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# International Journal for Research in Applied Science & Engineering Technology (IJRASET)

## Fuzzy HX Subring of a HX Ring

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**Abstract**— In this paper, we define the concept of a fuzzy HX ring and define a new algebraic structure of a fuzzy HX subring of a HX ring. We also discuss some related properties of it.

**Keywords**— HX ring, fuzzy HX ring, fuzzy set, fuzzy subring, fuzzy HX subring,.

### I. INTRODUCTION

In 1965, Zadeh [10] introduced the concept of fuzzy subset  $\mu$  of a set  $X$  as a function from  $X$  into the closed unit interval  $0$  and  $1$  and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [9] defined the idea of fuzzy subgroups and gave some of its properties. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1,2] gave the structures of HX ring on a class of ring. In this paper we define a new algebraic structure of a fuzzy HX subring of a HX ring and investigate some related properties.

### II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper,  $R = (R, +, \cdot)$  is a Ring,  $e$  is the additive identity element of  $R$  and  $xy$ , we mean  $x \cdot y$ .

### III. PROPERTIES OF FUZZY HX RING

In this section we define the concept of fuzzy HX ring and discuss some related results.

#### A. Definition

Let  $R$  be a ring. Let  $\mu$  be a fuzzy set defined on  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\emptyset\}$  be a HX ring. A fuzzy subset  $\lambda^\mu$  of  $\mathfrak{R}$  is called a fuzzy HX ring on  $\mathfrak{R}$  or a fuzzy ring induced by  $\mu$  if the following conditions are satisfied. For all  $A, B \in \mathfrak{R}$ ,

- i.  $\lambda^\mu(A - B) \geq \min \{ \lambda^\mu(A), \lambda^\mu(B) \}$ ,
- ii.  $\lambda^\mu(AB) \geq \min \{ \lambda^\mu(A), \lambda^\mu(B) \}$ ,

where  $\lambda^\mu(A) = \max \{ \mu(x) / \text{for all } x \in A \subseteq R \}$ .

#### B. Remark

For a fuzzy HX subring  $\lambda^\mu$  of a HX ring  $\mathfrak{R}$ , the following result is obvious.

- i.  $\lambda^\mu(A) \leq \lambda^\mu(0)$  and  $\lambda^\mu(A) = \lambda^\mu(-A)$ , for all  $A \in \mathfrak{R}$ .
- ii.  $\lambda^\mu(A - B) = 0$  implies that  $\lambda^\mu(A) = \lambda^\mu(B)$ .

#### C. Example

Let  $C^0 = C - \{0\}$ , where  $C$  is the set of all complex numbers.

For all  $a, b \in C^0$ , define the operators  $\oplus$  and  $\otimes$  on  $C^0$  as  $a \oplus b = ab$  and  $a \otimes b = |a|^{|b|}$ .

Clearly,  $(C^0, \oplus, \otimes)$  is a ring.

Define, a fuzzy set  $\mu$  on  $C^0$  as,

$$\mu(x) = \mu(a + ib) = \begin{cases} 0.8 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if } a < 0 \text{ and } b = 0 \\ 0.4 & \text{if } b \neq 0 \end{cases}$$

where,  $a$  is the real part of  $x$  lies in X-axis and  $b$  is the imaginary part of  $x$  lies in the Y-axis.

Then, Clearly  $\mu(x \otimes y) = \mu(x \oplus (-y)) \geq \min \{ \mu(x), \mu(y) \}$ ,

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$$\mu(x \otimes y) \geq \min \{ \mu(x), \mu(y) \}.$$

Clearly,  $\mu$  is a **fuzzy subring** on  $C^0$ .

Let  $I = (1, \infty)$  and  $H = \{ 1, -1, i, -i \}$

Define  $\mathfrak{R} = \{ a \oplus I / a \in H \}$ . For,

$$\begin{aligned} 1 \in H &\Rightarrow 1 \oplus I = 1 \cdot (1, \infty) = (1, \infty) \\ -1 \in H &\Rightarrow -1 \oplus I = -1 \cdot (1, \infty) = (-\infty, -1) \\ i \in H &\Rightarrow i \oplus I = i \cdot (1, \infty) = (i, \infty) \\ -i \in H &\Rightarrow -i \oplus I = -i \cdot (1, \infty) = (-\infty, -i). \end{aligned}$$

Now,  $\mathfrak{R} = \{ a \oplus I / a \in H \} = \{ (1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i) \} = \{ Q, A, B, C \}$ .

For any  $X, Y \in \mathfrak{R}$ , define the operations  $\oplus$  and  $\otimes$  on  $\mathfrak{R}$  as,

$$\begin{aligned} X \oplus Y &= XY = \{ xy / x \in X \text{ and } y \in Y \} \\ X \otimes Y &= |X|^{\ln|Y|} = \{ |x|^{\ln|y|} / x \in X \text{ and } y \in Y \} \end{aligned}$$

Then,

$\oplus$	Q	A	B	C
Q	Q	A	B	C
A	A	Q	C	B
B	B	C	A	Q
C	C	B	Q	A

$\otimes$	Q	A	B	C
Q	Q	Q	Q	Q
A	Q	Q	Q	Q
B	Q	Q	Q	Q
C	Q	Q	Q	Q

Clearly,  $(\mathfrak{R}, \oplus, \otimes)$  is a **HX ring** on  $(P_0(C^0), \oplus, \otimes)$ .

Define a fuzzy set  $\lambda^\mu: \mathfrak{R} \rightarrow [0, 1]$  as,

$$\begin{aligned} \lambda^\mu(Q) &= \text{Sup} \{ \mu(x) / x \in Q \} = 0.8 \\ \lambda^\mu(A) &= \text{Sup} \{ \mu(x) / x \in A \} = 0.6 \\ \lambda^\mu(B) &= \text{Sup} \{ \mu(x) / x \in B \} = 0.4 \\ \lambda^\mu(C) &= \text{Sup} \{ \mu(x) / x \in C \} = 0.4 \end{aligned}$$

Now,  $-Q = A; -A = Q; -B = C; -C = B$ .

$$\begin{aligned} Q \otimes Q &= Q \oplus A = A \\ Q \otimes A &= Q \oplus Q = Q \\ Q \otimes B &= Q \oplus C = C \\ Q \otimes C &= Q \oplus B = B \end{aligned}$$

$$\begin{aligned} A \otimes Q &= A \oplus A = Q \\ A \otimes A &= A \oplus Q = A \\ A \otimes B &= A \oplus C = B \\ A \otimes C &= A \oplus B = C \end{aligned}$$

$$\begin{aligned} B \otimes Q &= B \oplus A = C \\ B \otimes A &= B \oplus Q = B \\ B \otimes B &= B \oplus C = Q \\ B \otimes C &= B \oplus B = A \end{aligned}$$

$$C \otimes Q = C \oplus A = B$$

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$$C \oplus A = C \oplus Q = C$$

$$C \oplus B = C \oplus C = A$$

$$C \oplus C = C \oplus B = Q$$

For any  $X, Y \in \mathfrak{R}$ , we have,

$$\lambda^\mu(X \oplus Y) = \lambda^\mu(X \oplus (-Y)) \geq \min \{ \lambda^\mu(X), \lambda^\mu(Y) \},$$

$$\lambda^\mu(X \otimes Y) \geq \min \{ \lambda^\mu(X), \lambda^\mu(Y) \}.$$

Clearly,  $\lambda^\mu$  is a **fuzzy HX ring** on  $\mathfrak{R}$ .

### D. Theorem

If  $\mu$  is a fuzzy subring of a ring  $R$  then the fuzzy subset  $\lambda^\mu$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$ .

### Proof

Let  $\mu$  be a fuzzy subring of  $R$ .

$$\begin{aligned} \text{i. } \min\{\lambda^\mu(A), \lambda^\mu(B)\} &= \min\{\max\{\mu(x) / \text{for all } x \in A \subseteq R\}, \\ &\quad \max\{\mu(y) / \text{for all } y \in B \subseteq R\}\} \\ &= \min\{\mu(x_0), \mu(y_0)\} \\ &\leq \mu(x_0 - y_0), \text{ since } \mu \text{ is a fuzzy subring of } R \\ &\leq \max\{\mu(x-y) / \text{for all } x-y \in A-B \subseteq R\} \\ &\leq \lambda^\mu(A-B) \\ \lambda^\mu(A-B) &\geq \min\{\lambda^\mu(A), \lambda^\mu(B)\} \\ \text{ii. } \min\{\lambda^\mu(A), \lambda^\mu(B)\} &= \min\{\max\{\mu(x) / \text{for all } x \in A \subseteq R\}, \\ &\quad \max\{\mu(y) / \text{for all } y \in B \subseteq R\}\} \\ &= \min\{\mu(x_0), \mu(y_0)\} \\ &\leq \mu(x_0 y_0), \text{ since } \mu \text{ is a fuzzy subring of } R \\ &\leq \max\{\mu(xy) / \text{for all } xy \in AB \subseteq R\} \\ &\leq \lambda^\mu(AB) \\ \lambda^\mu(AB) &\geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}. \end{aligned}$$

Hence,  $\lambda^\mu$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$ .

### E. Remark

- 1) If  $\mu$  is not a fuzzy subring of  $R$  then the fuzzy set  $\lambda^\mu$  of  $\mathfrak{R}$  is a fuzzy HX subring of  $\mathfrak{R}$ , provided  $|X| \geq 2$  for all  $X \in \mathfrak{R}$ .
- 2) If  $\mu$  is a fuzzy subset of a ring  $R$  and  $\lambda^\mu$  be a fuzzy HX subring of  $\mathfrak{R}$ , such that  $\lambda^\mu(A) = \max\{\mu(x) / \text{for all } x \in A \subseteq R\}$ , then  $\mu$  may or may not be a fuzzy subring of  $R$ , which can be illustrated by the following example.

### F. Example

Let  $C^0 = C - \{0\}$ , where  $C$  is the set of all complex numbers.

For all  $a, b \in C^0$ , define the operators  $\oplus$  and  $\otimes$  on  $C^0$  as

$$a \oplus b = ab \text{ and } a \otimes b = |a|^{\ln|b|}.$$

Clearly,  $(C^0, \oplus, \otimes)$  is a ring.

Define, a fuzzy set  $\mu$  on  $C^0$  as,

$$\mu(x) = \mu(a + ib) = \begin{cases} 0.8 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if } a < 0 \text{ and } b = 0 \\ 0.4 & \text{if } b > 0 \\ 0.3 & \text{if } b < 0. \end{cases}$$

where,  $a$  is the real part of  $x$  lies in  $X$ -axis and  $b$  is the imaginary part of  $x$  lies in the  $Y$ -axis.

Let  $x = 2+3i$  and  $y = -2$ .

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Then,  $\mu (x \ominus y) = \mu (x \oplus (-y)) = \mu (-4 - 6i) = 0.3$ .

$$\min \{ \mu (x), \mu (y) \} = \min \{ 0.4, 0.6 \} = 0.4.$$

Hence,  $\mu (x \ominus y) \neq \min \{ \mu (x), \mu (y) \}$ ,

Clearly,  $\mu$  is not a fuzzy subring on  $C^0$ .

Let  $I = (1, \infty)$  and  $H = \{ 1, -1, i, -i \}$

Define  $\mathfrak{R} = \{ a \oplus I / a \in H \}$ .

$$\begin{aligned} \text{For, } 1 \in H &\Rightarrow 1 \oplus I = 1 \cdot (1, \infty) = (1, \infty) \\ -1 \in H &\Rightarrow -1 \oplus I = -1 \cdot (1, \infty) = (-\infty, -1) \\ i \in H &\Rightarrow i \oplus I = i \cdot (1, \infty) = (i, \infty) \\ -i \in H &\Rightarrow -i \oplus I = -i \cdot (1, \infty) = (-\infty, -i). \end{aligned}$$

Now,  $\mathfrak{R} = \{ a \oplus I / a \in H \} = \{ (1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i) \}$

Let  $\mathfrak{R} = \{ \{ (1, \infty), (-\infty, -1) \}, \{ (i, \infty), (-\infty, -i) \} \} = \{ Q, A \}$ .

For any  $X, Y \in \mathfrak{R}$ , define the operations  $\oplus$  and  $\otimes$  on  $\mathfrak{R}$  as,

$$X \oplus Y = XY = \{ xy / x \in X \text{ and } y \in Y \}$$

$$X \otimes Y = |X|^{\ln|Y|} = \{ |x|^{\ln|y|} / x \in X \text{ and } y \in Y \}$$

Then,

$\oplus$	Q	A
Q	Q	A
A	A	Q

$\otimes$	Q	A
Q	Q	Q
A	Q	Q

Clearly,  $(\mathfrak{R}, \oplus, \otimes)$  is a HX ring on  $(P_0(C^0), \oplus, \otimes)$ .

Define a fuzzy set  $\lambda^\mu: \mathfrak{R} \rightarrow [0, 1]$  as,

$$\lambda^\mu(Q) = \max \{ \mu(x) / x \in Q \} = 0.8$$

$$\lambda^\mu(A) = \max \{ \mu(x) / x \in A \} = 0.4$$

Now,  $-Q = Q; -A = A$ .

$$Q \oplus Q = Q \oplus Q = Q$$

$$Q \oplus A = Q \oplus A = A$$

$$A \oplus Q = A \oplus Q = A$$

$$A \oplus A = A \oplus A = Q$$

For any  $X, Y \in \mathfrak{R}$ , we have,

$$\lambda^\mu(X \oplus Y) = \lambda^\mu(X \oplus (-Y)) \geq \min \{ \lambda^\mu(X), \lambda^\mu(Y) \},$$

$$\lambda^\mu(X \otimes Y) \geq \min \{ \lambda^\mu(X), \lambda^\mu(Y) \}.$$

Clearly,  $\lambda^\mu$  is a fuzzy HX ring on  $\mathfrak{R}$ .

### G. Definition

Let  $R$  be a ring. Let  $\mu$  and  $\eta$  be any two fuzzy subsets of  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\emptyset\}$  be a HX ring on  $R$ . Let  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy subsets of  $\mathfrak{R}$ . The intersection of  $\lambda^\mu$  and  $\gamma^\eta$  is defined as  $(\lambda^\mu \cap \gamma^\eta)(A) = \min \{ \lambda^\mu(A), \gamma^\eta(A) \}$  for all  $A \in \mathfrak{R}$ .

### H. Theorem

Let  $\mu$  and  $\eta$  be any two fuzzy sets defined on  $R$ . Let  $\lambda^\mu$  and  $\gamma^\eta$  be any two fuzzy HX subrings of a HX ring  $\mathfrak{R}$  then the intersection of two fuzzy HX subrings,  $\lambda^\mu \cap \gamma^\eta$  is also a fuzzy HX subring of a HX ring  $\mathfrak{R}$ .

*Proof*

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Let  $A, B \in \mathfrak{R}$ .

$$\begin{aligned}
 1) \quad (\lambda^\mu \cap \gamma^\eta)(A-B) &= \min\{\lambda^\mu(A-B), \gamma^\eta(A-B)\} \\
 &\geq \min\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\
 &= \min\{\min\{\lambda^\mu(A), \gamma^\eta(A)\}, \min\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\
 &\geq \min\{(\lambda^\mu \cap \gamma^\eta)(A), (\lambda^\mu \cap \gamma^\eta)(B)\}. \\
 (\lambda^\mu \cap \gamma^\eta)(A-B) &\geq \min\{(\lambda^\mu \cap \gamma^\eta)(A), (\lambda^\mu \cap \gamma^\eta)(B)\}. \\
 2) \quad (\lambda^\mu \cap \gamma^\eta)(AB) &= \min\{\lambda^\mu(AB), \gamma^\eta(AB)\} \\
 &\geq \min\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\
 &= \min\{\min\{\lambda^\mu(A), \gamma^\eta(A)\}, \min\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\
 &\geq \min\{(\lambda^\mu \cap \gamma^\eta)(A), (\lambda^\mu \cap \gamma^\eta)(B)\}. \\
 (\lambda^\mu \cap \gamma^\eta)(AB) &\geq \min\{(\lambda^\mu \cap \gamma^\eta)(A), (\lambda^\mu \cap \gamma^\eta)(B)\}.
 \end{aligned}$$

Hence,  $\lambda^\mu \cap \gamma^\eta$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$ .

### I. Remark

- 1) The intersection of family of fuzzy HX subrings of a HX ring  $\mathfrak{R}$  is also fuzzy HX subring of  $\mathfrak{R}$ .
- 2) Let  $R$  be a ring. Let  $\mu$  and  $\eta$  be any two fuzzy sets of  $R$  and  $\mu \cap \eta$  is also a fuzzy set of  $R$  then  $\phi^{\mu \cap \eta}$  is a fuzzy HX subring of  $\mathfrak{R}$  induced by  $\mu \cap \eta$  of  $R$ .

### J. Theorem

If  $\lambda^\mu, \gamma^\eta, \phi^{\mu \cap \eta}$  are fuzzy HX subrings of a HX ring  $\mathfrak{R}$  induced by the fuzzy sets  $\mu, \eta, \mu \cap \eta$  of  $R$  then  $\phi^{\mu \cap \eta} = \lambda^\mu \cap \gamma^\eta$ .

*Proof*

Let  $\lambda^\mu$  and  $\gamma^\eta$  are fuzzy HX subrings of  $\mathfrak{R}$  then  $\lambda^\mu \cap \gamma^\eta$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$  by Theorem 3.8.  $\phi^{\mu \cap \eta}$  is a fuzzy HX subring of  $\mathfrak{R}$  induced by  $\mu \cap \eta$  of  $R$  by Theorem 3.4.

$$\begin{aligned}
 \phi^{\mu \cap \eta}(A) &= \max\{(\mu \cap \eta)(x) / \text{for all } x \in A \subseteq R\} \\
 &= \max\{\min\{\mu(x), \eta(x)\} / \text{for all } x \in A \subseteq R\} \\
 &= \min\{\max\{\mu(x) / \text{for all } x \in A \subseteq R\}, \\
 &\quad \max\{\eta(x) / \text{for all } x \in A \subseteq R\}\} \\
 &= \min\{\lambda^\mu(A), \gamma^\eta(A)\} \\
 \phi^{\mu \cap \eta}(A) &= (\lambda^\mu \cap \gamma^\eta)(A), \text{ for any } A \in \mathfrak{R}.
 \end{aligned}$$

Hence,  $\phi^{\mu \cap \eta} = \lambda^\mu \cap \gamma^\eta$ .

### K. Example

Let  $C^0 = C - \{0\}$ , where  $C$  is the set of all complex numbers.

For all  $a, b \in C^0$ , define the operators  $\oplus$  and  $\otimes$  on  $C^0$  as  $a \oplus b = ab$  and  $a \otimes b = |a|^{\ln|b|}$ .

Clearly,  $(C^0, \oplus, \otimes)$  is a ring.

Define, a fuzzy set  $\mu$  and  $\eta$  on  $C^0$  as,

$$\mu(x) = \mu(a + ib) = \begin{cases} 0.8 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.6 & \text{if } a < 0 \text{ and } b = 0 \\ 0.4 & \text{if } b \neq 0. \end{cases}$$

$$\eta(x) = \eta(a + ib) = \begin{cases} 0.7 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.5 & \text{if } a < 0 \text{ and } b = 0 \\ 0.3 & \text{if } b \neq 0. \end{cases}$$

where,  $a$  is the real part of  $x$  lies in X-axis and  $b$  is the imaginary part of  $x$  lies in the Y-axis.



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Clearly,  $\mu$  and  $\eta$  are fuzzy subrings on  $C^0$ .

Let  $I = (1, \infty)$  and  $H = \{1, -1, i, -i\}$ . Define  $\mathfrak{R} = \{a \oplus I / a \in H\}$ .

Now,  $\mathfrak{R} = \{a \oplus I / a \in H\} = \{(1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i)\} = \{Q, A, B, C\}$ .

For any  $X, Y \in \mathfrak{R}$ , define the operations  $\oplus$  and  $\otimes$  on  $\mathfrak{R}$  as,

$$\begin{aligned} X \oplus Y &= XY = \{xy / x \in X \text{ and } y \in Y\} \\ X \otimes Y &= |X|^{|\ln|Y||} = \{|x|^{|ln|y||} / x \in X \text{ and } y \in Y\} \end{aligned}$$

Clearly,  $(\mathfrak{R}, \oplus, \otimes)$  is a HX ring on  $(P_0(C^0), \oplus, \otimes)$ .

Define a fuzzy set  $\lambda^\mu: \mathfrak{R} \rightarrow [0, 1]$  as,

$$\begin{aligned} \lambda^\mu(Q) &= \text{Sup} \{ \mu(x) / x \in Q \} = 0.8 \\ \lambda^\mu(A) &= \text{Sup} \{ \mu(x) / x \in A \} = 0.6 \\ \lambda^\mu(B) &= \text{Sup} \{ \mu(x) / x \in B \} = 0.4 \\ \lambda^\mu(C) &= \text{Sup} \{ \mu(x) / x \in C \} = 0.4 \end{aligned}$$

Define a fuzzy set  $\gamma^\eta: \mathfrak{R} \rightarrow [0, 1]$  as,

$$\begin{aligned} \gamma^\eta(Q) &= \text{Sup} \{ \eta(x) / x \in Q \} = 0.7 \\ \gamma^\eta(A) &= \text{Sup} \{ \eta(x) / x \in A \} = 0.5 \\ \gamma^\eta(B) &= \text{Sup} \{ \eta(x) / x \in B \} = 0.3 \\ \gamma^\eta(C) &= \text{Sup} \{ \eta(x) / x \in C \} = 0.3. \end{aligned}$$

Clearly,  $\lambda^\mu$  and  $\gamma^\eta$  are fuzzy HX rings on  $\mathfrak{R}$ .

$$(\lambda^\mu \cap \gamma^\eta)(X) = \begin{cases} 0.7 & \text{if } X = Q \\ 0.5 & \text{if } X = A \\ 0.3 & \text{if } X = B, C. \end{cases}$$

Clearly,  $(\lambda^\mu \cap \gamma^\eta)$  is a fuzzy HX ring on  $\mathfrak{R}$ .

Now,

$$(\mu \cap \eta)(x) = (\mu \cap \eta)(a + ib) = \begin{cases} 0.7 & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.5 & \text{if } a < 0 \text{ and } b = 0 \\ 0.3 & \text{if } b \neq 0 \end{cases}$$

Clearly,  $(\mu \cap \eta)$  is a fuzzy subring of  $R$ .

Clearly,  $\phi^{\mu \cap \eta}$  is a fuzzy HX subring on  $\mathfrak{R}$ .

$$\phi^{\mu \cap \eta}(X) = \{ (\mu \cap \eta)(x) / x \in X \subseteq R \} = \begin{cases} 0.7 & \text{if } X = Q \\ 0.5 & \text{if } X = A \\ 0.3 & \text{if } X = B, C. \end{cases}$$

Hence,  $\phi^{\mu \cap \eta} = \lambda^\mu \cap \gamma^\eta$ .

### L. Definition

Let  $\mu$  and  $\eta$  are fuzzy subsets of  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\emptyset\}$  be a HX ring of  $R$ . Let  $\lambda^\mu$  and  $\gamma^\eta$  are fuzzy subsets of a HX ring  $\mathfrak{R}$ . The union of  $\lambda^\mu$  and  $\gamma^\eta$  is defined as,  $(\lambda^\mu \cup \gamma^\eta)(A) = \max\{\lambda^\mu(A), \gamma^\eta(A)\}$  for all  $A \in \mathfrak{R}$ .

### M. Theorem

Let  $\mu$  and  $\eta$  are fuzzy sets of  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\emptyset\}$  be a HX ring. If  $\lambda^\mu$  and  $\gamma^\eta$  are any two fuzzy HX subrings of  $\mathfrak{R}$  then  $(\lambda^\mu \cup \gamma^\eta)$  is also a fuzzy HX subring of  $\mathfrak{R}$ .

*Proof*

$$\begin{aligned} 1) (\lambda^\mu \cup \gamma^\eta)(A-B) &= \max\{\lambda^\mu(A-B), \gamma^\eta(A-B)\} \\ &\geq \max\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\ &= \min\{\max\{\lambda^\mu(A), \gamma^\eta(A)\}, \max\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\ &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \end{aligned}$$

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$$\begin{aligned}
 (\lambda^\mu \cup \gamma^\eta)(A-B) &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \\
 2) (\lambda^\mu \cup \gamma^\eta)(AB) &= \max\{\lambda^\mu(AB), \gamma^\eta(AB)\} \\
 &\geq \max\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\
 &= \min\{\max\{\lambda^\mu(A), \gamma^\eta(A)\}, \max\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\
 &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \\
 (\lambda^\mu \cup \gamma^\eta)(AB) &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}.
 \end{aligned}$$

Hence, union of two fuzzy HX subrings of a HX ring  $\mathfrak{R}$  is a fuzzy HX subring of  $\mathfrak{R}$ .

*N. Remark*

- 1) Union of family of fuzzy HX subrings of a HX ring  $\mathfrak{R}$  is also fuzzy HX subring of  $\mathfrak{R}$ .
- 2) Let R be a ring. Let  $\mu$  and  $\eta$  be any two fuzzy subsets of R then  $\phi^{\mu \cup \eta}$  is a fuzzy HX subring of  $\mathfrak{R}$  induced by the fuzzy subset  $\mu \cup \eta$  of R.

*O. Theorem*

Let R be a ring. Let  $\mu$  and  $\eta$  be any two fuzzy subsets of R. If  $\lambda^\mu, \gamma^\eta, \phi^{\mu \cup \eta}$  are fuzzy HX subrings of a HX ring  $\mathfrak{R}$  induced by  $\mu, \eta, \mu \cup \eta$  of R then  $\phi^{\mu \cup \eta} = \lambda^\mu \cup \gamma^\eta$ .

*Proof*

Let  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy HX subrings of  $\mathfrak{R}$  then  $\lambda^\mu \cup \gamma^\eta$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$  by Theorem 2.2.13.  $\phi^{\mu \cup \eta}$  is a fuzzy HX subring of  $\mathfrak{R}$  induced by  $\mu \cup \eta$  of R.

$$\begin{aligned}
 \phi^{\mu \cup \eta}(A) &= \max\{(\mu \cup \eta)(x) / \text{for all } x \in A \subseteq R\} \\
 &= \max\{\max\{\mu(x), \eta(x)\} / \text{for all } x \in A \subseteq R\} \\
 &= \max\{\max\{\mu(x) / \text{for all } x \in A \subseteq R\}, \max\{\eta(x) / \text{for all } x \in A \subseteq R\}\} \\
 &= \max\{\lambda^\mu(A), \gamma^\eta(A)\} \\
 \phi^{\mu \cup \eta}(A) &= (\lambda^\mu \cup \gamma^\eta)(A). \\
 \text{Therefore, } \phi^{\mu \cup \eta} &= \lambda^\mu \cup \gamma^\eta.
 \end{aligned}$$

*P. Definition*

Let  $\mu$  and  $\eta$  be fuzzy subsets of the rings  $R_1$  and  $R_2$  respectively. Let  $\lambda^\mu$  and  $\gamma^\eta$  be two fuzzy subsets of the HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively then the cartesian product of  $\lambda^\mu$  and  $\gamma^\eta$  is defined as  $(\lambda^\mu \times \gamma^\eta)(A, B) = \min\{\lambda^\mu(A), \gamma^\eta(B)\}$  for every  $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ .

*Q. Theorem*

Let  $\mu$  and  $\eta$  be any two fuzzy subsets of  $R_1$  and  $R_2$  respectively. Let  $\mathfrak{R}_1 \subset 2^{R_1} - \{\phi\}$  and  $\mathfrak{R}_2 \subset 2^{R_2} - \{\phi\}$  be any two HX rings. If  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy HX subrings of HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively, then  $\lambda^\mu \times \gamma^\eta$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}_1 \times \mathfrak{R}_2$ .

*Proof*

$$\begin{aligned}
 \text{Let } A, B \in \mathfrak{R}_1 \times \mathfrak{R}_2 \text{ where } A = (C,D), B = (E,F) \\
 1) (\lambda^\mu \times \gamma^\eta)(A - B) &= (\lambda^\mu \times \gamma^\eta)((C, D) - (E, F)) \\
 &= (\lambda^\mu \times \gamma^\eta)(C - E, D - F) \\
 &= \min\{\lambda^\mu(C - E), \gamma^\eta(D - F)\} \\
 &\geq \min\{\min\{\lambda^\mu(C), \lambda^\mu(E)\}, \min\{\gamma^\eta(D), \gamma^\eta(F)\}\} \\
 &= \min\{\min\{\lambda^\mu(C), \gamma^\eta(D)\}, \min\{\lambda^\mu(E), \gamma^\eta(F)\}\} \\
 &= \min\{(\lambda^\mu \times \gamma^\eta)(C,D), (\lambda^\mu \times \gamma^\eta)(E,F)\} \\
 &= \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\} \\
 (\lambda^\mu \times \gamma^\eta)(A - B) &\geq \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\}.
 \end{aligned}$$



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$$\begin{aligned}
 2) \quad (\lambda^\mu \times \gamma^\eta)(A \ B) &= (\lambda^\mu \times \gamma^\eta)((C,D), (E,F)) \\
 &= (\lambda^\mu \times \gamma^\eta)(CE, DF) \\
 &= \min\{\lambda^\mu(CE), \gamma^\eta(DF)\} \\
 &\geq \min\{\min\{\lambda^\mu(C), \lambda^\mu(E)\}, \min\{\gamma^\eta(D), \gamma^\eta(F)\}\} \\
 &= \min\{\min\{\lambda^\mu(C), \gamma^\eta(D)\}, \min\{\lambda^\mu(E), \gamma^\eta(F)\}\} \\
 &= \min\{(\lambda^\mu \times \gamma^\eta)(C,D), (\lambda^\mu \times \gamma^\eta)(E,F)\} \\
 &= \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\} \\
 (\lambda^\mu \times \gamma^\eta)(AB) &\geq \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\}
 \end{aligned}$$

Hence,  $\lambda^\mu \times \gamma^\eta$  is a fuzzy HX subring of a HX ring  $\mathfrak{R}$ .

### R. Theorem

Let  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy subsets of the HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively. Suppose that Q and  $Q^1$  are identity elements of  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively. If  $\lambda^\mu \times \gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$ , then at least one of the following statements must hold

- 1)  $\gamma^\eta(Q^1) \geq \lambda^\mu(A)$ , for all  $A \in \mathfrak{R}_1$
- 2)  $\lambda^\mu(Q) \geq \gamma^\eta(B)$ , for all  $B \in \mathfrak{R}_2$ .

### Proof

Let  $\lambda^\mu \times \gamma^\eta$  be a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$ . By contraposition, suppose that none of the statements (i) and (ii) holds then we can find  $A \in \mathfrak{R}_1$  and  $B \in \mathfrak{R}_2$  such that  $\lambda^\mu(A) > \gamma^\eta(Q^1)$  and  $\gamma^\eta(B) > \lambda^\mu(Q)$ .

$$\begin{aligned}
 \text{We have, } (\lambda^\mu \times \gamma^\eta)(A, B) &= \min\{\lambda^\mu(A), \gamma^\eta(B)\} \\
 &> \min\{\gamma^\eta(Q^1), \lambda^\mu(Q)\} \\
 &= \min\{\lambda^\mu(Q), \gamma^\eta(Q^1)\} \\
 &= (\lambda^\mu \times \gamma^\eta)(Q, Q^1) \\
 (\lambda^\mu \times \gamma^\eta)(A, B) &> (\lambda^\mu \times \gamma^\eta)(Q, Q^1).
 \end{aligned}$$

Thus,  $\lambda^\mu \times \gamma^\eta$  is not a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$ .

Hence, either  $\gamma^\eta(Q^1) \geq \lambda^\mu(A)$  for all  $A \in \mathfrak{R}_1$  or  $\lambda^\mu(Q) \geq \gamma^\eta(B)$ , for all  $B \in \mathfrak{R}_2$ .

### S. Theorem

Let  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy subsets of the HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively, such that  $\lambda^\mu(A) \leq \gamma^\eta(Q^1)$  for all  $A \in \mathfrak{R}_1$ ,  $Q^1$  being the identity element of  $\mathfrak{R}_2$ . If  $(\lambda^\mu \times \gamma^\eta)$  is a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$  then  $\lambda^\mu$  is a fuzzy HX subring of  $\mathfrak{R}_1$ .

### Proof

Let  $\lambda^\mu \times \gamma^\eta$  be a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$  and  $A, B \in \mathfrak{R}_1$  then  $(A, Q^1), (B, Q^1) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ .

Given,  $\lambda^\mu(A) \leq \gamma^\eta(Q^1)$  for all  $A \in \mathfrak{R}_1$ .

$$\begin{aligned}
 1) \quad \lambda^\mu(A-B) &= \min\{\lambda^\mu(A-B), \gamma^\eta(Q^1-Q^1)\} \\
 &= (\lambda^\mu \times \gamma^\eta)(A-B, Q^1-Q^1) \\
 &= (\lambda^\mu \times \gamma^\eta)((A, Q^1) - (B, Q^1)) \\
 &\geq \min\{(\lambda^\mu \times \gamma^\eta)(A, Q^1), (\lambda^\mu \times \gamma^\eta)(B, Q^1)\} \\
 &= \min\{\min\{\lambda^\mu(A), \gamma^\eta(Q^1)\}, \min\{\lambda^\mu(B), \gamma^\eta(Q^1)\}\} \\
 &= \min\{\lambda^\mu(A), \lambda^\mu(B)\} \\
 \lambda^\mu(A-B) &\geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}. \\
 2) \quad \lambda^\mu(AB) &= \min\{\lambda^\mu(AB), \gamma^\eta(Q^1 Q^1)\} \\
 &= (\lambda^\mu \times \gamma^\eta)(AB, Q^1 Q^1) \\
 &= (\lambda^\mu \times \gamma^\eta)((A, Q^1) \cdot (B, Q^1)) \\
 &\geq \min\{(\lambda^\mu \times \gamma^\eta)(A, Q^1), (\lambda^\mu \times \gamma^\eta)(B, Q^1)\} \\
 &= \min\{\min\{\lambda^\mu(A), \gamma^\eta(Q^1)\}, \min\{\lambda^\mu(B), \gamma^\eta(Q^1)\}\} \\
 &= \min\{\lambda^\mu(A), \lambda^\mu(B)\}
 \end{aligned}$$

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$$\lambda^\mu (AB) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}.$$

Hence,  $\lambda^\mu$  is a fuzzy HX subring of  $\mathfrak{R}_1$ .

### T. Theorem

Let  $\lambda^\mu$  and  $\gamma^\eta$  be fuzzy subsets of the HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively, such that  $\gamma^\eta(A) \leq \lambda^\mu(Q)$  for all  $A \in \mathfrak{R}_2$ ,  $Q$  being the identity element of  $\mathfrak{R}_1$ . If  $\lambda^\mu \times \gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$  then  $\gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_2$ .

*Proof*

Let  $\lambda^\mu \times \gamma^\eta$  be a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$  and  $A, B \in \mathfrak{R}_1$  then  $(A, Q), (B, Q) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ .

Given,  $\gamma^\eta(A) \leq \lambda^\mu(Q)$  for all  $A \in \mathfrak{R}_2$

$$\begin{aligned} 1) \quad \gamma^\eta(A-B) &= \min\{\lambda^\mu(Q-Q), \gamma^\eta(A-B)\} \\ &= (\lambda^\mu \times \gamma^\eta)(Q-Q, A-B) \\ &= (\lambda^\mu \times \gamma^\eta)((Q, A) - (Q, B)) \\ &\geq \min\{(\lambda^\mu \times \gamma^\eta)(Q, A), (\lambda^\mu \times \gamma^\eta)(Q, B)\} \\ &= \min\{\min\{\lambda^\mu(Q), \gamma^\eta(A)\}, \min\{\lambda^\mu(Q), \gamma^\eta(B)\}\} \\ &= \min\{\gamma^\eta(A), \gamma^\eta(B)\} \end{aligned}$$

$$\gamma^\eta(A-B) \geq \min\{\gamma^\eta(A), \gamma^\eta(B)\}.$$

$$\begin{aligned} 2) \quad \gamma^\eta(AB) &= \min\{\lambda^\mu(QQ), \gamma^\eta(AB)\} \\ &= (\lambda^\mu \times \gamma^\eta)(QQ, AB) \\ &= (\lambda^\mu \times \gamma^\eta)((Q, A) \cdot (Q, B)) \\ &\geq \min\{(\lambda^\mu \times \gamma^\eta)(Q, A), (\lambda^\mu \times \gamma^\eta)(Q, B)\} \\ &= \min\{\min\{\lambda^\mu(Q), \gamma^\eta(A)\}, \min\{\lambda^\mu(Q), \gamma^\eta(B)\}\} \\ &= \min\{\gamma^\eta(A), \gamma^\eta(B)\} \end{aligned}$$

$$\gamma^\eta(AB) \geq \min\{\gamma^\eta(A), \gamma^\eta(B)\}.$$

Hence,  $\gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_2$ .

### U. Corollary

Let  $\lambda^\mu$  and  $\gamma^\eta$  be two fuzzy subsets of the HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively. If  $\lambda^\mu \times \gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_1 \times \mathfrak{R}_2$ , then either  $\lambda^\mu$  is a fuzzy HX subring of  $\mathfrak{R}_1$  or  $\gamma^\eta$  is a fuzzy HX subring of  $\mathfrak{R}_2$ .

## IV. CONCLUSIONS

The concept of a fuzzy HX ring and algebraic structure of a fuzzy sub HX ring of a HX ring were introduced. Also some related properties were investigated. The purpose of this study is to implement the fuzzy set theory and ring theory in fuzzy sub HX ring of a HX ring.

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