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Families of Solutions of a Cubic Diophantine Equation

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Abstract: The ternary cubic equation $\alpha(x^2+y^2)+x+y+1=(2\alpha-1)xy+\alpha(\alpha+2)z^3$ such that $\alpha \in N$, is considered for determining its non-zero distinct integral solutions. Different patterns of integral solutions to the ternary cubic equation under consideration are obtained.
Keyword- Diophantine equations, Ternary cubic equation, integer solutions, lattice points.

I. INTRODUCTION

Number theory is, as the name suggests, is devoted to the study of numbers, first and foremost integers. The classical problems in Number Theory are often easy to formulate knowing some basic mathematics and this makes the area attractive. It is often only the formulation are simple, proofs are very long, complicated and requires excessive numerical work.

Diophantine equations are numerously rich because of its variety. The determination of integral solutions for cubic (homogeneous or non-homogeneous) Diophantine equations with three variables has been an interest to mathematicians since antiquity as can be seen from [1-3]. In this Communication, the non-homogeneous ternary cubic Diophantine equation represented by $\alpha(x^2+y^2)+x+y+1=(2\alpha-1)xy+\alpha(\alpha+2)z^3$ is considered for its non-zero distinct lattice points.

II. METHOD OF ANALYSIS

The ternary cubic Diophantine equation under consideration is

$$\alpha(x^2+y^2)+x+y+1=(2\alpha-1)xy+\alpha(\alpha+2)z^3 \tag{1}$$

Introduction of the transformations,

$$x=u+v,\ y=u-v \tag{2}$$

in (1) leads to

$$(u+1)^2+(4\alpha-1)v^2=\alpha(\alpha+2)z^3 \tag{3}$$

Equation (3) is solved through different methods and thus, we obtain different patterns of solutions to (1).

Pattern 1:

Take $$z=a^2+(4\alpha-1)b^2 \tag{4}$$

Write $$\alpha(\alpha+2)=\left((\alpha-1)+i\sqrt{(4\alpha-1)}\right)\left((\alpha-1)-i\sqrt{(4\alpha-1)}\right) \tag{5}$$

Using (4) and (5) in (3) and employing the method of factorization,

Write $$(u+1)+i\sqrt{(4\alpha-1)}\,v=\left((\alpha-1)+i\sqrt{(4\alpha-1)}\right)\left(a+i\sqrt{(4\alpha-1)}b\right)^3 \tag{6}$$

Equating real and imaginary parts of (6) on both sides we get

$$u+1=(\alpha-1)\left(a^3-3(4\alpha-1)ab^2\right)-3(4\alpha-1)a^2b+(4\alpha-1)^2b^3$$
$$v=a^3+(\alpha-1)3a^2b-(4\alpha-1)3ab^2-(4\alpha-1)(\alpha-1)b^3.$$

Substituting the values of u,v in (2)

$$x=\alpha a^3-9\alpha a^2b-3\alpha(4\alpha-1)ab^2+3\alpha(4\alpha-1)b^3-1$$

$$y=(\alpha-2)a^3-3(5\alpha-2)a^2b-3(\alpha-2)(4\alpha-1)ab^2+(4\alpha-1)(5\alpha-2)b^3-1$$

$$z=a^2+(4\alpha-1)b^2$$

Note 1:Equation (5) can be written as

$$\alpha(\alpha+2)=\left(-(\alpha-1)+i\sqrt{(4\alpha-1)}\right)\left(-(\alpha-1)-i\sqrt{(4\alpha-1)}\right)$$

Proceeding as above, we obtain

$x=-(\alpha-2)a^3-3(5\alpha-2)a^2b+3(\alpha-2)(4\alpha-1)ab^2+(4\alpha-1)(5\alpha-2)b^3-1$

$y=-\alpha a^3-9\alpha a^2b+3\alpha(4\alpha-1)ab^2+3\alpha(4\alpha-1)b^3-1$

$z=a^2+(4\alpha-1)b^2$

Pattern 2:

Equation (3) can be written as

$$(u+1)^2+(4\alpha-1)v^2=\alpha(\alpha+2)z^3*1$$

Write 1 as $$1=\frac{\left((2\alpha-1)+i\sqrt{(4\alpha-1)}\right)\left((2\alpha-1)-i\sqrt{(4\alpha-1)}\right)}{(2\alpha)^2}$$

and $$\left((u+1)+i\sqrt{(4\alpha-1)}\right)=\frac{\left((\alpha-1)+i\sqrt{(4\alpha-1)}\right)\left(a+i\sqrt{(4\alpha-1)}\,b\right)^3\left((2\alpha-1)+i\sqrt{(4\alpha-1)}\right)}{2\alpha} \quad (7)$$

Equating real and imaginary parts, we have

$$u+1=\frac{1}{2\alpha}[(2\alpha^2-7\alpha+2)a^3-3(3\alpha-2)(4\alpha-1)a^2b-3(2\alpha^2-7\alpha+2)(4\alpha-1)ab^2+(4\alpha-1)^2(3\alpha-2)b^3]$$

$$v=\frac{1}{2\alpha}[(3\alpha-2)a^3+3(2\alpha^2-7\alpha+2)a^2b-3(3\alpha-2)(4\alpha-1)ab^2-(2\alpha^2-7\alpha+2)(4\alpha-1)b^3]$$

Since our interest centers on finding integral solutions, replace a by $2\alpha A$ and b by $2\alpha B$ in the above equations. Thus the corresponding solutions to (1) are given by

$x=(2\alpha)^2[2\alpha(\alpha-2)A^3-6\alpha(5\alpha-2)A^2B-6\alpha(4\alpha^2-9\alpha+2)AB^2+2\alpha(4\alpha-1)(5\alpha-2)B^3]-1$

$y=(2\alpha)^2\left[(2\alpha^2-10\alpha+4)A^3-6(7\alpha^2-9\alpha+2)A^2B-6\left((4\alpha-1)(\alpha^2-5\alpha+2)\right)AB^2+(4\alpha-1)(14\alpha^2-18\alpha+4)B^3\right]-1$

$z=(2\alpha)^2[A^2+(4\alpha-1)B^2]$

Pattern 3:

Instead of (5), $\alpha(\alpha+2)$ can be written as

$$\alpha(\alpha+2)=\frac{\left((2\alpha+1)+i\sqrt{(4\alpha-1)}\right)\left((2\alpha+1)+i\sqrt{(4\alpha-1)}\right)}{2^2}$$

Proceeding as in pattern 1 and performing some algebra, we obtain the distinct non-zero integral solutions to (1) as

$x=2^2[2(\alpha+1)A^3-6(\alpha-1)A^2B-6(4\alpha^2+3\alpha-1)AB^2+2(4\alpha-1)(\alpha-1)B^3]-1$

$y=2^2[2\alpha A^3-18\alpha A^2B-6\alpha(4\alpha-1)AB^2+6\alpha(4\alpha-1)b^3]-1$

$z=2^2[A^2+1(4\alpha-1)B^2]$

Note 2:

Instead of (5), $\alpha(\alpha+2)$ can be written as

$$\alpha(\alpha+2)=\frac{\left(-(2\alpha+1)+i\sqrt{(4\alpha-1)}\right)\left(-(2\alpha+1)+i\sqrt{(4\alpha-1)}\right)}{2^2}$$

Proceeding as in pattern 1 and performing some algebra, we obtain the distinct non-zero integral solutions to (1) as

$x=2^2[-2\alpha A^3-18\alpha A^2B+6\alpha(4\alpha-1)AB^2+6\alpha(4\alpha-1)b^3]-1$

$y=2^2[-2(\alpha+1)A^3-6(\alpha-1)A^2B+6(4\alpha^2+3\alpha-1)AB^2+2(4\alpha-1)(\alpha-1)B^3]-1$

$z=2^2[A^2+(4\alpha-1)B^2]$

Pattern 4:

Equation (7) can be written as

$$\left((u+1)+i\sqrt{(4\alpha-1)}\right)=\left(\frac{(2\alpha+1)+i\sqrt{(4\alpha-1)}}{2}\right)\left(a+i\sqrt{(4\alpha-1)}\,b\right)^3\left(\frac{(2\alpha-1)+i\sqrt{(4\alpha-1)}}{2\alpha}\right)$$

Equating real and imaginary parts on both sides, we get

$$u+1=(\alpha-1)(a^3-3(4\alpha-1)a^2b)-(4\alpha-1)(3a^2b-(4\alpha-1)b^3)$$

$$v=a^3+3a^2b-3(4\alpha-1)ab^2-(4\alpha-1)b^3$$

Substituting the values of u,v in (2) we obtain the non-zero integral solutions to (1) as

$x = [\alpha(a^3 - 3(4\alpha - 1)ab^2) - 3\alpha(3a^2b - (4\alpha - 1)b^3)] - 1$
$y = [(\alpha - 2)(a^3 - 3(4\alpha - 1)ab^2) - (5\alpha - 2)(3a^2b - (4\alpha - 1)b^3)] - 1$
$z = a^2 + (4\alpha - 1)b^3$

III. CONCLUSION

In this paper, we have made an attempt to obtain a complete set of non-trivial distinct integral solutions for the non-homogeneous ternary cubic equation. To conclude, one may search for other choices of solutions to the considered cubic equation and further cubic equations with multi variables.

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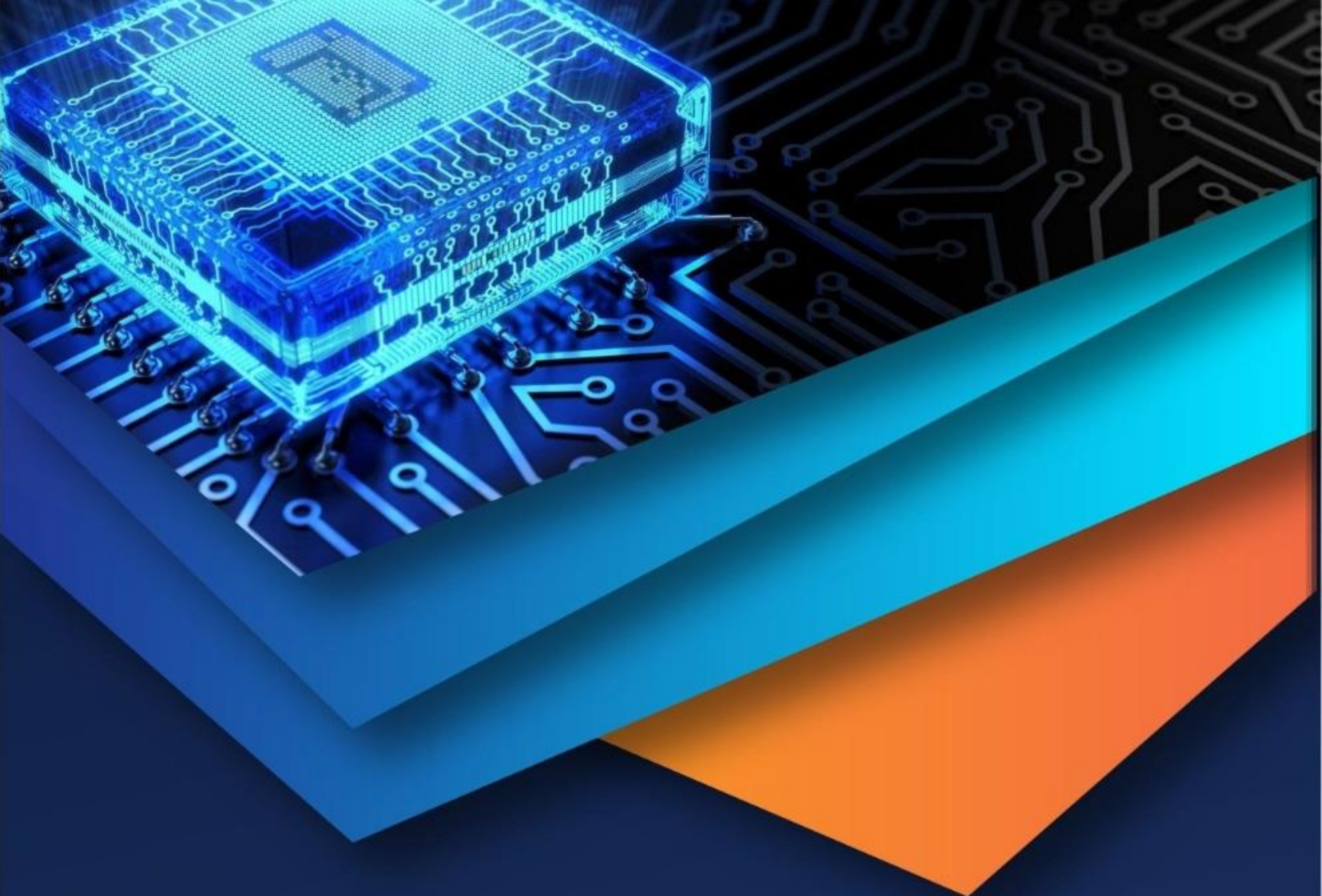

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