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Conquences of A Fixed Point Theorem for Quasi-Contractions of D*-Metric Spaces

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Abstract: The main aim of this paper is to prove the consequences of existence of fixed point on λ -generalized contraction of self mapping functions on D*-metric space. Key Words: D*-metric space, Quasi-contraction, self mappings,

I. INTRODUCTION

The notion of Quasi-contraction defined for selfmaps of metric spaces given by Lj. B. Ciric [3] has been extended to the selfmaps of D^* -metric spaces as follows:

A. Definition

Let f be a selfmap of a D*-metric space (X, D^*) and $x \in X$, $n \ge 1$ be an integer. The *orbit of* x under f of length n, denoted by $O_f(x:n)$, is defined by

$$O_f(x:n) = \left\{x, fx, f^2x, \ldots, f^nx\right\}$$

We define the diameter $\delta(A)$ of a set A in a D*-metric space (X, D^*) by $\delta(A) = \frac{Sup}{x, y \in A} \{D^*(x, y, y)\}$

The following Lemmas are use full in proving fixed point theorems of quasi-contractions on D*-metric spaces:

B. Definition

A selfmap f of a D*-metric space (X, D*) is called a Quasi-contraction, if there is a number q with $0 \le q < 1$ such that

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$$(fx, fy, fy) \le q \cdot \max \{ D^*(x, y, y), D^*(x, fx, fx), D^*(y, fy, fy), \}$$

$$D^*(x, fy, fy), D^*(y, fx, fx) \}$$

for all $x, y \in X$.

D

As already noted that for every λ -generalized contraction is a quasi-contraction. However the following example gives a quasi-contraction *f* on a *D**-metric space (*X*, *D**) which is not a λ -generalized contraction.

A. Definition

II. PRELIMINARY NOTES

Let f be a selfmap of a D*-metric space (X, D^*) and $x \in X$, $n \ge 1$ be an integer. The *orbit of* x under f of length n, denoted by $O_f(x:n)$, is defined by

$$O_f(x:n) = \left\{x, fx, f^2x, \dots, f^nx\right\}$$

We define the diameter $\delta(A)$ of a set A in a D*-metric space (X, D^*) by $\delta(A) = \frac{Sup}{x, y \in A} \{D^*(x, y, y)\}$

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The following Lemmas are use full in proving fixed point theorems of quasi-contractions on D^* -metric spaces: In this section we prove

B. Lemma

Suppose f is a quasi-contraction with constant q on a D*-metric space (X, D^*) and n be a positive integer. Then for each $x \in X$ and all integers $i, j \in \{1, 2, 3, ..., n\}$,

$$D^*(f^i x, f^j x, f^j x) \leq q \cdot \delta \left[O_f(x:n)\right] < \delta \left[O_f(x:n)\right].$$

Proof: Let $x \in X$ be arbitrary, $n \ge 1$ be an integer and $i, j \in \{1, 2, 3, ..., n\}$. Then $f^{i-1}x, f^{j-1}x, f^{i}x, f^{j}x \in O_f(x;n)$ and since f is a quasi-contraction,

$$D*(f^{i}x, f^{j}x, f^{j}x) = D*(ff^{i-1}x, ff^{j-1}x, ff^{j-1}x)$$

$$\leq q.\max\{D*(f^{i-1}x, f^{j-1}x, f^{j-1}x), D*(f^{i-1}x, f^{i}x, f^{i}x), D*(f^{j-1}x, f^{j}x, f^{j}x), D*(f^{j-1}x, f^{j}x, f^{j}x)\}$$

$$\leq q.Sup\{D*(u, v, v,): u, v \in O_f(x:n)\}$$

$$= q.\delta[O_f(x:n)]$$

C. Lemma

Suppose f is a quasi-contraction with constant q on a D^* -metric space (X, D^*) and $x \in X$, then for every positive integer n, there exists positive integer $k \le n$, such that

$$D^*(x, f^*x, f^*x) = \delta \left[O_f(x:n)\right]$$

Proof: If possible assume that the result is not true. This implies that there is positive integer m such that for all $k \le m$, we have $D^*(x, f^kx, f^kx) \ne \delta[O_f(x:m)]$. Since $O_f(x:m)$ contains x and f^kx for $k \le m$, it follows that

$$D^*(x, f^k x, f^k x) < \delta \left[O_f(x : m) \right]$$

Since $O_f(x:m)$ is closed, there exists $i, j \in \{1, 2, 3, ..., m\}$ such that $D^*(x, f^*x, f^*x) = \delta[O_f(x:m)]$, contradicting the Lemma 2.1. Therefore

$$D^*(x, f^k x, f^k x) = \delta [O_f(x:n)]$$
 for some $k \le n$.

D. Lemma

Suppose f is a quasi-contraction with constant q on a D^* -metric space (X, D^*) , then

$$\delta \Big[O_f (x:\infty) \Big] \leq \frac{1}{1-q} D^* (x, fx, fx) \text{ for all } x \in X.$$

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Proof: Let $x \in X$ be arbitrary. Since $O_f(x:1) \subseteq O_f(x:2) \subseteq \ldots \subseteq O_f(x:n) \subseteq O_f(x:n+1) \subseteq \ldots$, we get that

$$\delta \Big[O_f (x:1) \Big] \le \delta \Big[O_f (x:2) \Big] \le \ldots \le \delta \Big[O_f (x:n) \Big] \le \delta \Big[O_f (x:n+1) \Big] \le \ldots, \text{showing}$$
$$\lim_{n \to \infty} \delta \Big[O_f (x:n) \Big] = Sup \Big\{ \delta \Big[O_f (x:n) \Big] : n = 1, 2, 3, \ldots \Big\}.$$

Therefore to prove the Lemma, it is enough to show

III. MAIN RESULT

A. Theorem

Suppose f is a selfmap of a D^* -metric space (X, D^*) and X is f-orbitally complete. If there is a positive integer k such that f^k is a quasi-contraction with constant q. Then f has a unique fixed point $u \in X$. In fact,

$$u = \lim_{n \to \infty} f^n x \text{ for any } x \in X$$

and

$$D^*(f^n x, u, u) \leq \frac{q^n}{1-q} a(x) \text{ for all } x \in X, n \geq 1$$

where $a(x) = \max\left\{D^*(f^i x, f^{i+k} x, f^{i+k} x): i = 1, 2, 3, ...\right\}$ and $m = \left[\frac{n}{k}\right]$, the greatest integer not exceeding $\frac{n}{k}$.

Proof: Suppose f^k is a quasi-contraction of a D^* -metric space (X, D^*) . It has unique fixed point by Theorem 3.1. Let u be a fixed point of f^k . Then we claim that fu is also a fixed point of f^k . In fact,

$$f^{k}(fu) = f^{k+1}u = f(f^{k}u) = fu$$

By the uniqueness of fixed point of f^k , it follows that fu = u, showing that u is a fixed point of f. Uniqueness of the fixed point of f can be proved as in the Theorem 3.1.

To prove (3.3), let *n* be any integer. Then by the division algorithm, we have, n = mk + j, $0 \le j < k$, $m \ge 0$ Therefore $x \in X$, $f^n x = (f^k)^m f^j x$, since f^k is a quasi-contraction,

$$D^*(f^n x, u, u) \leq \frac{q^m}{1-q} D^*(f^j x, f^k f^j x, f^k f^j x)$$

$$\leq \frac{q^{m}}{1-q} \cdot \max\left\{ D^{*} \left(f^{i}x, f^{k}f^{i}x, f^{k}f^{i}x \right) : i = 0, 1, 2, \dots, k-1 \right\}$$

$$\leq \frac{q^{m}}{1-q} \cdot \max\left\{D^{*}\left(f^{i}x, f^{k+i}x, f^{k+i}x\right): i=0, 1, 2, \dots, k-1\right\}$$

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proving (3.3). Letting $m \to \infty$, we get that $\lim_{n \to \infty} f^n x = u$, since $q^m \to 0$ as $m \to \infty$, proving (3.2). This completes the

proof of the theorem.

B. Theorem

Let f be a quasi-contraction with constant q on a metric space (X, d) and X be f-orbitally complete, then f has a unique fixed point $u \in X$. In fact,

$$u = \lim_{n \to \infty} f^n x \text{ for all } x \in X$$

and

$$d(f^n x, u) \leq \frac{q^n}{1-q} d(x, fx)$$
 for all $x \in X$, $n \geq 1$.

Proof: If (X, d) is a *f*-orbitally complete metric space, then it can be proved that (X, D_1^*) is a *f*-orbitally complete D^* -metric space and hence *f*-orbitally complete for any selfmap *f* of *X*. Also if *f* is a quasi-contraction with constant *q* of (*X*, *d*), then the condition of quasi-contraction can be written as

$$D_{1}^{*}(fx, fy, fy) \leq q \cdot \max \left\{ D_{1}^{*}(x, y, y), D_{1}^{*}(x, fx, fx), D_{1}^{*}(y, fy, fy), D_{1}^{*}(x, fy, fy), D_{1}^{*}(y, fx, fx) \right\}$$

for all $x, y \in X$, since $D_1^*(x, y, y) = d(x, y)$; so that f is a quasi-contraction on (X, D_1^*) . Thus f is a quasi-contraction on the *f*-orbitally complete D^* -metric space (X, D_1^*) and hence the conclusions of Theorem 3.1 hold for f; which are the conclusions of the theorem.

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