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Cube Divisor Cordial Labeling For Some Graphs

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Abstract: Let G = {V (G), E (G)} be a simple graph and $f : V(G) \rightarrow \{1, 2, \cdots, |v|\}$ be a bijection. For each edge uv, assign the label 1. If either $[f(u)]^3 / f(v)$ or $[f(v)]^3 / f(u)$ and the label 0 otherwise f is called a cube divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$ .A graph with a square divisor cordial labeling is called a square divisor cordial graph. In this work, $W_n \oplus P_2$, $W_n \oplus W_m$, Merge graph and Bow graph a discussion is made on under Cube Divisor Cordial Labeling.
Keywords: Cube divisor cordial labeling, Wheel graph, Merge graph, Bow graph.

I. INTRODUCTION

A graph $G(V, E)$ is of vertices and edges. The vertex set $V(G)$ is non-empty set and the edge set $E(G)$ may be empty. Labelings of graphs subject to certain condition gave raise to enormous work which listed by J. A. Gallian [1], Cube Divisor Cordial Graph were introduced by K. K. Kanani and M. I. Bosmia [2] . Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \rightarrow 1, 2, \ldots, |V(G)|$ be a bijection. For each edge$e = uv$, assign the label 1 if $[f(u)]^3 / f(v)$ or $[f(v)]^3 / f(u)$ and the label 0 otherwise. The function f is called a Cube Divisor Cordial Labeling if $|e_f(0) - e_f(1)| \leq 1$. S. K. Vaidya and U. M. Prajapati introduced $W_n \oplus P_2$ admits some results on prime and K - prime labeling [5] , The graph $W_n \oplus W_m$ introduced S. K. Vaidya and U. M. Prajapati on some results on prime and K-prime labeling [5] , A. Solairaju and R. Raziya Begam proved the merge graph $S_n * S_{n-1}$ on edge-magic labeling of some graphs [3] , the bow graph $B_{m,m} + e$ proved R. Uma and N. Arun vigneshwari on Square sum labeling bow, bistar, and star related graphs [4] have been discussed in this paper.

II. DEFINITIONS

A. Cube Divisor Cordial Graph (CDCG)

Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \rightarrow 1, 2, \cdots, |V(G)|$ be a bijection. For each edge $e = uv$, assign the label 1 if $[f(u)]^3 \mid f(v)$ or $[f(v)]^3 \mid f(u)$ and the label 0 otherwise. The function f is called a Cube Divisor Cordial Labeling if $\left| e_f(0) - e_f(1) \right| \leq 1$. A graph with a Cube Divisor Cordial Labeling is called a Cube Divisor Cordial Graph**.**

B. $W_n \oplus P_2$

The graph obtained by identifying any of the rim vertices of a wheel W_n with an end vertex of a path graph P_2 is a prime graph.

C. $W_n \oplus W_m$

The graph $G = W_n \oplus W_m$i is the graph obtained by joining apex vertices of wheelsW_n and W_m to a new vertex w.

D. Merge graph

A merge graph$G_1 * G_2$ can be formed from two graphs G_1 and G_2 by merging a node of G_1 with a node of G_2. As an example let us consider T_3, a tree with three vertices and S_2 a star on three vertices then $T_3 * S_2$ is formed. Consider a vertex of T_3. Consider a vertex v_1 of S_2 .

E. Bow graph

A Bow graph is defined to be a double shell is which each shell has any order.

III. RESULTS

A. Theorem

The graph G obtained by identifying an apex vertex of a wheel graphW_n with an end vertex P_2 is a Cube Divisor Cordial Graph.

Proof:

Let $G = W_n \oplus P_2$. The order of G is $p = n + 2$ and size is $q = 2n + 1$. Let w_0 be the apex vertex

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$w_1, w_2, \dots, w_n$ be the consecutive rim verticesW_n of andv_1, v_2 be the vertices of path P_2 . Now, let us define the edge set E = $\{e_{0i}, e_{12}\}$, where $e_{0i} = (w_0, w_i)$ and$e_{12} = (v_1, v_2)$.
Let us define the function $f : V(G) \rightarrow \{1,2,3,\cdots,n+2\}$ as follows: $(w_0) = 1$;
$f(w_i) = i + 1, \forall 1 \leq i \leq n$;
$f(v_2) = n + 2$;
Also $|e_f(0) - e_f(1)| \leq 1$.
Since $e_f(0) = n$ and $e_f(1) = n + 1$. Hence, the graph $W_n \oplus P_2$ is Cube Divisor Cordial Graph.

B. Example

The Cube Divisor Cordial Labeling of $W_n \oplus P_2$ is shown in Fig 3.1.

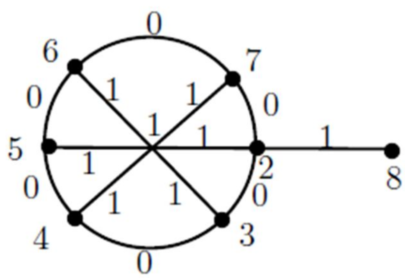

Fig 3.1 CDCL of $W_n \oplus P_2$

Therefore by the definition of Cube Divisor Cordial Labeling, $|e_f(0) - e_f(1)| \leq 1$.
$|7 - 6| \leq 1$.
Hence, the graph $W_n \oplus P_2$ is a Cube Divisor Cordial Labeling graph.

C. Theorem

The graph G obtained by identifying an apex vertex of a wheel graph $W_n \oplus W_m$ for$= n$, where $m, n = even$.
Proof:
Let the common apex vertex of a graph G be $w_0$and the consecutive rim vertices of wheel W_m and W_n . The vertex set of W_m and $W_n$are $\{u_1, u_2, \dots, u_m\}$ and $\{v_1, v_2, \dots, v_n\}$. The order of G is $p = n + m + 1$ and the size is $q = 2(n + m)$. Theedge set $E = \{e_{0i}, e_{0j}, e_{ii+1}, e_{jj+1}\}$ where $1 \leq i, j \leq n$. where
$e_{0i} = (w_0, u_i)$; $e_{0j} = (w_0, v_j)$;
$e_{ii+1} = (u_i, u_{i+1})$; $e_{jj+1} = (v_j, v_{j+1})$;
Define the function $f : V(G) \rightarrow \{1,2,3,\cdots,m+n+1\}$; $f(w) = 1$;
$f(u_i) = i + 1, \forall 1 \leq i \leq m$;
$f(v_j) = n + j + 1, \forall 1 \leq j \leq n$;
$e_f(0) = n + m$ and $e_f(1) = n + m$.
$|e_f(0) - e_f(1)| \leq 1$.
$|n + m - (n + m)| \leq 1$. This is the condition ofCDCG.
Hence, $W_n \oplus W_m$ is Cube Divisor Cordial Graph.

D. Example

The Cube Divisor Cordial Labeling of $W_4 \oplus W_4$is shown in Fig 3.2.
$e_f(0) = 8$ and $e_f(1) = 8$.
By the definition of Cube Divisor Cordial Labeling, $|e_f(0) - e_f(1)| \leq 1$.
$|8 – 8| \leq 1$.

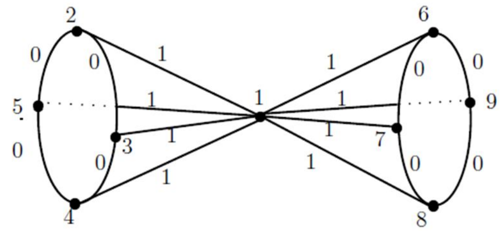

Fig 3.2 CDCL of $W_4 \oplus W_4$

Hence, the graph $W_n \oplus W_m$ is a Cube Divisor Cordial Labeling graph.

E. Theorem

Merge graph of $S_n * S_{n-1}$ is a Cube Divisor Cordial Graph.

Proof:

Let G = {V, E} is a graph obtained by merging the apex vertex of star graph $S_{n-1}$with one of the pendent vertex of the star graph S_n . Then the order of G is $p = 2n$and the size is $q = 2n - 1$. Then the vertex set $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$. The apex vertex of S_n and S_{n-1} are v_1 and $u_1$respectively. The edge set of $S_n$is = $\{e_{0i}\}$, where$e_{0i} = (v_0, v_i)\ \forall 1 \le i \le n$ and edge set of S_{n-1} is

$E = \{e_{nj}\}$, where $e_{nj} = (v_n, u_j)\ \forall 1 \le j \le n - 1$. Let us define the function

$f : V(G) \to \{1, 2, 3, \cdots, 2n - 1\}$ by,

$f(v_0) = 1$; $f(v_n) = 2$;

$f(v_i) = 2i + 2\ \forall\ 1 \le i \le n$;

$f(u_j) = 2j + 1\ \forall\ 1 \le j \le n - 1$.

Also $e_f(0) = n - 1$and $e_f(1) = n$, such that

Therefore by the definition of Cube Divisor Cordial Graph, $|e_f(0) - e_f(1)| \le 1$.

Hence, the merge graph $S_n * S_{n-1}$ of is a Cube Divisor Cordial Graph.

F. Example

Cube Divisor Cordial labeling of merge graph $S_5 * S_4$ is shown in Fig 3.3

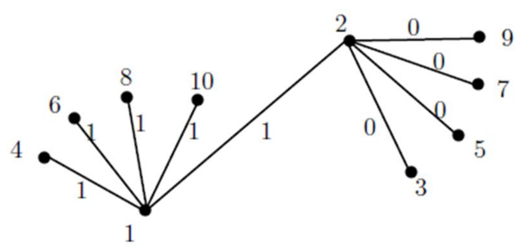

Fig 3.3 CDCL of $S_5 * S_4$

ef (0) = 4 and ef (1) = 5.

Therefore by the definition of Cube Divisor Cordial Labeling, $e_f(0) - e_f(1)| \le 1$.

$|5 - 4| \le 1$.

Hence, the merge graph of $S_5 * S_4$ is a Cube Divisor Cordial Labeling.

G. Theorem

The Bow graph $B_{m,m} + e$ is a Cube Divisor Cordial Graph $(If\ m = n)$.

Proof:

Let the graph $G = B_{m,m} + e$ be a bow graph. Let the vertex set $V(G) = \{w_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n, u_0\}$, where w_0 is the apex vertex, $\{v_1, v_2, \dots, v\_m\}$ are the vertices of first shell adjacent to w_0 is the apex vertex, $\{v_1, v_2, \dots, v_m\}$are the vertices of first shell adjacent to w_0 and $\{u_1, u_2, \dots, u_n\}$ are the vertices of second shell adjacent to w_0. Then, the order of G is $p = m + n +$

2 and the size of G is $q = 2(m + n) - 1$. Let us define the edge set $(G) = \{e_{0i}, e_{0j}, e_{n0}\}$, where $e_{0i} = (w_0, v_i)$; $e_{0j} = (w_0, u_j)$; $e_{n0} = (u_n, u_0)$.

Define the vertex labeling function $f : V(G) \to \{1, 2, 3, \cdots, m + n + 2\}$ as follows: $f(w_0) = 1$;

$f(v_i) = i + 1, \forall 1 \leq i \leq n$;

$f(u_j) = n + j + 1, \forall 1 \leq j \leq n$;

$f(u_0) = 2n + 2$;

$e_f(0) = m + n - 1$, $e_f(1) = m + n$.

$|e_f(0) - e_f(1)| \leq 1$..

$|m + n - 1 - (m + n)| \leq 1$

Hence, the Bow graph $B_{m,m} + e$ is a Cube Divisor Cordial Graph.

H. Example

Cube Divisor Cordial Labeling of $B_{4,4} + e$ is shown in Fig 3.4.

$e_f(0) = m + n - 1$, $e_f(1) = m + n$;

$e_f(0) = 7$ and $e_f(1) = 8$.

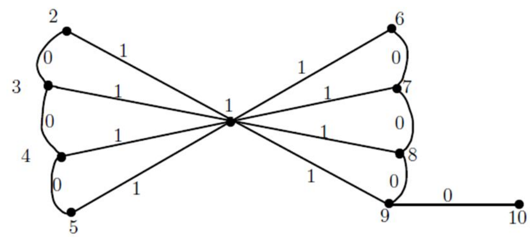

Fig 3.4 SDCL of $B_{4,4} + e$

Therefore by the definition of Cube Divisor Cordial Labeling, $|e_f(0) - e_f(1)| \leq 1$.

$|8-7| \leq 1$. Hence, the Bow graph $B_{4,4}$ + e is a Cube Divisor Cordial Labeling.

IV. CONCLUSION

The overview of Cube Divisor Cordial Graphs is the current interest due to its diversified applications. Here we investigate some results corresponding to labeled graphs. Similar work can be carried out for the other graphs also. The complied information related to CDCL will be useful for researchers to get some idea related to their field.

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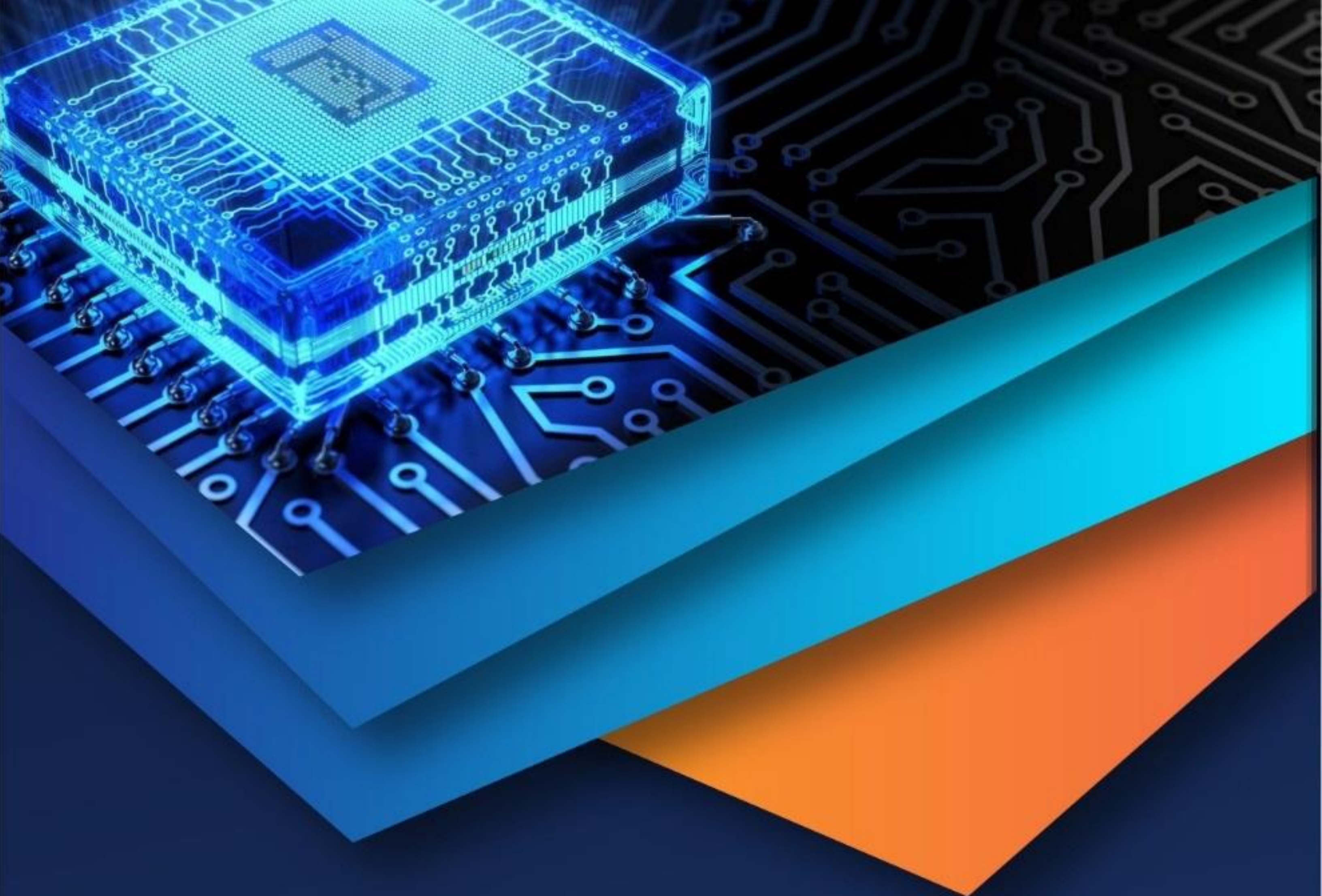

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