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Construction of control chart based on six sigma initiatives for Cumulative – Sum

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Abstract – A control chart is a statistical device used for the study and control of repetitive process. W.A. Shewhart [6] of Bell Telephone Laboratories suggested control charts based on the 3 sigma limits. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing Six Sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola. If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart [6], then no point fall outside the control limits because of the improvement in the quality of the process. In this paper an attempt is made to construct a control chart based on six sigma initiatives for cumulative – sum specially designed for the companies applying Six Sigma initiatives in their organization. Suitable Table – IV is also constructed and presented for the engineers to take quick decisions.
Keywords – Control Chart, Process control, Six sigma, Six sigma quality level.

I. INTRODUCTION

The concept of Six Sigma was introduced by the engineer M.Harry who analyzed variations in outcomes of the company's internal procedures and realized that by measuring variations it will be possible to improve the working of the system. The procedure was aimed at taking action to improve the overall performance. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities. Radhakrishnan [3] suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan [4,5] constructed control charts based on six sigma initiatives for defects, mean, average fraction defectives, number of defectives, X bar using standard deviation, Exponentially Weighted Moving Average (EWMA), proportion defectives – number of defectives, Fraction defectives, Standard deviation, Standard deviation with variable sample size, average number of nonconformities per multiple units, number of defects - average number of defects per unit and range. The control charts originated by W.A. Shewhart [6] was based on 3 sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in the process, then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is required to monitor the outcomes of the companies, which adopt six sigma initiatives.

In this paper an attempt is made to construct a control chart based on six sigma initiatives for Cumulative - Sum. Suitable Table – IV is also constructed and presented for the engineers to take quick decisions.

II. CONCEPTS AND TERMINOLOGIES

A. V-Mask

It is a new frame used in the Cumulative sum (CUSUM) to detect an out control point.

B. Scaling factor ($v_{6\sigma}$)

It is a scaling factor related to the geometry of the control chart based on six sigma initiatives for cumulative - sum and the mask dimensions.

$$v_{6\sigma} = 2.0\,\sigma_{6\sigma}, \text{ where } \sigma_{6\sigma} = \sigma_{ss}/\sqrt{n}$$

C. Lead distance & half of the V-Mask (d &θ)

These are the parameters to construct the V-mask.

$d = \frac{E(\alpha)}{\delta^2}$ and $\theta = \tan^{-1}\left(\frac{D}{2y}\right)$ suggested by Eugene L. Grant and Richard S. Leavenworth [1].

where

d is the distance from the vertex of these angles to the locator point P in the V mask.

θ is the angel between the horizontal and each control limit line

$D = \delta\sigma / \sqrt{n}$ is the actual of the shift magnitude, either plus or minus, that must be detected with virtual certainty

y is a scaling related to the geometry of the control chart and the mask dimensions

E (α) is a factor that is a function of the acceptable Type I error probability.

TABLE I
FUNCTION OF THE ACCEPTABLE TYPE I ERROR PROBABILITY

Type I Error	0.10	0.05	0.02	0.01	0.0027	3.4×10^{-6}
E (α)	5.991	7.378	9.210	10.597	13.215	26.570

D. Magnitude of shift ($D_{6\sigma}$)

It is shift in the process mean for cumulative sum control chart based on six sigma initiatives.

$$D_{6\sigma} = \delta\sigma_{6\sigma}$$

III. CONSTRUCTION OF CONTROL CHART BASED ON SIX SIGMA INITIATIVES FOR CUMULATIVE – SUM

The horizontal scale represents the counter i on subgroups; the vertical scale represents the cumulative sum of subgroup average deviations from an aimed – at or standard value $\bar{X}$. The two – sided control limits for $\bar{X}$ are the edges A $\bar{A}$ and B $\bar{B}$, which are the lower and upper control limits, respectively.

A. V – Mask

Two elements are needed to construct the mask. They are d (distance parameter) and θ (angel between the horizontal and each control limit line). Fix the tolerance level (TL) and process capability (CP) to determine the process standard deviation (σ_{SS}). By using σ_{SS} , we get the value of $\sigma_{6\sigma}$ i.e. $\sigma_{6\sigma} = \frac{\sigma_{SS}}{\sqrt{n}}$. Apply the value of $\sigma_{6\sigma}$ in the distance parameter

$$d = \frac{E(\alpha)}{\delta^2} = E(3.4\times10^{-6})\left(\frac{\sigma_{6\sigma}}{D_{6\sigma}}\right).$$

For a specified TL and Cp of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a C program and presented in Table - IV for various combinations of TL and Cp. The value of θ, the angel between the horizontal and each control limit line are obtained using $\theta = \tan^{-1}\left(\frac{D}{2y}\right)$.

IV. CONDITIONS FOR APPLICATION

A. Human involvement should be less in the manufacturing process

B. The company adopts Six sigma quality initiatives in its processes

V. EXAMPLE

The example provided by Montgomery, D.C, [2] is considered here. Piston rings for an automotive engine are produced by a forging process. We wish to establish statistical control of the inside diameter of the rings manufactured by this process using cumulative

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sum control chart. Twenty – five samples, each of size four, have been taken when we think the process is in control. The inside diameter measurement data from these samples are:

Table II
INSIDE DIAMETER MEASUREMENT DATA

Sample Number	Observations
1-4	47 32 44 35
5-8	33 33 34 34
9-12	34 34 31 34
13-16	12 21 24 47
17-20	35 23 38 40
21-24	19 37 31 27
25-28	23 45 26 37
29-32	33 12 29 43
33-36	25 22 37 33
37-40	29 32 30 13
41-44	40 18 30 11
45-48	21 18 36 34
49-52	26 35 31 29
53-56	52 29 21 18
57-60	26 20 30 20
61-64	19 1 30 30
65-68	28 34 39 17
69-72	29 25 24 30
73-76	21 37 32 25
77-80	24 22 16 35
81-84	28 39 23 21
85-88	41 32 46 12
89-92	14 23 41 42
93-96	32 28 46 27
97-100	42 34 22 34

TABLE III
PARAMETERS IN V-MASK FOR CUSUM ARE SUGGESTED BASED ON 3σ

Sub group Number	Numbers of drawings	Average $\bar{X}$	$\left(\bar{X}_i - \bar{X}_0\right)$	$\Sigma\left(\bar{X}_i - \bar{X}_0\right)$
1	1-4	39.50	9.50	9.50
2	5-8	33.50	3.50	13.00
3	9-12	33.25	3.25	16.25
4	13-16	26.00	-4.00	12.25
5	17-20	34.00	4.00	16.25
6	21-24	28.50	-1.50	14.75
7	25-28	32.75	2.75	17.50
8	29-32	29.25	-0.75	16.75
9	33-36	29.25	-0.75	16.00
10	37-40	26.00	-4.00	12.00
11	41-44	24.75	-5.25	6.75
12	45-48	27.25	-2.75	4.00
13	49-52	30.25	0.25	4.25

14	53-56	30.00	0.00	4.25
15	57-60	24.00	-6.00	-1.75
16	61-64	20.00	-10.00	-11.75
17	65-68	29.50	-0.50	-12.25
18	69-72	27.00	-3.00	-15.25
19	73-76	28.75	-1.25	-16.50
20	77-80	24.25	-5.75	-22.25
21	81-84	27.75	-2.25	-24.50
22	85-88	32.75	2.75	-21.75
23	89-92	30.00	0.00	-21.75
24	93-96	33.25	3.25	-18.50
25	97-100	33.00	3.00	-15.50

A. Three sigma based Cumulative – Sum Control Chart

The Cumulative – Sum Control Chart (CUSUM Chart) is formed by plotting the quantity $C_i = \sum_{j=1}^{i} (\bar{X}_j - \bar{X}_0)$ against the sample i and C_i is called the Cumulative Sum up to and including the ith sample.

$$d = \frac{E(\alpha)}{\delta^2} = E(0.0027)\left(\frac{\sigma_{\bar{X}}}{D}\right)$$

$$= 13.216\left(\frac{4.73}{7.5}\right)^2$$

$$d \qquad = 5.26$$

Units on horizontal axis and

$$\theta = \tan^{-1}\left(\frac{D}{2y}\right) = \tan^{-1}\left(\frac{7.5}{2 \times 4}\right)$$

$$= \tan^{-1}\left(\frac{7.5}{8}\right)$$

$$\theta \qquad = 43^0$$

From the resulting Figure 1, it is clear that the process is in control, since the entire sample numbers lie inside the control limits.

B. CUSUM Chart with V-Mask based on Six Sigma initiatives

For a given TL = 19.5 & Cp = 2.5, it is found from the Table- 4 the value of σ_{SS} is 1.3 and $\sigma_{6\sigma} = \frac{\sigma_{SS}}{\sqrt{n}} = \frac{1.3}{\sqrt{4}} = 0.65$.

Then the parameters for CUSUM chart with V-Mask based on six sigma initiatives are

$$d = \frac{E(\alpha)}{\delta^2} = E(3.4 \times 10^{-6})\left(\frac{\sigma_{6\sigma}}{D_{6\sigma}}\right)$$

$$= 26.570\left(\frac{0.65}{1.03}\right)^2$$

$$d \qquad = 10.58$$

Units on horizontal axis and

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$$\theta = \tan^{-1}\left(\frac{D_{6\sigma}}{2v_{6\sigma}}\right) = \tan^{-1}\left(\frac{1.03}{2\times 1.3}\right)$$

$$= \tan^{-1}\left(\frac{1.03}{2.6}\right)$$

$$\theta = 22^{0}$$

Table IV

σ_{SS} Values for a specified Cp and TL

TL Cp	15.5	16.5	17.5	18.5	19.5
	2.58	2.75	2.92	3.08	3.25
1.1	2.35	2.5	2.65	2.80	2.95
1.2	2.15	2.29	2.43	2.57	2.70
1.3	1.99	2.12	2.24	2.37	2.50
1.4	1.84	1.96	2.08	2.20	2.32
1.5	1.72	1.83	1.94	2.06	2.17
1.6	1.61	1.72	1.82	1.93	2.03
1.7	1.52	1.62	1.72	1.81	1.91
1.8	1.44	1.53	1.62	1.71	1.81
1.9	1.36	1.45	1.54	1.62	1.71
2.0	1.29	1.38	1.46	1.54	1.63
2.1	1.23	1.31	1.39	1.47	1.55

From the resulting Figure 2, it is clear that the sample numbers 7, 8, 9 and 10 goes above the upper control limit. Therefore the process does not exhibit statistical control.

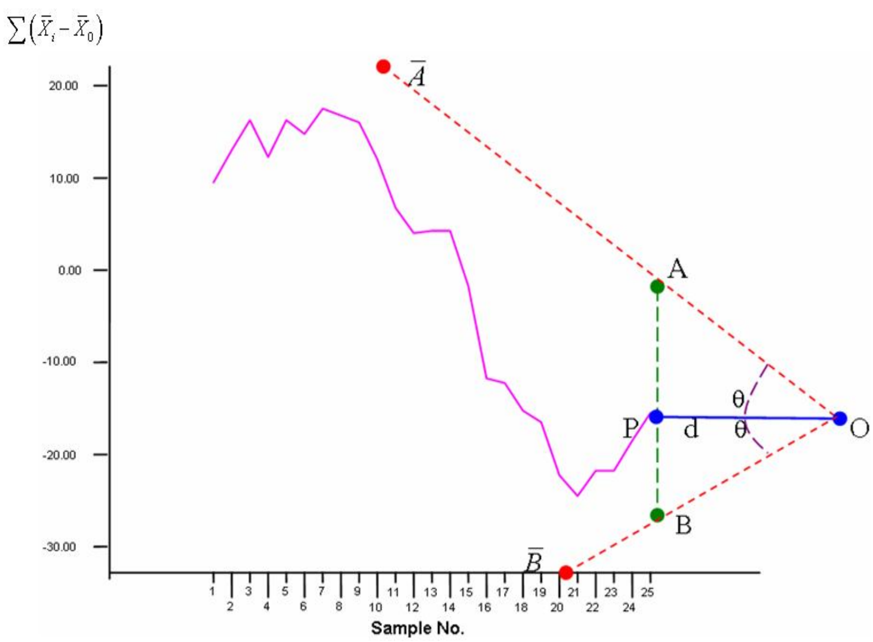

Fig 1: V – mask for three sigma based CUSUM chart

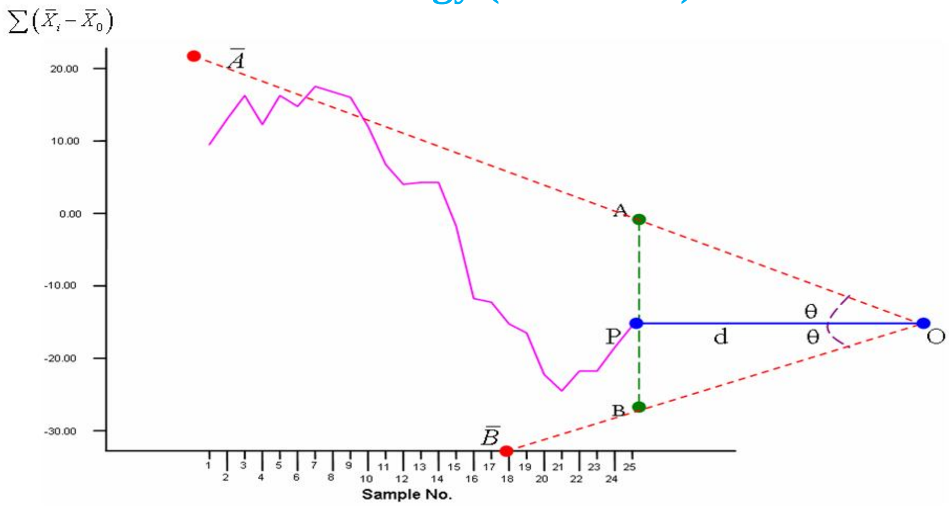

Figure 2: V – mask for CUSUM chart based on six sigma initiatives

VI. CONCLUSION

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for cumulative sum with an example. It is found that the process was not in control even when six sigma initiatives are adopted. It is very clear from the comparison that when the process is centered with reduced variation many points fall outside the control limits than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this paper will be very useful for the companies practicing Six Sigma initiatives in their process. These charts will replace the existing Shewhart [6] control charts in future when all the companies started implementing six sigma initiatives in their organization.

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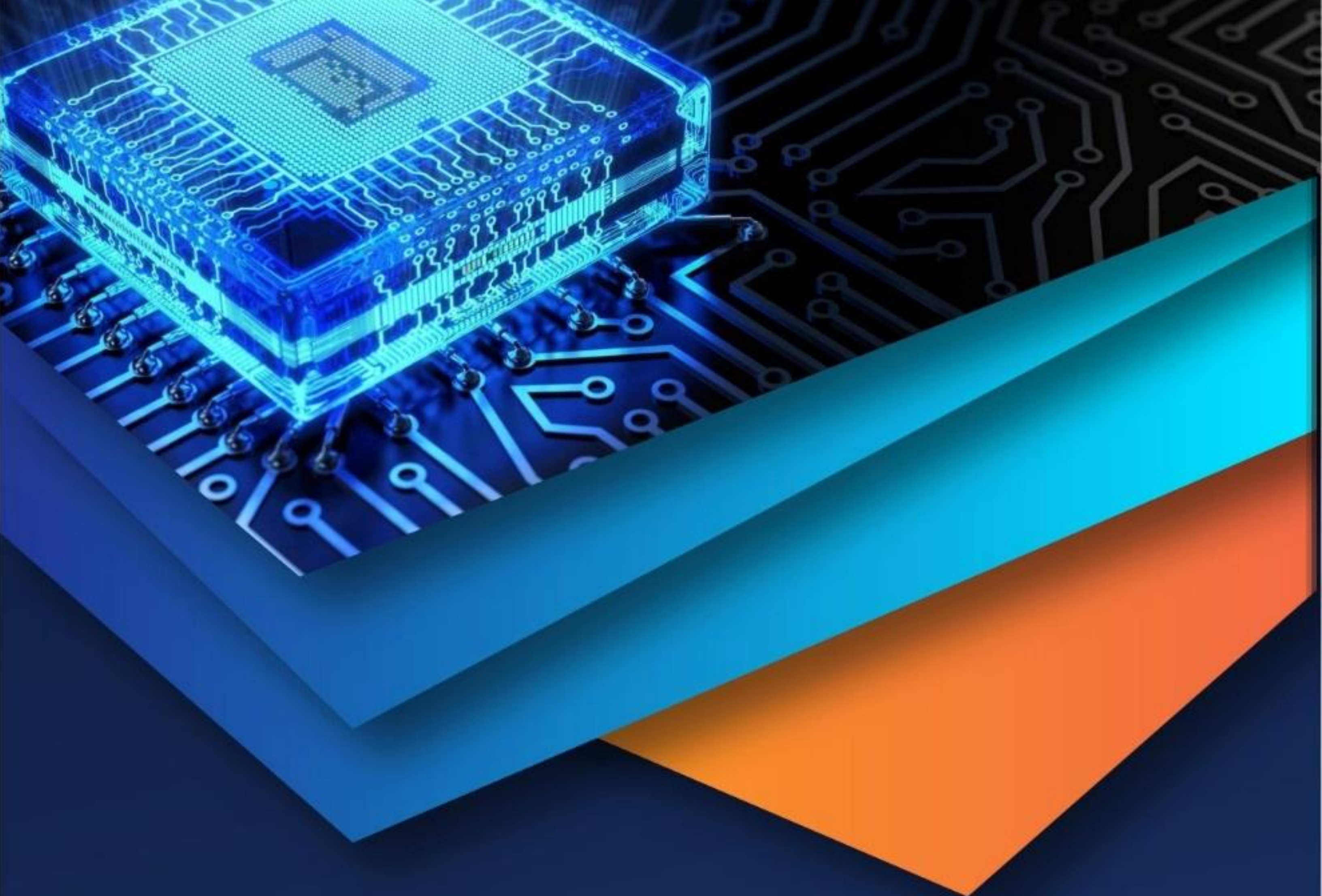

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