iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 4 **Issue:** X **Month of publication:** October 2016

DOI:

www.ijraset.com
IC Value: 13.98
Volume 4 Issue X, October 2016
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Integral Solutions of an Infinite Cone $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$

Manju Somanath[1], J. Kannan[2], K. Raja[3]
[1,3]Assistant Professor, Department of Mathematics, National College, Trichy, Tamil nadu, India
[2]Research Scholar, Department of Mathematics, National College, Trichy, Tamil nadu, India

Abstract: The quadratic Diophantine equation representing an infinite cone given by $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$, $\alpha \in N$, is analyzed for its non-zero distinct integer points. Few different patterns of integer points satisfying the infinite cone under consideration are obtained.
Keyword- Diophantine equations, integer solutions, infinite cone, lattice points.

I. INTRODUCTION

Number theory is, as the name suggests, is devoted to the study of numbers, first and foremost integers. The classical problems in Number Theory are often easy to formulate knowing some basic mathematics and this makes the area attractive. It is often only the formulation are simple, proofs are very long, complicated and requires excessive numerical work.
A great deal of number theory also arises from the study of the solutions in integers of a polynomial equation $f(x_1, x_2, \dots x_n) = 0$, called the Diophantine equation. They have fewer equations than unknown variables and involve integers that work correctly for all equations. Quadratic Diophantine equations are extremely important in Number theory. There are several Diophantine equations that have no solutions, trivial solutions, finitely or infinitely many solutions. This communication concerns with interesting ternary quadratic equation $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$, $\alpha \in N$, representing an infinite cone for determining its infinitely many non-zero lattice points.

II. METHOD OF ANALYSIS

The ternary quadratic equation studied for its non-zero distinct integer solutions is given by

$$\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2 \qquad (1)$$

Pattern 1:

Introducing the linear transformations

$$x = u + v, \quad y = u - v \qquad (2)$$

(2) in (1), it leads to

$$u^2 = (4\alpha - 1)(z^2 - v^2) \qquad (3)$$

Equation (3) can be written as

$$\frac{u}{(4\alpha-1)(z+v)} = \frac{z-v}{u} = \frac{p}{q} \ (say), \ q \neq 0. \qquad (4)$$

which is equivalent to the system of equations

$$qu + (4\alpha - 1)pv - (4\alpha - 1)pz = 0$$

$$pu + qv - qz = 0$$

Volume 4 Issue X, October 2016
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

On employing the method of cross multiplication, we get

$$x = 2(4\alpha-1)pq - p^2(4\alpha-1) + q^2$$

$$y = p^2(4\alpha-1) + 2(4\alpha-1)pq - q^2$$

$$z = p^2(4\alpha-1) + q^2$$

Pattern 2:

Equation (3) can be written as

$$\frac{u}{(z+v)} = \frac{(4\alpha-1)(z-v)}{u} = \frac{p}{q}\ (say), q \neq 0. \tag{5}$$

Proceeding as above, we obtain

$$x = 2pq(4\alpha-1) - p^2 + q^2(4\alpha-1)$$

$$y = p^2 + 2pq(4\alpha-1) - q^2(4\alpha-1)$$

$$z = q^2(4\alpha-1) + p^2$$

Pattern 3:

Equation (3) also can be also written as

$$\frac{u}{(4\alpha-1)(z-v)} = \frac{z+v}{u} = \frac{p}{q}\ (say)\quad q \neq 0 \tag{6}$$

Proceeding as above, we obtain

$$x = q^2 - 2(4\alpha-1)pq - (4\alpha-1)p^2$$

$$y = p^2(4\alpha-1) - q^2 - 2(4\alpha-1)pq$$

$$z = -p^2(4\alpha-1) - q^2$$

Pattern 4:

Equation (3) also can be also written as

$$\frac{u}{(z-v)} = \frac{(4\alpha-1)(z+v)}{u} = \frac{p}{q}\ (say)\quad q \neq 0$$

Proceeding as above, we obtain

$$x = q^2(4\alpha-1) - 2(4\alpha-1)pq - p^2$$

$$y = p^2 - q^2(4\alpha-1) - 2(4\alpha-1)pq$$

$$z = -p^2 - q^2(4\alpha-1)$$

Pattern 5:

Equation (3) can be written as

$$u^2 + (4\alpha-1)v^2 = (4\alpha-1) * z^2 \tag{7}$$

Volume 4 Issue X, October 2016
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Assume $$z = a^2 + (4\alpha - 1)b^2 \quad (8)$$

Considering $(4\alpha - 1) as$ $$(4\alpha - 1) = \left(i\sqrt{4\alpha - 1}\right)\left(-i\sqrt{4\alpha - 1}\right) \quad (9)$$

Substituting (8) and (9) in (7) and applying the method of factorization

$$u + i\sqrt{(4\alpha - 1)}\, v = i\sqrt{(4\alpha - 1)}\left(a + i\sqrt{4\alpha - 1}\, b\right)^2 \quad (10)$$

Equating the real and imaginary parts, we have

$$u = -2(4\alpha - 1)ab; \quad v = a^2 - (4\alpha - 1)\, b^2 \quad (11)$$

Using (11), (8) and (2), we obtain the integral solutions to (1) as presented below:

$$x = a^2 - (4\alpha - 1)b^2 - 2(4\alpha - 1)ab$$

$$y = (4\alpha - 1)b^2 - a^2 - 2(4\alpha - 1)ab$$

$$z = a^2 + (4\alpha - 1)b^2.$$

Pattern 6:
Equation (3) can be written as

$$u^2 - (4\alpha - 1) * z^2 = -(4\alpha - 1)v^2 \quad (12)$$

Assume $$v = a^2 - (4\alpha - 1)b^2 \quad (13)$$

Considering $-(4\alpha - 1)$ as

$$-(4\alpha - 1) = \left(\sqrt{4\alpha - 1}\right)\left(-\sqrt{4\alpha - 1}\right) \quad (14)$$

Substituting (13) and (14) in (12) and applying the method of factorization

$$u + \sqrt{(4\alpha - 1)}\, z = \sqrt{(4\alpha - 1)}\left(a + \sqrt{4\alpha - 1}\, b\right)^2 \quad (15)$$

Equating the rational and irrational parts, we have

$$u = 2(4\alpha - 1)ab; \quad z = a^2 + (4\alpha - 1)\, b^2 \quad (16)$$

Using (13), (14) and (16), we obtain the integral solutions to (1) as presented below:

$$x = a^2 + (4\alpha - 1)(2ab - b^2)$$

$$y = (4\alpha - 1)(2ab + b^2) - a^2$$

$$z = a^2 + (4\alpha - 1)b^2$$

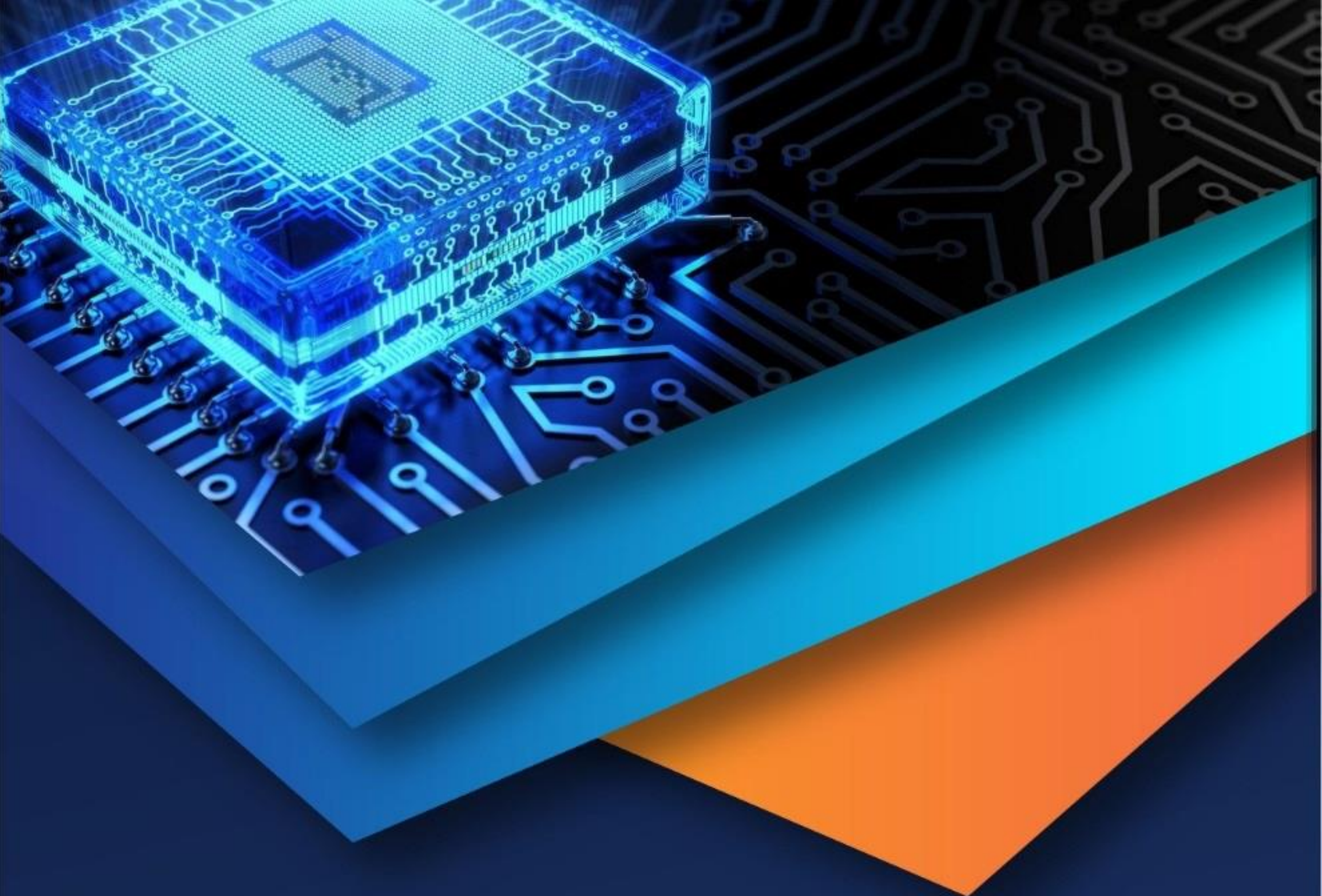
III. CONCLUSION

The ternary quadratic Diophantine equations are rich in variety. One may search for other choices of Diophantine equations to find their corresponding integer solutions.

REFERENCES

[1] Ivan Niven, Herbert S. Zuckermann and Hugh L. Montgomery, an introduction to the Theory of Numbers, John Wiley & Sons Inc, New York, 2004.
[2] Andre weil, Number Theory : An Approach through History, From Hammurapito to Legendre, Bikahsuser, Boston, 1987.
[3] Bibhotibhusan Batta and Avadhesh Narayanan Singh, History of Hindu Mathematics, Asia Publishing House, 1983.

[4] Boyer. C. B., History of mathematics, John Wiley & sons lnc., New York, 1968.
[5] L.E. Dickson, History of Theory of Numbers, Vol.2, Chelsea Publishing Company, New York, 1952.
[6] Davenport, Harold (1999), The higher Arithmetic: An introduction to the Theory of Numbers (7th ed.) Cambridge University Press.
[7] John Stilwell, Mathematics and its History, Springer Verlag, New York, 2004.
[8] James Matteson, M.D. "A Collection of Diophantine problems with solutions" Washington, Artemas Martin, 1888.
[9] Tituandreescu, DorinAndrica, "An introduction to Diophantine equations" Springer Publishing House, 2002.
[10] R. Steiner, Class number bounds and Catalan's equation, Math. Comp. 67 (223) (1998) Pp.1317–1322.
[11] R.J. Stroeker, R. Tijdeman, Diophantine equations, Computational Methods in Number Theory, MC Track 155, Central Math Comp Sci, Amsterdam, 1982, pp. 321–369.
[12] R. Styer, Small two variable exponential Diophantine equations, Math. Comp. 60 (202) (1993) 811–816.
[13] Conway J H and Guy R K, The book of numbers, Springer Science and Business Media, 2006
[14] .T.N. Shorey, R. Tijdeman, Exponential Diophantine Equations, Cambridge University Press, Cambridge, 1986.
[15] Manju Somanath, J. Kannan, K.Raja, "Lattice Points of an infinite cone $x^2 + y^2 = 85z^2$ " International Journal of Recent Innovation Engineering and Research Vol. 1, Issue 5, Pp.14-16, September 2016.
[16] M. A .Gopalan, Manju Somanath, K.Geetha, On Ternary Quadratic Diophantine Equation $z^2 = 50x^2 + y^2$,International Journal for research in emerging Science and technology, Vol.3, Issue3, Pp.644-648,February2016.
[17] M. A .Gopalan, Manju Somanath, K.Geetha, On Ternary Quadratic Diophantine Equation $z^2 = 7x^2 + 9y^2$, Bulletin of Mathematics and Statistics research,Vol.2,issue1,Pp.1-8,2014.
[18] On the ternary quadratic equation $5(x^2 + y^2) - 9xy = 19z^2$, IJIRSET, Vol.2 No.6,pp.2008-2010. June 2013.
[19] Manju Somanath, Sangeetha G, Gopalan M. A., Relations among special figurate numbers through equation $y^2 = 10x^2 + 1$, impact J, Sci. Tech. Vol.5,No.1,pp.57-60, 2011.
[20] Gopalan M.A., Sangeetha V and Manju Somanath, Observations on the ternary quadratic equation $x^2 = 24\alpha^2 + y^2$, Bulletin of Society for Mathematical Services & Standards, Vol. 3, No. 2, pp.88-91, 2014.
[21] Gopalan M. A., and Palanikumar R, Observations on $y^2 = 12x^2 + 1$, Antarctica J. Math. Vol.8,No.2, pp.149-152,2011.
[22] Gopalan M. A., Vidhyalakshmi S., Sumathi G., Lattice points on the elliptic paraboloid $9x^2 + 4y^2 = z$, Advances in Theoretical and Applied Mathematics, Vol.7 .No.43, pp.79-85,2012.
[23] Integral points on the homogeneous cone $z^2 = 5x^2 + 11y^2$, Discovery Science, Vol.3, No.7 pp.5-8, Jan.2013.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY