$.

Clearly, μ is a **fuzzy subring** on C^0.

Let $I = (1 , \infty)$ and $H = \{ 1 ,-1, i, -i \}$

Define $\Re = \{ a \oplus I / a \in H \}$. For,

$1 \in H \Rightarrow 1 \oplus I = 1 \cdot (1 , \infty) = (1 , \infty)$

$-1 \in H \Rightarrow -1 \oplus I = -1 \cdot (1 , \infty) = (-\infty , -1)$

$i \in H \Rightarrow i \oplus I = i \cdot (1 , \infty) = (i , \infty)$

$-i \in H \Rightarrow -i \oplus I = -i \cdot (1 , \infty) = (-\infty , -i)$.

Now, $\Re = \{ a \oplus I / a \in H \} = \{(1 , \infty), (-\infty , -1), (i , \infty), (-\infty , -i) \} = \{ Q, A, B, C\}$.

For any $X, Y \in \Re$, define the operations $\oplus$ and $\otimes$ on $\Re$ as,

$X \oplus Y = XY = \{xy / x\in X \text{ and } y \in Y \}$

$X \otimes Y = | X |^{\ln | Y |} = \{ | x |^{\ln | y |} / x\in X \text{ and } y \in Y \}$

Then,

$\oplus$	Q	A	B	C
Q	Q	A	B	C
A	A	Q	C	B
B	B	C	A	Q
C	C	B	Q	A

$\otimes$	Q	A	B	C
Q	Q	Q	Q	Q
A	Q	Q	Q	Q
B	Q	Q	Q	Q
C	Q	Q	Q	Q

Clearly, $(\Re , \oplus , \otimes)$ is a **HX ring** on $(P_0(C^0) , \oplus , \otimes)$.

Define a fuzzy set $\lambda^\mu : \Re \rightarrow [0 , 1]$ as,

$\lambda^\mu (Q) = \text{Sup} \{ \mu (x) / x \in Q \} = 0.8$

$\lambda^\mu (A) = \text{Sup} \{ \mu (x) / x \in A \} = 0.6$

$\lambda^\mu (B) = \text{Sup} \{ \mu (x) / x \in B \} = 0.4$

$\lambda^\mu (C) = \text{Sup} \{ \mu (x) / x \in C \} = 0.4$

Now, $- Q = A ; - A = Q ; - B = C ; - C = B$.

$Q \ominus Q = Q \oplus A = A$
$Q \ominus A = Q \oplus Q = Q$
$Q \ominus B = Q \oplus C = C$
$Q \ominus C = Q \oplus B = B$

$A \ominus Q = A \oplus A = Q$
$A \ominus A = A \oplus Q = A$
$A \ominus B = A \oplus C = B$
$A \ominus C = A \oplus B = C$

$B \ominus Q = B \oplus A = C$
$B \ominus A = B \oplus Q = B$
$B \ominus B = B \oplus C = Q$
$B \ominus C = B \oplus B = A$

$C \ominus Q = C \oplus A = B$

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ISSN: 2321-9653
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$C \ominus A = C \oplus Q = C$
$C \ominus B = C \oplus C = A$
$C \ominus C = C \oplus B = Q$

For any X, Y $\in \mathfrak{R}$, we have,

$\lambda^{\mu}(X \ominus Y) = \lambda^{\mu}(X \oplus (-Y)) \geq \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$,
$\lambda^{\mu}(X \otimes Y) \geq \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$.

Clearly, λ^{μ} is a **fuzzy HX ring** on $\mathfrak{R}$.

D. *Theorem*

If μ is a fuzzy subring of a ring R then the fuzzy subset λ^{μ} is a fuzzy HX subring of a HX ring $\mathfrak{R}$.

Proof

Let μ be a fuzzy subring of R.

i. $\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} = \min\{\max\{\mu(x) \,/\, \text{for all } x \in A \subseteq R\}, \max\{\mu(y) \,/\, \text{for all } y \in B \subseteq R\}\}$
$= \min\{\mu(x_0), \mu(y_0)\}$
$\leq \mu(x_0 - y_0)$, since μ is a fuzzy subring of R
$\leq \max\{\mu(x-y) \,/\, \text{for all } x-y \in A-B \subseteq R\}$
$\leq \lambda^{\mu}(A-B)$
$\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

ii. $\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} = \min\{\max\{\mu(x) \,/\, \text{for all } x \in A \subseteq R\}, \max\{\mu(y) \,/\, \text{for all } y \in B \subseteq R\}\}$
$= \min\{\mu(x_0), \mu(y_0)\}$
$\leq \mu(x_0 y_0)$, since μ is a fuzzy subring of R
$\leq \max\{\mu(xy) \,/\, \text{for all } xy \in AB \subseteq R\}$
$\leq \lambda^{\mu}(AB)$
$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$.

Hence, λ^{μ} is a fuzzy HX subring of a HX ring $\mathfrak{R}$.

E. *Remark*

1) If μ is not a fuzzy subring of R then the fuzzy set λ^{μ} of $\mathfrak{R}$ is a fuzzy HX subring of $\mathfrak{R}$, provided $|X| \geq 2$ for all $X \in \mathfrak{R}$.
2) If μ is a fuzzy subset of a ring R and λ^{μ} be a fuzzy HX subring of $\mathfrak{R}$, such that $\lambda^{\mu}(A) = \max\{\mu(x) \,/\, \text{for all } x \in A \subseteq R\}$, then μ may or may not be a fuzzy subring of R, which can be illustrated by the following example.

F. *Example*

Let $C^0 = C - \{0\}$, where C is the set of all complex numbers.

For all a, b $\in C^0$, define the operators $\oplus$ and $\otimes$ on C^0 as

$a \oplus b = ab$ and $a \otimes b = |a|^{\ln|b|}$.

Clearly, $(C^0, \oplus, \otimes)$ is a ring.

Define, a fuzzy set μ on C^0 as,

$$\mu(x) = \mu(a+ib) = \begin{cases} 0.8 & \text{if} \quad a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if} \quad a < 0 \text{ and } b = 0 \\ 0.4 & \text{if} \quad b > 0 \\ 0.3 & \text{if} \quad b < 0. \end{cases}$$

where, a is the real part of x lies in X-axis and b is the imaginary part of x lies in the Y-axis.

Let x = 2+3i and y = -2.

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ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Then, $\mu (x \Theta y) = \mu (x \oplus (-y)) = \mu (-4 - 6i) = 0.3.$

$\min \{ \mu (x) , \mu (y) \} = \min \{ 0.4 , 0.6 \} = 0.4.$

Hence, $\mu (x \Theta y) \not\geq \min \{ \mu (x) , \mu (y) \},$

Clearly, μ is a not a fuzzy subring on C^0.

Let $I = (1 , \infty)$ and $H = \{ 1 ,-1, i, -i \}$

Define $\Re = \{ a \oplus I / a \in H \}$.

For, $1 \in H \Rightarrow 1 \oplus I = 1. (1 , \infty) = (1 , \infty)$

$-1 \in H \Rightarrow -1 \oplus I = -1. (1 , \infty) = (-\infty , -1)$

$i \in H \Rightarrow i \oplus I = i. (1 , \infty) = (i , \infty)$

$-i \in H \Rightarrow -i \oplus I = -i. (1 , \infty) = (-\infty , -i)$.

Now, $\Re = \{ a \oplus I / a \in H \} = \{(1 , \infty), (-\infty , -1), (i , \infty), (-\infty , -i) \}$

Let $\Re = \{\{(1 , \infty), (-\infty , -1)\},\{(i , \infty), (-\infty , -i) \}\} = \{ Q, A \}$.

For any $X, Y \in \Re$, define the operations $\oplus$ and $\otimes$ on $\Re$ as,

$X \oplus Y = XY = \{xy / x\in X \text{ and } y \in Y \}$

$X \otimes Y = | X |^{\ln | Y |} = \{ | x |^{\ln | y |} / x\in X \text{ and } y \in Y \}$

Then,

$\oplus$	Q	A
Q	Q	A
A	A	Q

$\otimes$	Q	A
Q	Q	Q
A	Q	Q

Clearly, $(\Re , \oplus , \otimes)$ is a HX ring on $(P_0(C^0) , \oplus , \otimes)$.

Define a fuzzy set $\lambda^\mu : \Re \to [0 , 1]$ as,

$\lambda^\mu (Q) = \max \{ \mu (x) / x \in Q \} = 0.8$

$\lambda^\mu (A) = \max \{ \mu (x) / x \in A \} = 0.4$

Now, $- Q = Q ; - A = A$.

$Q \Theta Q = Q \oplus Q = Q$

$Q \Theta A = Q \oplus A = A$

$A \Theta Q = A \oplus Q = A$

$A \Theta A = A \oplus A = Q$

For any $X, Y \in \Re$, we have,

$\lambda^\mu(X \Theta Y) = \lambda^\mu(X \oplus (-Y)) \geq \min \{ \lambda^\mu(X) , \lambda^\mu(Y) \},$

$\lambda^\mu(X \otimes Y) \geq \min \{ \lambda^\mu(X) , \lambda^\mu (Y) \}.$

Clearly, λ^μ is a fuzzy HX ring on $\Re$.

G. Definition

Let R be a ring. Let μ and η be any two fuzzy subsets of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring on R . Let λ^μ and γ^η be fuzzy subsets of $\Re$. The intersection of λ^μ and γ^η is defined as $(\lambda^\mu \cap \gamma^\eta) (A) = \min\{ \lambda^\mu (A), \gamma^\eta(A)\}$ for all $A \in \Re$.

H. Theorem

Let μ and η be any two fuzzy sets defined on R. Let λ^μ and γ^η be any two fuzzy HX subrings of a HX ring $\Re$ then the intersection of two fuzzy HX subrings, $\lambda^\mu \cap \gamma^\eta$ is also a fuzzy HX subring of a HX ring $\Re$.

Proof

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Let A , B $\in \Re$.

1) $(\lambda^{\mu}\cap \gamma^{\eta})(A–B) = \min\{\lambda^{\mu}(A–B), \gamma^{\eta}(A–B)\}$
$\geq \min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\},\min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\}$
$= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\},\min\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\}$
$\geq \min\{(\lambda^{\mu}\cap \gamma^{\eta})(A), (\lambda^{\mu}\cap \gamma^{\eta})(B)\}$.

$(\lambda^{\mu}\cap \gamma^{\eta})(A–B) \geq \min\{(\lambda^{\mu}\cap \gamma^{\eta})(A), (\lambda^{\mu}\cap \gamma^{\eta})(B)\}$.

2) $(\lambda^{\mu}\cap \gamma^{\eta})(AB) = \min\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\}$
$\geq \min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\},\min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\}$
$= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\},\min\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\}$
$\geq \min\{(\lambda^{\mu}\cap \gamma^{\eta})(A), (\lambda^{\mu}\cap \gamma^{\eta})(B)\}$.

$(\lambda^{\mu}\cap \gamma^{\eta})(AB) \geq \min\{(\lambda^{\mu}\cap \gamma^{\eta})(A), (\lambda^{\mu}\cap \gamma^{\eta})(B)\}$.

Hence, $\lambda^{\mu}\cap \gamma^{\eta}$ is a fuzzy HX subring of a HX ring $\Re$.

I. Remark

1) The intersection of family of fuzzy HX subrings of a HX ring $\Re$ is also fuzzy HX subring of $\Re$.
2) Let R be a ring. Let μ and η be any two fuzzy sets of R and $\mu \cap \eta$ is also a fuzzy set of R then $\varphi^{\mu\cap\eta}$ is a fuzzy HX subring of $\Re$ induced by $\mu \cap \eta$ of R.

J. Theorem

If λ^{μ}, γ^{η}, $\varphi^{\mu\cap\eta}$ are fuzzy HX subrings of a HX ring $\Re$ induced by the fuzzy sets μ, η, $\mu \cap \eta$ of R then $\varphi^{\mu\cap\eta} = \lambda^{\mu}\cap \gamma^{\eta}$.

Proof

Let λ^{μ} and γ^{η} are fuzzy HX subrings of $\Re$ then $\lambda^{\mu}\cap \gamma^{\eta}$ is a fuzzy HX subring of a HX ring $\Re$ by Theorem 3.8.
$\varphi^{\mu\cap\eta}$ is a fuzzy HX subring of $\Re$ induced by $\mu \cap \eta$ of R by Theorem 3.4.

$$\varphi^{\mu\cap\eta}(A) = \max\{(\mu\cap\eta)(x) / \text{for all } x\in A \subseteq R\}$$
$$= \max\{\min\{\mu(x), \eta(x)\} / \text{for all } x\in A \subseteq R\}$$
$$= \min\{\max\{\mu(x) / \text{for all } x\in A \subseteq R\}, \max\{\eta(x) / \text{for all } x\in A \subseteq R\}\}$$
$$= \min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}$$
$$\varphi^{\mu\cap\eta}(A) = (\lambda^{\mu}\cap \gamma^{\eta})(A), \text{ for any } A \in \Re.$$

Hence, $\varphi^{\mu\cap\eta} = \lambda^{\mu}\cap \gamma^{\eta}$.

K. Example

Let $C^0 = C – \{0\}$, where C is the set of all complex numbers.

For all a, b $\in C^0$, define the operators $\oplus$ and $\otimes$ on C^0 as $a \oplus b = ab$ and $a \otimes b = |a|^{\ln|b|}$.

Clearly ,(C^0, $\oplus$, $\otimes$) is a ring.

Define, a fuzzy set μ and η on C^0 as ,

$$\mu(x) = \mu(a+ib) = \begin{cases} 0.8 & \text{if} \quad a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if} \quad a < 0 \text{ and } b = 0 \\ 0.4 & \text{if} \quad b \neq 0. \end{cases}$$

$$\eta(x) = \eta(a+ib) = \begin{cases} 0.7 & \text{if} \quad a \geq 0 \text{ and } b = 0 \\ 0.5 & \text{if} \quad a < 0 \text{ and } b = 0 \\ 0.3 & \text{if} \quad b \neq 0. \end{cases}$$

where, a is the real part of x lies in X-axis and b is the imaginary part of x lies in the Y-axis.

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Clearly, μ and η are fuzzy subrings on C^0.

Let $I = (1, \infty)$ and $H = \{1, -1, i, -i\}$.Define $\Re = \{a \oplus I / a \in H\}$.

Now, $\Re = \{a \oplus I / a \in H\} = \{(1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i)\} = \{Q, A, B, C\}$.

For any X, Y $\in \Re$, define the operations $\oplus$ and $\otimes$ on $\Re$ as,

$$X \oplus Y = XY = \{xy / x \in X \text{ and } y \in Y\}$$

$$X \otimes Y = |X|^{\ln|Y|} = \{|x|^{\ln|y|} / x \in X \text{ and } y \in Y\}$$

Clearly, $(\Re, \oplus, \otimes)$ is a HX ring on $(P_0(C^0), \oplus, \otimes)$.

Define a fuzzy set $\lambda^\mu : \Re \to [0, 1]$ as,

$$\lambda^\mu(Q) = \text{Sup}\{\mu(x) / x \in Q\} = 0.8$$

$$\lambda^\mu(A) = \text{Sup}\{\mu(x) / x \in A\} = 0.6$$

$$\lambda^\mu(B) = \text{Sup}\{\mu(x) / x \in B\} = 0.4$$

$$\lambda^\mu(C) = \text{Sup}\{\mu(x) / x \in C\} = 0.4$$

Define a fuzzy set $\gamma^\eta : \Re \to [0, 1]$ as,

$$\gamma^\eta(Q) = \text{Sup}\{\eta(x) / x \in Q\} = 0.7$$

$$\gamma^\eta(A) = \text{Sup}\{\eta(x) / x \in A\} = 0.5$$

$$\gamma^\eta(B) = \text{Sup}\{\eta(x) / x \in B\} = 0.3$$

$$\gamma^\eta(C) = \text{Sup}\{\eta(x) / x \in C\} = 0.3.$$

Clearly, λ^μ and γ^η are fuzzy HX rings on $\Re$.

$$(\lambda^\mu \cap \gamma^\eta)(X) = \begin{cases} 0.7 & \text{if } X = Q \\ 0.5 & \text{if } X = A \\ 0.3 & \text{if } X = B, C. \end{cases}$$

Clearly, $(\lambda^\mu \cap \gamma^\eta)$ is a fuzzy HX ring on $\Re$.

Now,

$$(\mu \cap \eta)(x) = (\mu \cap \eta)(a + ib) = \begin{cases} 0.7 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.5 & \text{if } a < 0 \text{ and } b = 0 \\ 0.3 & \text{if } b \neq 0 \end{cases}$$

Clearly, $(\mu \cap \eta)$ is a fuzzy subring of R.

Clearly, $\varphi^{\mu \cap \eta}$ is a fuzzy HX subring on $\Re$.

$$\varphi^{\mu \cap \eta}(X) = \{(\mu \cap \eta)(x) / x \in X \subseteq R\} = \begin{cases} 0.7 & \text{if } X = Q \\ 0.5 & \text{if } X = A \\ 0.3 & \text{if } X = B, C. \end{cases}$$

Hence, $\varphi^{\mu \cap \eta} = \lambda^\mu \cap \gamma^\eta$.

L. Definition

Let μ and η are fuzzy subsets of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring of R. Let λ^μ and γ^η are fuzzy subsets of a HX ring $\Re$. The union of λ^μ and γ^η is defined as, $(\lambda^\mu \cup \gamma^\eta)(A) = \max\{\lambda^\mu(A), \gamma^\eta(A)\}$ for all $A \in \Re$.

M. Theorem

Let μ and η are fuzzy sets of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^μ and γ^η are any two fuzzy HX subrings of $\Re$ then $(\lambda^\mu \cup \gamma^\eta)$ is also a fuzzy HX subring of $\Re$.

Proof

1) $$\begin{aligned} (\lambda^\mu \cup \gamma^\eta)(A-B) &= \max\{\lambda^\mu(A-B), \gamma^\eta(A-B)\} \\ &\geq \max\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\ &= \min\{\max\{\lambda^\mu(A), \gamma^\eta(A)\}, \max\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\ &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \end{aligned}$$

$(\lambda^{\mu} \cup \gamma^{\eta})(A–B) \geq \min\{(\lambda^{\mu} \cup \gamma^{\eta})(A), (\lambda^{\mu} \cup \gamma^{\eta})(B)\}$.

2) $(\lambda^{\mu} \cup \gamma^{\eta})(AB) = \max\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\}$

$\geq \max\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\}$

$= \min\{\max\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}, \max\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\}$

$\geq \min\{(\lambda^{\mu} \cup \gamma^{\eta})(A), (\lambda^{\mu} \cup \gamma^{\eta})(B)\}$.

$(\lambda^{\mu} \cup \gamma^{\eta})(AB) \geq \min\{(\lambda^{\mu} \cup \gamma^{\eta})(A), (\lambda^{\mu} \cup \gamma^{\eta})(B)\}$.

Hence, union of two fuzzy HX subrings of a HX ring $\mathfrak{R}$ is a fuzzy HX subring of $\mathfrak{R}$.

N. *Remark*

1) Union of family of fuzzy HX subrings of a HX ring $\mathfrak{R}$ is also fuzzy HX subring of $\mathfrak{R}$.
2) Let R be a ring. Let μ and η be any two fuzzy subsets of R then $\varphi^{\mu \cup \eta}$ is a fuzzy HX subring of $\mathfrak{R}$ induced by the fuzzy subset $\mu \cup \eta$ of R.

O. *Theorem*

Let R be a ring. Let μ and η be any two fuzzy subsets of R. If λ^{μ}, γ^{η}, $\varphi^{\mu \cup \eta}$ are fuzzy HX subrings of a HX ring $\mathfrak{R}$ induced by μ, η, $\mu \cup \eta$ of R then $\varphi^{\mu \cup \eta} = \lambda^{\mu} \cup \gamma^{\eta}$.

Proof

Let λ^{μ} and γ^{η} be fuzzy HX subrings of $\mathfrak{R}$ then $\lambda^{\mu} \cup \gamma^{\eta}$ is a fuzzy HX subring of a HX ring $\mathfrak{R}$ by Theorem 2.2.13.

$\varphi^{\mu \cup \eta}$ is a fuzzy HX subring of $\mathfrak{R}$ induced by $\mu \cup \eta$ of R.

$\varphi^{\mu \cup \eta}(A) = \max\{(\mu \cup \eta)(x) \,/\, \text{for all } x \in A \subseteq R\}$

$= \max\{\max\{\mu(x), \eta(x)\} \,/\, \text{for all } x \in A \subseteq R\}$

$= \max\{\max\{\mu(x) \,/\, \text{for all } x \in A \subseteq R\}, \max\{\eta(x) \,/\, \text{for all } x \in A \subseteq R\}\}$

$= \max\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}$

$\varphi^{\mu \cup \eta}(A) = (\lambda^{\mu} \cup \gamma^{\eta})(A)$.

Therefore, $\varphi^{\mu \cup \eta} = \lambda^{\mu} \cup \gamma^{\eta}$.

P. *Definition*

Let μ and η be fuzzy subsets of the rings R_1 and R_2 respectively. Let λ^{μ} and γ^{η} be two fuzzy subsets of the HX rings $\mathfrak{R}_1$ and $\mathfrak{R}_2$ respectively then the cartesian product of λ^{μ} and γ^{η} is defined as $(\lambda^{\mu} \times \gamma^{\eta})(A, B) = \min\{\lambda^{\mu}(A), \gamma^{\eta}(B)\}$ for every $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$.

Q. *Theorem*

Let μ and η be any two fuzzy subsets of R_1 and R_2 respectively. Let $\mathfrak{R}_1 \subset 2^{R1} - \{\phi\}$ and $\mathfrak{R}_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings. If λ^{μ} and γ^{η} be fuzzy HX subrings of HX rings $\mathfrak{R}_1$ and $\mathfrak{R}_2$ respectively,
then $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX subring of a HX ring $\mathfrak{R}_1 \times \mathfrak{R}_2$.

Proof

Let $A, B \in \mathfrak{R}_1 \times \mathfrak{R}_2$ where $A = (C,D)$, $B = (E,F)$

1) $(\lambda^{\mu} \times \gamma^{\eta})(A - B) = (\lambda^{\mu} \times \gamma^{\eta})((C, D) - (E, F))$

$= (\lambda^{\mu} \times \gamma^{\eta})(C - E, D - F)$

$= \min\{\lambda^{\mu}(C - E), \gamma^{\eta}(D - F)\}$

$\geq \min\{\min\{\lambda^{\mu}(C), \lambda^{\mu}(E)\}, \min\{\gamma^{\eta}(D), \gamma^{\eta}(F)\}\}$

$= \min\{\min\{\lambda^{\mu}(C), \gamma^{\eta}(D)\}, \min\{\lambda^{\mu}(E), \gamma^{\eta}(F)\}\}$

$= \min\{(\lambda^{\mu} \times \gamma^{\eta})(C,D), (\lambda^{\mu} \times \gamma^{\eta})(E,F)\}$

$= \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\}$

$(\lambda^{\mu} \times \gamma^{\eta})(A–B) \geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\}$.

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2) $(\lambda^{\mu} \times \gamma^{\eta})(A\,B) = (\lambda^{\mu} \times \gamma^{\eta})((C,D),(E,F))$

$= (\lambda^{\mu} \times \gamma^{\eta})(CE, DF)$

$= \min\{\lambda^{\mu}(CE), \gamma^{\eta}(DF)\}$

$\geq \min\{\min\{\lambda^{\mu}(C), \lambda^{\mu}(E)\}, \min\{\gamma^{\eta}(D), \gamma^{\eta}(F)\}\}$

$= \min\{\min\{\lambda^{\mu}(C), \gamma^{\eta}(D)\}, \min\{\lambda^{\mu}(E), \gamma^{\eta}(F)\}\}$

$= \min\{(\lambda^{\mu} \times \gamma^{\eta})(C,D), (\lambda^{\mu} \times \gamma^{\eta})(E,F)\}$

$= \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\}$

$(\lambda^{\mu} \times \gamma^{\eta})(AB) \geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\}$

Hence, $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX subring of a HX ring $\Re$.

R. Theorem

Let λ^{μ} and γ^{η} be fuzzy subsets of the HX rings $\Re_1$ and $\Re_2$ respectively. Suppose that Q and Q^1 are identity elements of $\Re_1$ and $\Re_2$ respectively. If $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX subring of $\Re_1 \times \Re_2$, then atleast one of the following statements must hold

1) $\gamma^{\eta}(Q^1) \geq \lambda^{\mu}(A)$, for all $A \in \Re_1$
2) $\lambda^{\mu}(Q) \geq \gamma^{\eta}(B)$, for all $B \in \Re_2$.

Proof

Let $\lambda^{\mu} \times \gamma^{\eta}$ be a fuzzy HX subring of $\Re_1 \times \Re_2$. By contraposition, suppose that none of the statements (i) and (ii) holds then we can find $A \in \Re_1$ and $B \in \Re_2$ such that $\lambda^{\mu}(A) > \gamma^{\eta}(Q^1)$ and $\gamma^{\eta}(B) > \lambda^{\mu}(Q)$.

We have, $(\lambda^{\mu} \times \gamma^{\eta})(A,B) = \min\{\lambda^{\mu}(A), \gamma^{\eta}(B)\}$

$> \min\{\gamma^{\eta}(Q^1), \lambda^{\mu}(Q)\}$

$= \min\{\lambda^{\mu}(Q), \gamma^{\eta}(Q^1)\}$

$= (\lambda^{\mu} \times \gamma^{\eta})(Q, Q^1)$

$(\lambda^{\mu} \times \gamma^{\eta})(A,B) > (\lambda^{\mu} \times \gamma^{\eta})(Q, Q^1)$.

Thus, $\lambda^{\mu} \times \gamma^{\eta}$ is not a fuzzy HX subring of $\Re_1 \times \Re_2$.

Hence, either $\gamma^{\eta}(Q^1) \geq \lambda^{\mu}(A)$ for all $A \in \Re_1$ or $\lambda^{\mu}(Q) \geq \gamma^{\eta}(B)$, for all $B \in \Re_2$.

S. Theorem

Let λ^{μ} and γ^{η} be fuzzy subsets of the HX rings $\Re_1$ and $\Re_2$ respectively, such that $\lambda^{\mu}(A) \leq \gamma^{\eta}(Q^1)$ for all $A \in \Re_1$, Q^1 being the identity element of $\Re_2$. If $(\lambda^{\mu} \times \gamma^{\eta})$ is a fuzzy HX subring of $\Re_1 \times \Re_2$ then λ^{μ} is a fuzzy HX subring of $\Re_1$.

Proof

Let $\lambda^{\mu} \times \gamma^{\eta}$ be a fuzzy HX subring of $\Re_1 \times \Re_2$ and $A, B \in \Re_1$ then $(A, Q^1), (B, Q^1) \in \Re_1 \times \Re_2$.

Given, $\lambda^{\mu}(A) \leq \gamma^{\eta}(Q^1)$ for all $A \in \Re_1$.

1) $\lambda^{\mu}(A-B) = \min\{\lambda^{\mu}(A-B), \gamma^{\eta}(Q^1 - Q^1)\}$

$= (\lambda^{\mu} \times \gamma^{\eta})(A-B, Q^1 - Q^1)$

$= (\lambda^{\mu} \times \gamma^{\eta})((A, Q^1) - (B, Q^1))$

$\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(A, Q^1), (\lambda^{\mu} \times \gamma^{\eta})(B, Q^1)\}$

$= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(Q^1)\}, \min\{\lambda^{\mu}(B), \gamma^{\eta}(Q^1)\}\}$

$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$.

2) $\lambda^{\mu}(AB) = \min\{\lambda^{\mu}(AB), \gamma^{\eta}(Q^1 Q^1)\}$

$= (\lambda^{\mu} \times \gamma^{\eta})(AB, Q^1 Q^1)$

$= (\lambda^{\mu} \times \gamma^{\eta})((A, Q^1) \cdot (B, Q^1))$

$\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(A, Q^1), (\lambda^{\mu} \times \gamma^{\eta})(B, Q^1)\}$

$= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(Q^1)\}, \min\{\lambda^{\mu}(B), \gamma^{\eta}(Q^1)\}\}$

$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

Volume 4 Issue VI, June 2016
ISSN: 2321-9653

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$.

Hence, λ^{μ} is a fuzzy HX subring of $\mathfrak{R}_1$.

T. Theorem

Let λ^{μ} and γ^{η} be fuzzy subsets of the HX rings $\mathfrak{R}_1$ and $\mathfrak{R}_2$ respectively, such that $\gamma^{\eta}(A) \leq \lambda^{\mu}(Q)$ for all $A \in \mathfrak{R}_2$, Q being the identity element of $\mathfrak{R}_1$. If $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX sub ring of $\mathfrak{R}_1 \times \mathfrak{R}_2$ then γ^{η} is a fuzzy HX subring of $\mathfrak{R}_2$.

Proof

Let $\lambda^{\mu} \times \gamma^{\eta}$ be a fuzzy HX subring of $\mathfrak{R}_1 \times \mathfrak{R}_2$ and $A, B \in \mathfrak{R}_1$ then $(A, Q),(B, Q) \in \mathfrak{R}_1 \times \mathfrak{R}_2$.

Given, $\gamma^{\eta}(A) \leq \lambda^{\mu}(Q)$ for all $A \in \mathfrak{R}_2$

1)
$$\begin{aligned}\gamma^{\eta}(A-B) &= \min\{\lambda^{\mu}(Q-Q), \gamma^{\eta}(A-B)\}\\ &= (\lambda^{\mu} \times \gamma^{\eta})(Q-Q, A-B)\\ &= (\lambda^{\mu} \times \gamma^{\eta})((Q, A)-(Q, B))\\ &\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(Q, A), (\lambda^{\mu} \times \gamma^{\eta})(Q, B)\}\\ &= \min\{\min\{\lambda^{\mu}(Q),\gamma^{\eta}(A)\},\min\{\lambda^{\mu}(Q), \gamma^{\eta}(B)\}\}\\ &= \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\end{aligned}$$

$$\gamma^{\eta}(A-B) \geq \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}.$$

2)
$$\begin{aligned}\gamma^{\eta}(AB) &= \min\{\lambda^{\mu}(QQ), \gamma^{\eta}(AB)\}\\ &= (\lambda^{\mu} \times \gamma^{\eta})(QQ, AB)\\ &= (\lambda^{\mu} \times \gamma^{\eta})((Q, A)\cdot(Q, B))\\ &\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(Q, A), (\lambda^{\mu} \times \gamma^{\eta})(Q, B)\}\\ &= \min\{\min\{\lambda^{\mu}(Q),\gamma^{\eta}(A)\},\min\{\lambda^{\mu}(Q),\gamma^{\eta}(B)\}\}\\ &= \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\end{aligned}$$

$$\gamma^{\eta}(AB) \geq \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}.$$

Hence, γ^{η} is a fuzzy HX subring of $\mathfrak{R}_2$.

U. Corollary

Let λ^{μ} and γ^{η} be two fuzzy subsets of the HX rings $\mathfrak{R}_1$ and $\mathfrak{R}_2$ respectively. If $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX subring of $\mathfrak{R}_1 \times \mathfrak{R}_2$, then either λ^{μ} is a fuzzy HX subring of $\mathfrak{R}_1$ or γ^{η} is a fuzzy HX subring of $\mathfrak{R}_2$.

IV. CONCLUSIONS

The concept of a fuzzy HX ring and algebraic structure of a fuzzy sub HX ring of a HX ring were introduced. Also some related properties were investigated. The purpose of this study is to implement the fuzzy set theory and ring theory in fuzzy sub HX ring of a HX ring.

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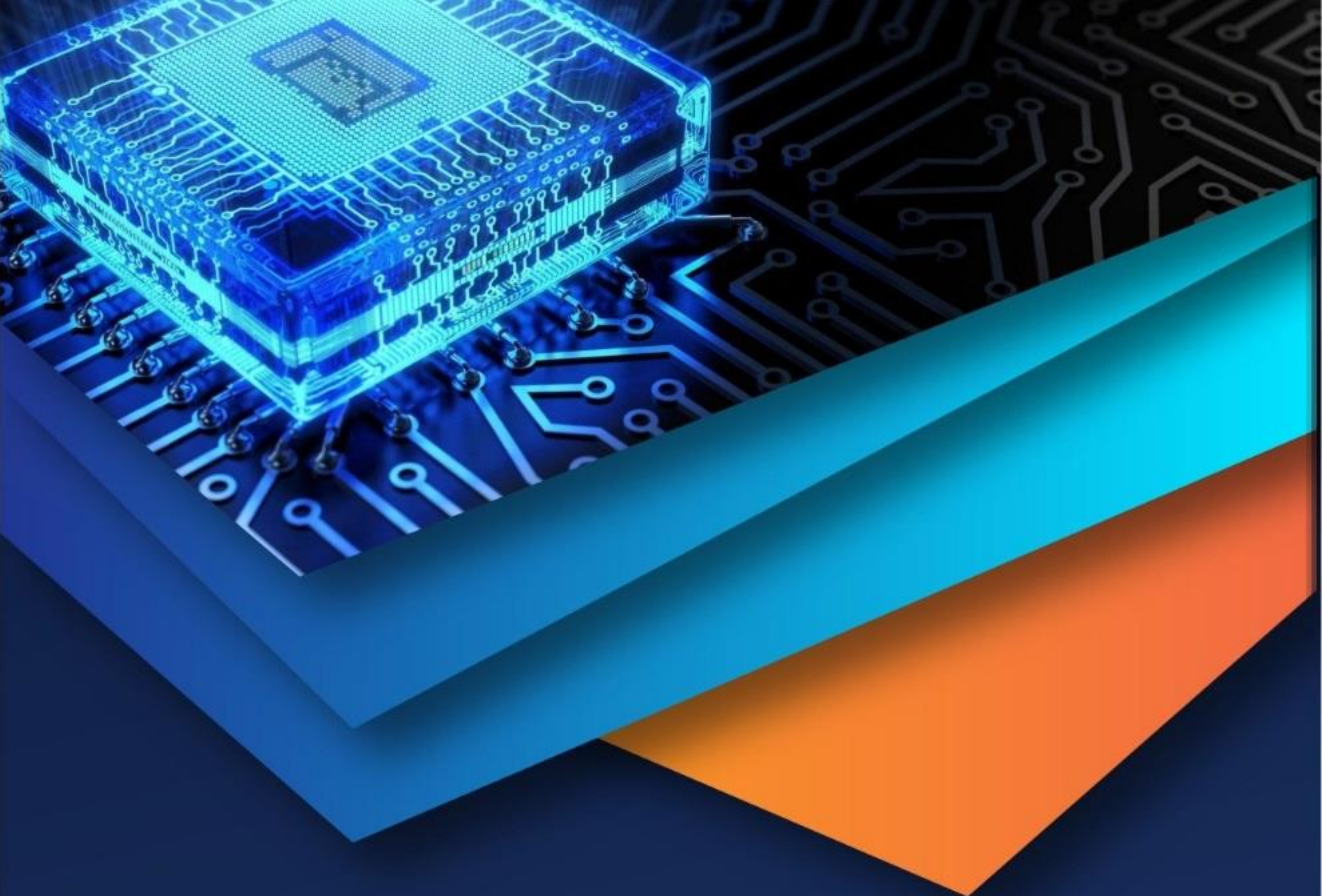

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