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Synthetic Control Chart for Monitoring Parameters of the Weibull Distribution

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Abstract- This paper proposes the synthetic control chart for monitoring the parameters of the Weibull distribution. Performance of the proposed control chart is measured using average run length and is compared with the t-chart (control chart for monitoring Weibull distribution). Comparison study shows that the proposed control chart performs significantly better than the t-chart.
Key-Wards: Control chart, synthetic, Weibull distribution, high-quality, average run length.

I. INTRODUCTION

For high-quality processes where the defect rate is very low, e.g., parts per million (ppm), time-between-events (TBE) control charts have several advantages over the ordinary control charts. Most existing TBE control charts are based on the homogeneous Poisson process assumption, so that the distribution of TBE is an exponential. However, the exponential distribution is not suitable in many applications, especially when the failure rate is not constant. To monitor processes for which the exponential assumption is violated, Weibull distribution is a good alternative and it is a simple generalization of the exponential distribution. Due to its flexibility it has been widely used by various authors to model the failure times.

The Weibull distribution has two parameters: a shape parameter and a scale parameter. The shape parameter represents the failure rate. When shape parameter is less than one, the failure rate decreases over time. For shape parameter equal to one, the Weibull distribution has a constant failure rate and the Weibull distribution reduces to the exponential distribution. In case where shape parameter is greater than one, the failure rate increases with time. When shape parameter equal to 3.4, the Weibull distribution behaves similarly to the normal distribution. This range of flexibility is one of the reasons why the Weibull distribution is widely applied.

In the literature related to the Weibull distribution, Nelson [3] considered the Weibull distribution for monitoring the median of the quality characteristics. Zang et al. [7] studied the economic design of $\overline{X}$ control chart for monitoring system with Weibull in-control times. Ramalhoto and Morais [5] proposed the control chart for monitoring Weibull distribution with three parameters under the assumption that the shape and scale parameters are to be known. Ranjan et al. [6] studied the control chart procedures for monitoring the inter-arrival times. For detecting a shift of a percentile of a Weibull population Nichols and Padgett [4] proposed a bootsrap control chart for Weibull percentiles. Alireza et al. [1] proposed the Shewhart control charts for monitoring reliability with Weibull lifetimes. Muhammad and Chi-Hyuck [2] studied the control chart for Weibull distribution under truncated life tests.

The purpose of this article is to improve the performance of the high-yield processes using Weibull control variable. In this article synthetic control chart is developed for monitoring parameters of Weibull distribution. Performance of the proposed synthetic control chart is compared with the t-chart. The rest of the article is organized as follows.

In Section 2, brief review of t-chart is presented. Section 3 gives conforming run length control chart. Section 4 describes the synthetic control chart for monitoring parameters of the Weibull distribution and its design. In Section 5, comparison study of the proposed control chart and numerical example are presented. Section 6 gives concluding remarks.

II. CONTROL CHART for MONITORING WEIBULL DISTRIBUTION OR t- CHART

The density function of Weibull distribution is given by

$$f(x) = \left(\frac{\beta}{\theta}\right)\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^{\beta}\right); \quad x \geq 0$$

$$= \qquad 0 \qquad ; \quad x < 0$$

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Where, $\beta > 0$ is the shape parameter and $\theta > 0$ is the scale parameter. The cumulative distribution function of Weibull distribution is

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\theta} \right)^{\beta} \right); \quad x \geq 0$$
$$= \quad 0 \quad ; \quad x < 0$$

A control chart for process monitoring of time-between-event, or t chart, should have exact probability limits due to the skewness of the Weibull distribution. The control limits of Weibull distributions are given by

$$Upper\ control\ \lim it\ (UCL) = \theta_0 \left[Ln\left(\frac{2}{\alpha} \right) \right]^{1/\beta_0},$$

$$Central\ \lim it\ (CL) = \theta_0 \left[Ln(2) \right]^{1/\beta_0}, \text{ and}$$

$$Lower\ control\ \lim it\ (LCL) = \theta_0 \left[Ln\left(\frac{2}{2-\alpha} \right) \right]^{1/\beta_0}.$$

If a manufacturer is only interested in detecting upward shifts in order to safeguard the quality of the process, only UCL will be sufficient and which is given by

$$UCL = \theta_0 \left[Ln\left(\frac{1}{\alpha} \right) \right]^{1/\beta_0}$$

Where □□is the acceptable false alarm probability, □0 and □0 are the in-control shape and scale parameter respectively. In the following, the false alarm probability is fixed at □□= 0.0027 which is equivalent to three sigma limits for $\overline{X}$ chart under normal distribution assumption.

The Weibull TBE chart or t-chart detects upward shifts in the process when $X \geq UCL$ which means that the process has deteriorated and $X < LCL$ indicates that the process quality has improved.

The average run length (ARL) of upper-sided chart for Weibull distribution is given by

$$ARL = \frac{1}{P(X \geq UCL)},$$

$$ARL = \frac{1}{\exp\left(-\left(\frac{x}{\theta} \right)^{\beta} \right)}.$$

III. THE CONFORMING RUN LENGTH CONTROL CHART

The conforming run length control chart was proposed by Bourke (1991). The Conforming run length (*CRL*) is the number of inspected units between two consecutive nonconforming units including ending nonconforming unit. CRL has a geometric distribution and its probability mass function is given by

$$P(CRL) = p(1-p)^{CRL}, \quad CRL = 1, 2, 3, \ldots$$

The cumulative probability function of CRL is

$$F(CRL) = 1 - (1-p)^{CRL},$$

If $CRL \leq L$, an upward process shift is signaled. Therefore, for detection of an upward process shift (increase in p), a single lower control limit L of CRL chart is sufficient and it is given by

$$L = \frac{\ln(1-\alpha_{CRL})}{\ln(1-p_0)},$$

where, α_{CRL} is the type-I error probability of the CRL chart and p_0 is the in-control fraction nonconforming. L must be rounded to an integer.

For the CRL chart, ARL_{CRL} is the average number of the CRL samples required to detect out-of-control signal and it is given by

$$ARL_{CRL} = \frac{1}{p\left(1-(1-p)^L\right)}.$$

Following section gives the synthetic control chart for monitoring Weibull distribution parameters.

IV. THE SYNTHETIC WEIBULL TIME BETWEEN EVENT CONTROL CHART

In literature, Wu and Spedding (2000) proposed the synthetic control chart for detecting small shifts in a process mean. For the proposed synthetic control chart procedure given by Wu and Spedding is followed. The synthetic control chart is proposed by combining the operations of the t-chart and conforming run length chart. The operations of synthetic control chart are outlined below.

A. Determine t-chart based UCL (>0) and the CRL based lower control limit L.
B. Take a sample of n units for inspection and record the TBE X.
C. If $X < UCL$, a sample is a conforming one and control flow goes back to step (2). Otherwise, a sample is a nonconforming one and control flow continues to the next step.
D. Check number of samples between the current and previous nonconforming samples. This number is taken as CRL value for synthetic chart.
E. If $CRL > L$, then the process is said to be under control and control flow goes back to the step (2). Otherwise the process is taken as out-of-control and control flow continues to the next step.
F. Take action to locate and remove the assignable causes and then go back to step (2).

Generally, the synthetic control chart works the same way as the ordinary CRL chart, except that each unit in the CRL chart is replaced by a sample of n units in the synthetic control chart. In the CRL chart, a fraction nonconforming (p) is the probability that nonconforming unit occurs. However, in the synthetic control chart P corresponds to a nonconforming sample occurs.

Let $ARL_s(\delta)$ be the out-of-control ARL of the synthetic t-chart and calculated as follows:

$$ARL_S(\delta) = \frac{1}{P[1-(1-P)^L]}, \qquad (1)$$

Where δ is shift in shape parameter or scale parameter of the Weibull distribution.

The above equation (1) can be used to determine two design parameters L and UCL in synthetic control chart. If $ARL_s(\delta_0)$ is set at the value specified by the user, then any set of values of L and UCL satisfying the above equation (1) will result in a synthetic chart that can meet the requirement for $ARL_s(\delta_0)$ or false alarm rate.

V. DESIGN of the SYNTHETIC WEIBULL TIME BETWEEN EVENT CONTROL CHART

The design of the synthetic control chart is based on ARL; that is, $ARL_s(\delta_0)$ should be large so that the false alarm rate can be kept low, while $ARL_s(\delta)$ should be small so that a prompt detection of the process can be held. To design a synthetic control chart requires obtaining suitable values of *L* and *UCL*. For this, we have to consider two special cases of $ARL_s(\delta)$.

a) $ARL_s(\delta^*)$: The design shift δ^* is the magnitude considered large enough to seriously impair the quality of the products; thus the corresponding $ARL_s(\delta^*)$ should be as small as possible.

b) $ARL_s(\delta_0)$: $ARL_s(\delta_0)$ is decided by the requirement of the false alarm rate. The in-control ARL of the synthetic t-chart for β_0 and θ_0 is given by

$$ARL_S(\delta_0) = \frac{1}{P(\delta_0)[1-(1-P(\delta_0))^L]}, \qquad (2)$$

Where,

$$P(\delta_0) = P(X \geq UCL),$$
$$P(\delta_0) = 1 - P(X < UCL),$$

The synthetic control chart is properly designed by solving an optimization problem.
The objective function to be minimize is

$$ARL_S(\delta^*) = \frac{1}{P(\delta^*)[1-(1-P(\delta^*))^L]},$$

subject to the equality constraint

$$ARL_S(\delta_0) = \frac{1}{P(\delta_0)[1-(1-P(\delta_0))^L]}.$$

The optimal design procedure for the proposed chart, described in this section is similar to the design presented in Wu and Spedding (2000). To design the proposed synthetic chart, two optimal parameters (L, UCL) have to be selected. The objective function to be minimized is the out-of-control $ARL_s(\delta^*)$ with an optimal shift size (which is considered large enough to seriously impair the process quality) subject to a specified in-control ARL.
The optimal design procedure of the synthetic t-chart is described below:

A. Specify β_0, θ_0 and $ARL_s(0)$.
B. Initialize L as 1.
C. Obtain UCL from equation (2) by solving numerically.
D. Calculate $ARL_s(\delta^*)$ from the current UCL and L, using equation (2).
E. If $ARL_s(\delta^*)$ has been improved (reduced), increase L by one and then go back to step (3). Otherwise, go to the next step.
F. Take the current L and UCL as the final values in the synthetic control chart.

To illustrate the design of the synthetic t-chart, consider the case when $\beta_0 = 1.5$, $\theta_0 = 2$, $\delta^* = \theta^* = 4.2$ and $ARL_s(\delta_0) = 370$. Table 1 shows that each set of (*L*, *UCL*) results in different $ARL_s(\delta^*)$. The ARL first declines and then increases. The $ARL_s(\delta^*)$ reaches its minimum at 3.9904 when *L* = 9 and *UCL* = 5.056. So in this case, the design parameters of the synthetic t-chart are *L* = 9 and *UCL* = 5.056.

Table1: Different sets of values of *L* and *UCL* for the synthetic t-chart.

L	UCL	$ARL_s(\delta^*)$
1	4.12	6.9806
2	4.428	5.2460
3	4.602	4.6151
4	4.724	4.3122
5	4.817	4.1493
6	4.892	4.0598
7	4.955	4.0137
8	5.009	3.9937
9	**5.056**	**3.9904**
10	5.098	3.9989
11	5.136	4.0155
12	5.17	4.0362
13	5.202	4.0619
14	5.231	4.0888
15	5.258	4.1173

VI. COMPARISON STUDY Of The PROPOSED CONTROL CHART

ARL is used as a performance measure criterion for comparison of the proposed synthetic control chart and t-chart. ARL is an average number of TBE points required to signal an out-of-control status. The Weibull distribution has two parameters and a change in any of them could cause an out-of-control signal in a process. The change in the parameters affects the ARL of the control charts based on Weibull distribution. There are three possible ways in which the parameters of Weibull distribution can change and they are investigated in the following.

Case 1: The change in the scale parameter

Table 2 and Table 3 give the ARL of t-chart and synthetic t-chart when there is a shift only in scale parameter and fixed shape parameter with the in-control $\theta_0 = 0.5,\ 2.$

Table 2: ARL values of t-chart and synthetic t-chart when $\beta = 0.5,\ \theta_0 = 0.5$, L=12, UCL=8.639 and $ARL_s(\delta_0) = 370$.

Shift in scale parameter (θ)	ARL of t-chart	ARL of synthetic t-chart
0.5	370.00	370.09
1	65.46	38.16
1.5	30.39	15.81
2	19.24	9.91
2.5	14.08	7.40
3	11.18	6.06
3.5	9.35	5.22
4	8.09	4.65
4.5	7.18	4.24
5	6.49	3.92
6	5.51	3.47
8	4.39	2.93
10	3.75	2.61

Table 3: ARL values of t-chart and synthetic t-chart when $\beta = 1.5,\ \theta_0 = 2$, L=9, UCL=5.056 and $ARL_s(\delta_0) = 370$.

Shift in scale parameter (θ)	ARL of t-chart	ARL of synthetic t-chart
2	370.03	370.01
2.2	168.31	133.30
2.4	89.89	60.54
2.6	54.04	32.65
2.8	35.51	20.04
3	25.00	13.57
3.6	11.57	6.22
4	8.09	4.52
5	4.46	2.81
6	3.12	2.18

From Table 2 and Table 3, we observed that the ARL values of the synthetic t-chart are significantly less than the ARL values of the t-chart for all increasing shifts in the scale parameter. Therefore, the synthetic t-chart performs significantly better than the t-chart.

Case 2: The Change in the Shape Parameter

Table 4 and Table 5 give the ARL of t-chart and the synthetic t-chart for fixed scale parameter and there is only shift in shape parameter. In this case, for computing ARL values the scale parameter is fixed at $\theta = 0.5,\ 2$ and in-control

Table 4: ARL values of t-chart and synthetic t-chart when $\beta_0 = 1.5,\ \theta = 2$, L=9, UCL=5.056 and $ARL_s(\delta_0) = 370$

Shift in shape parameter (β)	ARL of t-chart	ARL of synthetic t-chart
0.1	3.08	3.08
0.2	3.55	3.47
0.3	4.17	3.99
0.4	4.98	4.68
0.5	6.10	5.63
0.6	7.66	6.96
0.7	9.89	8.90
0.8	13.20	11.81
0.9	18.26	16.37
1	26.32	23.78
1.1	39.71	36.40
1.2	63.09	58.97

1.3	106.26	101.62
1.4	191.11	187.14
1.5	**370.03**	**370.09**
1.6	778.57	790.06
1.7	1798.82	1830.88
1.8	4617.72	4634.64
1.9	13346.58	12904.88
2	44085.27	39837.31

Table 5: ARL values of t-chart and synthetic t-chart when $\beta_0 = 0.5,\ \theta = 0.5$, L=12, UCL=8.639 and $ARL_s(\delta_0) = 370$

Shift in shape parameter (β)	ARL of t-chart	ARL of synthetic t-chart
0.1	4.17	3.88
0.2	7.66	6.55
0.3	18.26	15.01
0.4	63.09	54.71
0.42	85.62	75.97
0.44	118.84	108.26
0.46	168.98	158.49
0.48	246.59	238.67
0.5	**370.00**	**370.10**
0.52	572.04	591.69
0.54	913.24	976.48
0.56	1509.08	1665.69
0.58	2587.68	2941.08
0.6	4617.05	5383.46

From Table 4 and Table 5, observed that the as shape parameter decreases from its in-control value the both control charts performs better. For decreasing shifts in the shape parameter from its in-control value proposed synthetic t-chart performs significantly better than the t-chart. As shape parameter increases from its in-control value, the ARL of both the control charts goes on increasing from its in-control ARL. To detect increasing shifts in the shape parameter from its in-control value, both the control charts are insensitive. In this case, performance of the proposed synthetic t-chart is worth for increasing shifts in shape parameter from its in-control value.

Case 3: The change in both the shape and scale parameters

Third case is a change in both the parameters shape as well as scale with in-control $\beta_0 = 0.5,\ \theta_0 = 0.5.$ Table 6 and Table 7 give ARL of t-chart and the synthetic t-chart for shifts in both the parameters.

Table 6: ARL values of t-chart and synthetic t-chart when $\beta_0 = 0.5,\ \theta_0 = 0.5$, L=12, UCL=8.639 and $ARL_s(\delta_0) = 370$

Shift in scale parameter (θ)	Shift in shape parameter (β)	ARL of t-chart	ARL of synthetic t-chart
0.5	0.5	370.00	370.10
1	0.6	261.56	141.08
1.5	0.7	264.95	90.57
2	0.8	288.98	65.10
2.5	0.9	316.62	48.06
3	1	339.77	35.60
5	1.1	52.63	7.06
6	1.2	36.94	4.99
7	1.3	26.77	3.81
10	1.4	8.90	2.26
15	1.5	3.52	1.55

From Table 6, it is seen that the synthetic t-chart performs significantly better than the t-chart for moderate to large shifts in the scale parameter and small shifts in the shape parameter. It is also remarked that the if shifts are of s equal shifts in both the parameters scale and shape, both the control charts are insensitive to detect out-of-control signal and ARL values are goes on increasing. Table 7 shows the ARL values of the proposed synthetic control chart and t-chart when there is shift of equal magnitude in both the parameters.

Table 7: ARL values of t-chart and synthetic t-chart when $\beta_0 = 0.5,\ \theta_0 = 0.5$, L=12, UCL=8.639 and $ARL_s(\delta_0) = 370$

Shift in scale parameter (θ)	Shift in shape parameter (β)	ARL of t-chart	ARL of synthetic t-chart
0.5	0.5	370.0026	370.1027
0.6	0.6	1926.001	1741.386
0.7	0.7	13527.53	9374.093
0.8	0.8	132438.8	56447.17

VII. CONCLUSIONS

In this article, the upper-sided synthetic t- chart is proposed to detect a signal, when scale or shape parameters shifts from their in-control value. Proposed synthetic control chart performs significantly better than the t-chart to detect all positive shifts in scale parameter. When there is shift in shape parameter in positive direction from the in-control value of shape parameter the synthetic chart performance is poor. For decreasing shifts in shape parameter, proposed synthetic control chart performs better. In case when there is a shift in both the parameters scale and shape, the proposed synthetic control chart performs significantly better than the t-chart. However, when there is shift of equal magnitude in both the parameters, both the control charts are insensitive to detect a signal. ARL values of the proposed synthetic t-chart are at least 40% less than the t-chart In general synthetic control chart has a higher power of detecting an out-of-control signal. The proposed chart is simple and easy for implementation.

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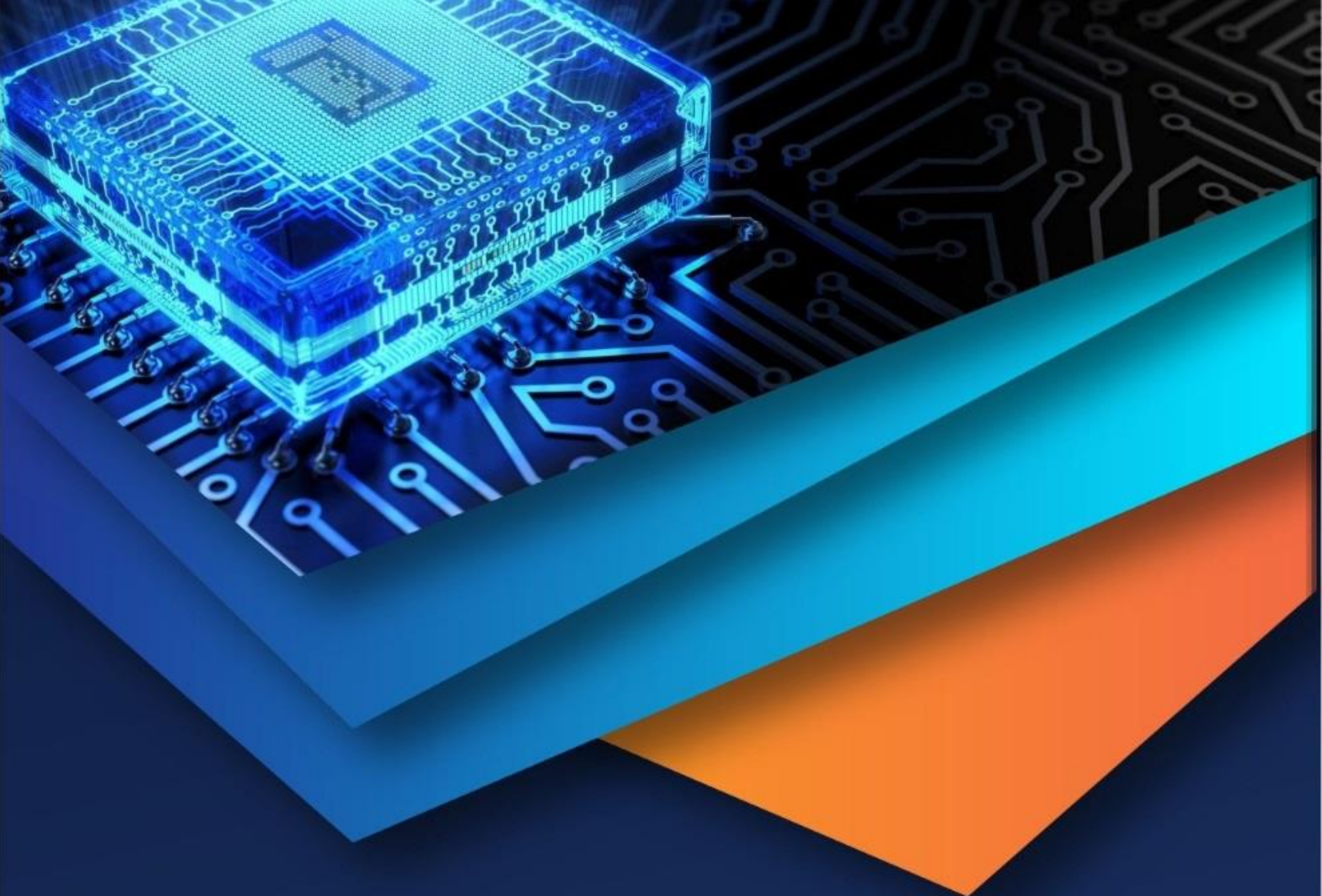

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