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Integral Solutions of an Infinite Cone $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$

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Abstract: ***The quadratic Diophantine equation representing an infinite cone given by $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$, $\alpha \in N$, is analyzed for its non-zero distinct integer points. Few different patterns of integer points satisfying the infinite cone under consideration are obtained.***
Keyword- Diophantine equations, integer solutions, infinite cone, lattice points.

I. INTRODUCTION

Number theory is, as the name suggests, is devoted to the study of numbers, first and foremost integers. The classical problems in Number Theory are often easy to formulate knowing some basic mathematics and this makes the area attractive. It is often only the formulation are simple, proofs are very long, complicated and requires excessive numerical work.

A great deal of number theory also arises from the study of the solutions in integers of a polynomial equation $f(x_1, x_2, \dots x_n) = 0$, called the Diophantine equation. They have fewer equations than unknown variables and involve integers that work correctly for all equations. Quadratic Diophantine equations are extremely important in Number theory. There are several Diophantine equations that have no solutions, trivial solutions, finitely or infinitely many solutions. This communication concerns with interesting ternary quadratic equation $\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2$, $\alpha \in N$, representing an infinite cone for determining its infinitely many non-zero lattice points.

II. METHOD OF ANALYSIS

The ternary quadratic equation studied for its non-zero distinct integer solutions is given by

$$\alpha(x^2 + y^2) = (2\alpha - 1)xy + (4\alpha - 1)z^2 \quad (1)$$

Pattern 1:

Introducing the linear transformations

$$x = u + v, \quad y = u - v \quad (2)$$

(2) in (1), it leads to

$$u^2 = (4\alpha - 1)(z^2 - v^2) \quad (3)$$

Equation (3) can be written as

$$\frac{u}{(4\alpha-1)(z+v)} = \frac{z-v}{u} = \frac{p}{q} \ (say), \ q \neq 0. \quad (4)$$

which is equivalent to the system of equations

$$qu + (4\alpha - 1)pv - (4\alpha - 1)pz = 0$$

$$pu + qv - qz = 0$$

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On employing the method of cross multiplication, we get

$$x = 2(4\alpha - 1)pq - p^2(4\alpha - 1) + q^2$$

$$y = p^2(4\alpha - 1) + 2(4\alpha - 1)pq - q^2$$

$$z = p^2(4\alpha - 1) + q^2$$

Pattern 2:

Equation (3) can be written as

$$\frac{u}{(z+v)} = \frac{(4\alpha-1)(z-v)}{u} = \frac{p}{q} \ (say), q \neq 0. \quad (5)$$

Proceeding as above, we obtain

$$x = 2pq(4\alpha - 1) - p^2 + q^2(4\alpha - 1)$$

$$y = p^2 + 2pq(4\alpha - 1) - q^2(4\alpha - 1)$$

$$z = q^2(4\alpha - 1) + p^2$$

Pattern 3:

Equation (3) also can be also written as

$$\frac{u}{(4\alpha-1)(z-v)} = \frac{z+v}{u} = \frac{p}{q} \ (say) \quad q \neq 0 \quad (6)$$

Proceeding as above, we obtain

$$x = q^2 - 2(4\alpha - 1)pq - (4\alpha - 1)p^2$$

$$y = p^2(4\alpha - 1) - q^2 - 2(4\alpha - 1)pq$$

$$z = -p^2(4\alpha - 1) - q^2$$

Pattern 4:

Equation (3) also can be also written as

$$\frac{u}{(z-v)} = \frac{(4\alpha-1)(z+v)}{u} = \frac{p}{q} \ (say) \quad q \neq 0$$

Proceeding as above, we obtain

$$x = q^2(4\alpha - 1) - 2(4\alpha - 1)pq - p^2$$

$$y = p^2 - q^2(4\alpha - 1) - 2(4\alpha - 1)pq$$

$$z = -p^2 - q^2(4\alpha - 1)$$

Pattern 5:

Equation (3) can be written as

$$u^2 + (4\alpha - 1)v^2 = (4\alpha - 1) * z^2 \quad (7)$$

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Assume $z = a^2 + (4\alpha - 1)b^2$ (8)

Considering $(4\alpha - 1)as$ $(4\alpha - 1) = \left(i\sqrt{4\alpha - 1}\right)\left(-i\sqrt{4\alpha - 1}\right)$ (9)

Substituting (8) and (9) in (7) and applying the method of factorization

$$u + i\sqrt{(4\alpha - 1)}\, v = i\sqrt{(4\alpha - 1)}\, \left(a + i\sqrt{4\alpha - 1}\, b\right)^2 \quad (10)$$

Equating the real and imaginary parts, we have

$$u = -2(4\alpha - 1)ab; \quad v = a^2 - (4\alpha - 1)\, b^2 \quad (11)$$

Using (11), (8) and (2), we obtain the integral solutions to (1) as presented below:

$$x = a^2 - (4\alpha - 1)b^2 - 2(4\alpha - 1)ab$$

$$y = (4\alpha - 1)b^2 - a^2 - 2(4\alpha - 1)ab$$

$$z = a^2 + (4\alpha - 1)b^2.$$

Pattern 6:
Equation (3) can be written as

$$u^2 - (4\alpha - 1) * z^2 = -(4\alpha - 1)v^2 \quad (12)$$

Assume $v = a^2 - (4\alpha - 1)b^2$ (13)

Considering $-(4\alpha - 1)$ as

$$-(4\alpha - 1) = \left(\sqrt{4\alpha - 1}\right)\left(-\sqrt{4\alpha - 1}\right) \quad (14)$$

Substituting (13) and (14) in (12) and applying the method of factorization

$$u + \sqrt{(4\alpha - 1)}\, z = \sqrt{(4\alpha - 1)}\, \left(a + \sqrt{4\alpha - 1}\, b\right)^2 \quad (15)$$

Equating the rational and irrational parts, we have

$$u = 2(4\alpha - 1)ab; \quad z = a^2 + (4\alpha - 1)\, b^2 \quad (16)$$

Using (13), (14) and (16), we obtain the integral solutions to (1) as presented below:

$$x = a^2 + (4\alpha - 1)(2ab - b^2)$$

$$y = (4\alpha - 1)(2ab + b^2) - a^2$$

$$z = a^2 + (4\alpha - 1)b^2$$

III. CONCLUSION

The ternary quadratic Diophantine equations are rich in variety. One may search for other choices of Diophantine equations to find their corresponding integer solutions.

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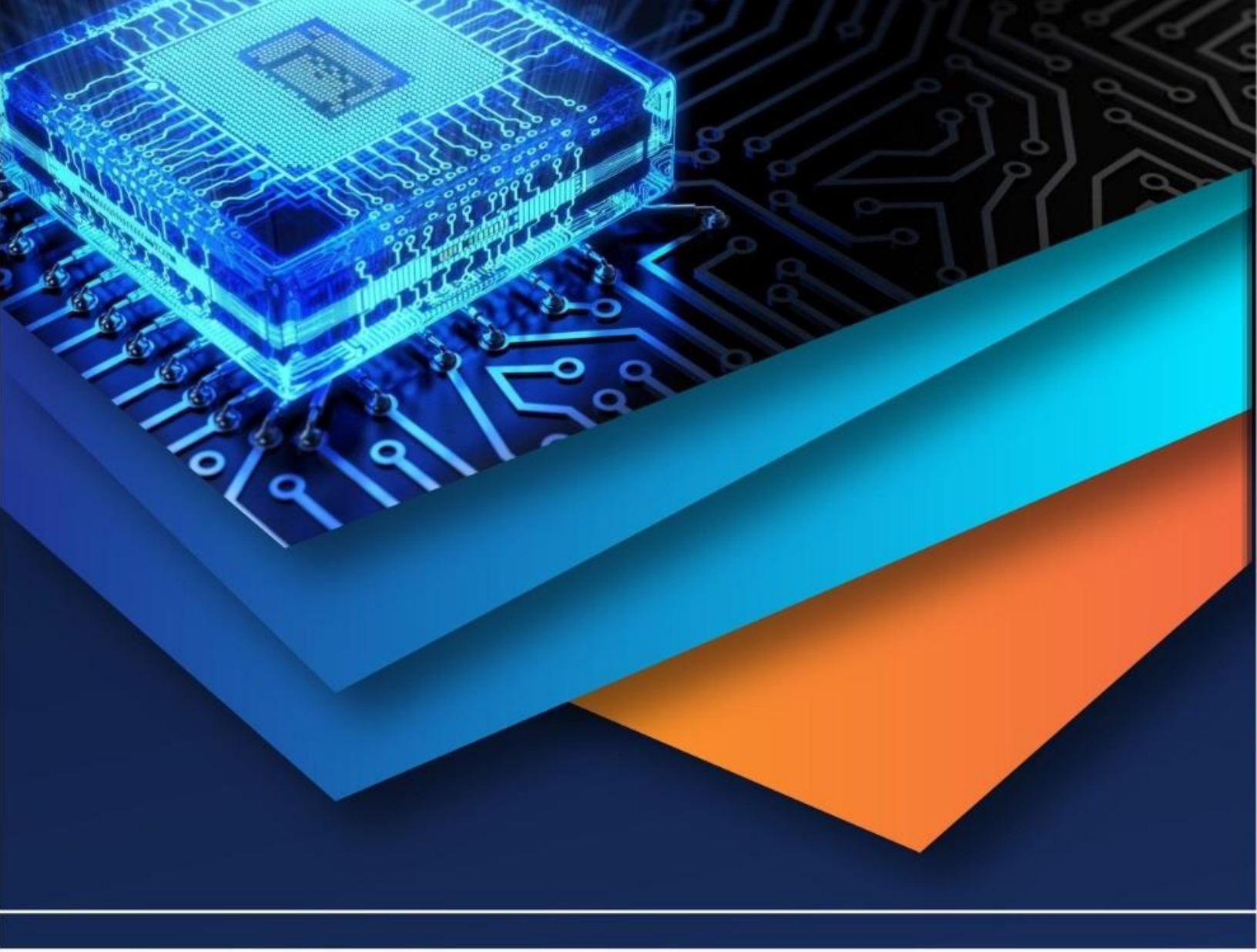

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