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# Special Rectangles and Narcissistic Numbers of Order 3 And 4 

G.Janaki ${ }^{1}$, P.Saranya ${ }^{2}$<br>${ }^{1,2}$ Department of Mathematics, Cauvery College for women, Trichy-620018


#### Abstract

We search for infinitely many rectangles such that $x^{2}+y^{2}+3 A-S^{2}+k^{2}+S K=$ Narcissistic numbers of order 3 and 4 respectively, in which $x, y$ represents the length and breadth of the rectangle. Also the total number of rectangles satisfying the relation under consideration as well as primitive and non-primitive rectangles are also present. Keywords-Rectangle, Narcissistic numbers of order 3 and 4, primitive,non-primitive.


## I. INTRODUCTION

The older term for number theory is arithmetic, which was superseded as number theory by early twentieth century. The first historical find of an arithmetical nature is a fragment of a table, the broken clay tablet containing a list of Pythagorean triples. Since then the finding continues.
For more ideas and interesting facts one can refer [1].In [2] one can get ideas on pairs of rectangles dealing with non-zero integral pairs representing the length and breadth of rectangle. [3,4] has been studied for knowledge on rectangles in connection with perfect squares, Niven numbers and kepriker triples.[5-10] was referred for connections between Special rectangles and polygonal numbers, jarasandha numbers and dhuruva numbers
Recently in $[11,12]$ special pythagorean triangles in connections with Narcissistic numbers are obtained.
In this communication, we search for infinitely many rectangles such that $x^{2}+y^{2}+3 A-S^{2}+k^{2}+S K=$ Narcissistic numbers of order 3 and 4 respectively, in which $x, y$ represents the length and breadth of the rectangle.
Also the total number of rectangles satisfying the relation under consideration as well as primitive and non-primitive rectangles are also present.

## II. NOTATIONS

A-Area of the rectangle
S-Semi-perimeter of the rectangle

## III. BASIC DEFINITIONS

## Definition 1:Narcissistic Numbers

An n -digit number which is the sum of $\mathrm{n}^{\text {th }}$ power of its digits is called an n -narcissistic number. It is also known as Armstrong number.
Definition 2:Primitive Rectangle
A rectangle is said to be primitive if the generators $\mathrm{u}, \mathrm{v}$ are of opposite parity and $\operatorname{gcd}(\mathrm{u}, \mathrm{v})=1$, where

$$
x=u+v ; y=u-v \text { and } u>v>0
$$

## IV. METHOD OF ANALYSIS

Let x , y be two non-zero distinct positive integers representing the length and breadth of a rectangle R.Let $k \geq 0$ be any given integer.
The problem under consideration is to solve the equation
$x^{2}+y^{2}+3 A-S^{2}+k^{2}+S k=$ Narcissistic Number
To solve (1), let us introduce the linear transformation $x=u+v$ and $y=u-v(u>v>0)$
Therfore (1) reduces to
$(u+k)^{2}-v^{2}=$ Narcissist ic Number
Case1:
Consider the $3^{\text {rd }}$ order Narcissistic Number 153.

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Therefore (3) becomes,
$(u+k)^{2}-v^{2}=153$
Applying the method of factorization, we have

| $\mathrm{u}+\mathrm{k}$ | 13 | 27 | 77 |
| :---: | :---: | :---: | :---: |
| v | 4 | 24 | 76 |

From the above mentioned values, the following results are observed.
TABLE I

| k | No.of Rectangles related to 153 | Observations |
| :---: | :---: | :--- |
| 0 | 3 | 2 rectangles are primitive and one is non-primitive. |
| 1,2 | 2 | For $\mathrm{k}=1$, both the rectangles are non-primitive. <br> For $\mathrm{k}=2$, both the rectangles are primitive. |
| $3-8$ | 1 | For $\mathrm{k}=3,5,7,8$, the rectangles are non- primitive. <br> For $\mathrm{k}=4,6$, the rectangles are primitive. |

## Case2:

Consider the $3^{\text {rd }}$ order Narcissistic Number 371.
Therefore (3) becomes,
$(u+k)^{2}-v^{2}=371$
Applying the method of factorization, we have

| $\mathrm{u}+\mathrm{k}$ | 30 | 186 |
| :---: | :---: | :---: |
| v | 23 | 185 |

From the above mentioned values, the following results are observed.
TABLE II

| k | No.of Rectangles related to 371 | Observations |
| :---: | :---: | :--- |
| 0 | 2 | Both the rectangles are primitive |
| $1-6$ | 1 | For $\mathrm{k}=1,3,5$, the rectangles are non-primitive <br> For $\mathrm{k}=2,4,6$, the rectangles are primitive. |

## Case3:

Consider the $3^{\text {rd }}$ order Narcissistic Number 407.
Therefore (3) becomes,
$(u+k)^{2}-v^{2}=407$
Applying the method of factorization, we have

| $\mathrm{u}+\mathrm{k}$ | 24 | 204 |
| :---: | :---: | :---: |
| v | 13 | 203 |

From the above mentioned values, the following results are observed.

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TABLE III

| k | No.of Rectangles related to 407 | Observations |
| :---: | :---: | :--- |
| 0 | 2 | Both the rectangles are primitive |
| $1-10$ | 1 | For $\mathrm{k}=1,3,5,7,9$, the rectangles are non-primitive <br> For $\mathrm{k}=2,4,6,8,10$, the rectangles are primitive. |

## Case 4:

Consider the $4^{\text {th }}$ order Narcissistic Number 8208.
Therefore (3) becomes,
$(u+k)^{2}-v^{2}=8208$
Applying the method of factorization, we have

| $\mathrm{u}+\mathrm{k}$ | 92 | 93 | 103 | 127 | 132 | 183 | 237 | 348 | 517 | 687 | 1028 | 2053 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| v | 16 | 21 | 49 | 89 | 96 | 159 | 219 | 336 | 509 | 681 | 1024 | 2051 |

From the above mentioned values, the following results are observed.
TABLE IV

| k | No.of Rectangles related to 8208 | Observations |
| :---: | :---: | :---: |
| 0,1 | 12 | For $\mathrm{k}=0$, all the rectangles are non-primitive For $\mathrm{k}=1$, all the rectangles are primitive |
| 2,3 | 11 | For $\mathrm{k}=2$, all the rectangles are non-primitive <br> For $\mathrm{k}=3$, there are 5 primitive and 6 non-primitive rectangles |
| 4,5 | 10 | For $\mathrm{k}=4$, all the rectangles are non-primitive <br> For $\mathrm{k}=5$, there are 8 primitive and 2 non-primitive rectangles |
| 6,7 | 9 | For $\mathrm{k}=6$, all the rectangles are non-primitive For $\mathrm{k}=7$, all the rectangles are primitive |
| 8-11 | 8 | For $\mathrm{k}=8,10$ all the rectangles are non-primitive <br> For $\mathrm{k}=9$, there are 3 primitive and 5 non-primitive rectangles <br> For $\mathrm{k}=11$ all the rectangles are primitive |
| 12-17 | 7 | For $\mathrm{k}=12,14,16$ all the rectangles are non-primitive <br> For $\mathrm{k}=13,17$ all the rectangles are primitive <br> For $\mathrm{k}=15$,there are 3 primitive and 4 non-primitive rectangles |
| 18-23 | 6 | For $\mathrm{k}=18,20,22$ all the rectangles are non-primitive <br> For $\mathrm{k}=19,23$ there are 5 primitive and 1 non-primitive rectangles <br> For $\mathrm{k}=21$,there are 3 primitive and 3 non-primitive rectangles |
| 24-35 | 5 | For $\mathrm{k}=24,26,28,30,32,34$ all the rectangles are non-primitive <br> For $\mathrm{k}=25,29,31,35$ all the rectangles are primitive <br> For $k=27$,there are 3 primitive and 2 non-primitive rectangles <br> For $k=33$,there are 2 primitive and 3 non-primitive rectangles |
| 36,37 | 4 | For $\mathrm{k}=36$, all the rectangles are non-primitive <br> For $\mathrm{k}=37$, there are 3 primitive and 1 non-primitive rectangles |
| 38-53 | 3 | For $\mathrm{k}=38,40,42,44,46,48,50,52$ all the rectangles are non-primitive For $\mathrm{k}=41,43,49,53$ all the rectangles are primitive |


|  |  | For $\mathrm{k}=39,45,47,51$ there are 2 primitive and 1 non-primitive rectangles |
| :--- | :---: | :--- |
| $54-71$ | 2 | For $\mathrm{k}=54,56,58,60,62,64,66,68,70$ all the rectangles are non- <br> primitive <br> For $\mathrm{k}=55,59,61,67,71$ all the rectangles are primitive <br> For $\mathrm{k}=57,63,65,69$ there are 1 primitive and 1 non-primitive rectangles |
| $72-75$ | 1 | For $\mathrm{k}=72,74$ the rectangles is non-primitive <br> For $\mathrm{k}=73,75$ the rectangles is primitive |

## V. CONCLUSION

To conclude, one may search for the connections between the rectangles and Narcissistic numbers of higher order and other number patterns.

## REFERENCES

[1] Dickson L. E., (1952) History of Theory of Numbers, Vol. 11, Chelsea Publishing Company, New York.
[2] J. N. Kapur, Dhuruva numbers, Fascinating world of Mathematics and Mathematical sciences, Trust society, Vol 17, 1997.
[3] M. A. Gopalan and A. Vijayasankar, "Observations on a Pythagorean problem", Acta Ciencia Indica, Vol.XXXVI M, No 4, 517-520, 2010.
[4] M. A. Gopalan, A. Gnanam and G. Janaki, "A Remarkable Pythagorean problem", Acta Ciencia Indica, Vol.XXXIII M, No 4, 1429-1434, 2007
[5] M. A. Gopalan and A. Gnanam, "Pythagorean triangles and Polygonal numbers", International Journal of Mathematical Sciences, Vol 9, No. 1-2, 211-215, 2010
[6] M. A. Gopalan and G. Janaki, "Pythagorean triangle with perimeter as Pentagonal number", Antartica J. Math., Vol 5(2), 15-18, 2008
[7] M. A. Gopalan and G. Janaki, "Pythagorean triangle with nasty number as a leg", Journal of Applied Mathematical Analysis and Applications, Vol 4, No 1-2, 13-17, 2008
[8] G.Janaki and S.Vidhya, "Rectangle with area as a special polygonal number", International Journal of Engineering Research, Vol-4, Issue-1, 88-91, 2016
[9] G.Janaki and C.Saranya, "Special Pairs of Pythagorean Triangles and Jarasandha Numbers", American International Journal of Research in Science, Technology, Engineering \& Mathematics, issue-13, 118-120, Dec 2015-Feb 2016
[10] M. A. Gopalan, Vidhyalakshmi.S and Shanthi, "A connection between rectangle and dhuruva Numbers of digits 3 and 5", International Journal of Recent Scientific Research Vol. 7, Issue, 2, pp. 9234-9236, March, 2016
[11] G.Janaki and P.Saranya, "Special pairs of Pythagorean triangles and narcissistic number", International Journal of Multidisciplinary Research and Development, Volume 3; Issue 4; April 2016; Page No. 106-108
[12] G.Janaki and P.Saranya, "Special Pythagorean Triangles in Connection with the Narcissistic Numbers of Order 3 and 4", American International Journal of Research in Science, Technology, Engineering \& Mathematics, Volume-2, Issue 14, March-May 2016.

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