Thermophoresis Effect on MHD Mass Transfer Flow Past a Vertical Porous Plate Embedded in a Porous Medium in a Slip Flow Regime with Thermal Radiation and Chemical Reaction

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Abstract: Thermophoresis effect on MHD mass transfer flow past a vertical porous plate embedded in a porous medium in a Slip Flow Regime with Thermal Radiation and Chemical Reaction is investigated. The analytical solutions of momentum, energy and concentration equations are obtained by Perturbation technique. The velocity, temperature and concentration profiles are computed. The dimensionless Skin friction co-efficient, Nusselt number and Sherwood number are also estimated. The effects of few physical parameters Prandtl number Pr, Grashof number for heat transfer Gr, Grashof number for mass transfer Gm, Suction parameter S and radiative parameter R on velocity, temperature and concentration profiles are analyzed through graphs.

Keywords: MHD, Porous Medium, Radiation, Slip flow, Soret Effect.

I. INTRODUCTION

Thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient, also known as Soret effect or Thermophoresis. It is often applied to aerosol mixtures, but commonly refers to the phenomenon in all phases of matter. The Soret effect normally applies to liquid mixtures, which behave according to different, less well-understood mechanisms than gaseous mixtures. The heat transfer problem with a convective surface boundary condition has attracted the interest of many researchers, since it is more general and realistic especially in several engineering and industrial processes such as transpiration cooling process, material drying, polymers, cosmetics and toiletries, laser pulse heating, ground water flow etc.

Anjalidevi S.P and Kandasamy R. [1] has studied Effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. The Slip flow heat transfer in Rectangular Micro channel was considered by S.Yu and T.A.Ahem[2]. S.Das, M.Jana and R.N.Jana[3] studied the Radiation effect on natural convection near a vertical plate embedded in porous medium with ramped wall temperature. Recently Patil et al. [4] studied double diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and chemical reaction. Soret effects due to natural convection between heated inclined plates were investigated by Raju et al. [5]. In their study Reddy et al. [6-8], considered effect of thermal diffusion on heat and mass transfer flow problems in different geometries. Ravikumar et al. [9] investigated, heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source. MHD transient free convection and chemically reactive flow past a porous vertical plate with radiation and temperature gradient dependent heat source in slip flow regime was investigated by Rao et al. [10]. The classical model for radiation effect introduced by Cogley et al. [11] is used. Perturbation technique is applied to convert the governing non-linear partial differential equations in to a system of ordinary differential equations which are solved analytically.

Motivated by above cited work, in this chapter we have considered an unsteady MHD free convection flow of a viscous fluid past a vertical porous plate embedded with porous medium in the presence of Thermophoresis Effect. In obtaining the solution, the terms regarding radiation effect and temperature gradient dependent heat source are taken into account in energy equation. The permeability of the porous medium and the suction velocity are considered to be exponentially decreasing function of time.

II. MATHEMATICAL FORMULATION

We consider a two-dimensional unsteady flow of a laminar incompressible flow, electrically conducting and heat...
generating/absorbing fluid with mass transfer, past a semi-infinite vertical porous plate embedded in a porous medium in the presence of thermal radiation and chemical reaction with thermophoresis. The X axis is taken along the plate, Y axis perpendicular to it and directed in the fluid region and Z axis along the width of the plate. A uniform magnetic field $B_0$ in the presence of radiation is imposed transversely is applied perpendicular to the plate. There is no applied voltage and under the assumption of the induced magnetic field is neglected, results that absence of any electrical field and the magnetic Reynolds number is small. The radiative heat flux in the X-direction is negligible in comparison to that in Y-direction. Under the above assumptions the governing equations are given as

Momentum equation

$$\frac{\partial \tilde{v}^*}{\partial \tilde{y}^*} = 0$$

Energy equation

$$\frac{\partial \tilde{u}^*}{\partial \tilde{t}} + \tilde{v}^* \frac{\partial \tilde{u}^*}{\partial \tilde{y}^*} = \frac{1}{\rho} \frac{\partial \tilde{\rho}^*}{\partial \tilde{y}^*} + g \beta (T^* - T_{w^*}) + g\beta (C^* - C_{w^*}^*) + \nu \frac{\partial^2 \tilde{u}^*}{\partial \tilde{y}^*^2} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\nu u^*}{K^*}$$

Species continuity equation

$$\frac{\partial C^*}{\partial \tilde{t}} + \tilde{v}^* \frac{\partial C^*}{\partial \tilde{y}^*} = D_m \frac{\partial^2 C^*}{\partial \tilde{y}^*^2} + K^* + \frac{D_n K_l}{T_m} \frac{\partial^2 T^*}{\partial \tilde{y}^*^2}$$

where $u$ and $v$ are the velocity components in the x-direction and y-direction respectively, $v$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\beta$ is the volumetric coefficient of thermal expansion, $T^*$ and $T_{w^*}$ are the fluid temperature within the boundary layer and in the free-stream respectively, while $\beta^*$ is the volumetric coefficient of mass expansion, $C^*$ and $C_{w^*}$ are the fluid temperature within the boundary layer and in the free-stream respectively, $\sigma$ is the electric conductivity, $B_0$ is the uniform magnetic field strength (magnetic induction), $\rho$ is the density of the fluid, $q_r$ is the heat flux, $C_p$ is the specific heat at constant pressure, $Q_0$ is the rate of heat generation/absorption and $D_m$ is the chemical molecular diffusivity.

By using Cogley et al. (8) the radiative heat flux is given by

$$\frac{\partial q_r^*}{\partial \tilde{y}^*} = 4(T^* - T_{w^*})I^*$$

where $I^* = \int K_{\lambda \omega} \frac{\partial e_{bb^*}}{\partial T^*} d\lambda$, $K_{\lambda \omega}$ is the absorption co-efficient at the wall and $e_{bb^*}$ is the Planck’s function.

The corresponding boundary conditions for the model

$$y^* = 0: u^* = u_{\text{Slip}}^* = h^* \frac{\partial \tilde{u}^*}{\partial \tilde{y}^*}, T^* = T_{w^*}^* + \varepsilon (T_w^* - T_{w^*}^*) e^* + C^* = C_{w^*}^* + \varepsilon (C_w^* - C_{w^*}^*) e^*$$

$$y^* \to \infty: u^* \to U_x^* = U_0 (1 + \varepsilon e^*), T^* \to T_{\infty}^*, C^* \to C_{\infty}^*$$

Where, $T_w^*$ and $C_{w^*}^*$ are the dimensional temperature and species concentration at the wall respectively. $h^*$ is the characteristic dimension of the flow fluid.

From the continuity equation, it is clear that the suction velocity normal to the plate is a function of time only and we shall take it in the form as

$$v^* = -V_0 (1 + \varepsilon e^*$$

Where $A$ is a real positive constant, $\varepsilon$ and $\varepsilon A$ are small quantities less than unity and $V_0$ is a scale of suction velocity which is a non-zero positive constant.

Outside the boundary layer, Equation (2) gives
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\[
\frac{1}{\rho} \frac{dp^*}{dx^*} = \frac{dU^*}{dt} + \frac{\sigma B_2^2}{\rho} U^* + \nu U^* \frac{d^2U^*}{dx^2} + \sigma \epsilon \frac{\partial U^*}{\partial t}
\]

(9)

Now, we introduce the non-dimensional parameters as follows

\[
\begin{aligned}
& u = \frac{U^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad y = \frac{y^*V_0}{v}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad t = \frac{t^*V_0}{v}, \quad n = \frac{n^*v}{V_0^2}, \\
& \theta = \frac{\theta^*}{\theta_\infty}, \quad \varphi = \frac{\varphi^*}{\varphi_\infty}, \quad \alpha = \frac{\kappa v^2}{v^2}, \quad K = \frac{\kappa v}{v^2} \\
& Q = \frac{Q_0 v}{\rho C_p V_0^2}, \quad \Theta = \frac{\Theta_0 v}{\rho C_p V_0^2}, \quad G_m = \frac{G_m v}{U_0 V_0^2}, \quad \Pr = \frac{\mu C_p}{k} \\
& R = \frac{4v^2}{\rho C_p V_0^2}, \quad M = \frac{\sigma B_2^2 v}{p V_0^2}, \quad Sc = \frac{D_m}{v}
\end{aligned}
\]

After substituting boundary conditions and dimensionless parameters the governing equations (2) - (5) reduced to

\[
\begin{aligned}
& \frac{\partial u}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial u}{\partial y} - \frac{dU_\infty}{dt} + \frac{\partial}{\partial y} \left( \frac{U_\infty}{U_0} \right) + \Theta \theta + G_m \varphi + N(U_\infty - U) \\
& \frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left( \frac{\Theta}{\Theta_0} \right) - R \theta - Q \theta \\
& \frac{\partial \varphi}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \varphi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \varphi}{\partial y^2} - K \varphi - S_r \frac{\partial^2 \theta}{\partial y^2}
\end{aligned}
\]

(11) - (13)

Where \( N = M \frac{1}{\alpha} \)

The boundary conditions (6) and (7) in the dimensionless form can be written as

\[
\begin{aligned}
& u = u_{slp} = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \epsilon e^{nt}, \quad \varphi = 1 + \epsilon e^{nt} \quad \text{at} \ y = 0 \\
& u \to U_\infty = 1 + \epsilon e^{nt}, \quad \theta \to 0, \quad \varphi \to 0 \quad \text{as} \ y \to 0
\end{aligned}
\]

(14) - (15)

III. SOLUTION OF THE PROBLEM

Equations (11) to (13) are non-linear partial differential equations are reduced to a set of ordinary differential equations and these can be solved analytically by Perturbation method. These are done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

\[
\begin{aligned}
& u = u_0(y) + \epsilon e^{nt} u_1(y) + O(\epsilon^2) \\
& \theta = \Theta_0(y) + \epsilon e^{nt} \Theta_1(y) + O(\epsilon^2) \\
& \varphi = \varphi_0(y) + \epsilon e^{nt} \varphi_1(y) + O(\epsilon^2)
\end{aligned}
\]

(16) - (18)

Substituting (16) to (18) in Equations (11) to (13) and equating the harmonic and non-harmonic terms and also neglecting the coefficients of \( O(\epsilon^2) \). We get the pairs of equations for \((u_0, \Theta_0, \varphi_0)\) and \((u_1, \Theta_1, \varphi_1)\).

\[
\begin{aligned}
& u_0'' + u_0' - Nu_0 = -N - G \Theta_0 - G_m \varphi_0 \\
& u_1'' + u_1' - (N + n) u_1 = -Au_0 - Cr \Theta_1 - G_m \varphi_1 - (N + n) \\
& \Theta_0'' + Pr\Theta_0' - Pr(R + Q) \Theta_0 = 0 \\
& \Theta_1'' + Pr\Theta_1' - Pr(R + Q + n) \Theta_1 = -AP\Theta_0' \\
& \varphi_0'' + Sc\varphi_0' - ISc \varphi_0 = -SrSc \Theta_0''
\end{aligned}
\]

(19) - (23)
Here primes denote differentiation with respect to y.
The corresponding boundary conditions are
\[ u_0 = h u'_0, \quad u_1 = h u'_1, \quad \theta_0 = 1, \quad \phi_0 = 1, \]
\[ \theta_1 = 1, \quad \phi_1 = 1 \quad \text{at} \quad y = 0 \] (25)
\[ u_0 = 1, \quad u_1 = 1, \quad \theta_0 = 0, \quad \phi_0 = 0, \]
\[ \theta_1 \to 0, \quad \phi_1 \to 0 \quad \text{as} \quad y \to \infty \] (26)

The solution of Equations of (19) - (24), which satisfying boundary conditions are given by
\[ u_0(y) = 1 + L_{t_0} e^{-m_3 y} + L_{t_0} e^{-m_3 y} + (L_{t_0} + L_{t_2}) e^{-m_1 y} \]
\[ u_1(y) = 1 + L_{t_2} e^{-m_2 y} + L_{t_1} e^{-m_3 y} + (L_{t_1} + L_{t_3}) e^{-m_2 y} + L_{t_3} e^{-m_2 y} + L_{t_2} e^{-m_1 y} \]
\[ \theta_0(y) = e^{-m_1 y} \]
\[ \theta_1(y) = (1 - L_{t_2}) e^{-m_2 y} + L_{t_2} e^{-m_2 y} \]
\[ \phi_0(y) = (1 - L_{t_2}) e^{-m_2 y} + L_{t_2} e^{-m_2 y} \]
\[ \phi_1(y) = L_{t_3} e^{-m_2 y} + L_{t_2} e^{-m_2 y} + L_{t_4} e^{-m_2 y} + L_{t_4} e^{-m_2 y} \] (36)

By virtue of equations (15) - (17), we obtain for the velocity, temperature and concentration as follows
\[ u(y, t) = (1 + L_{t_0} e^{-m_3 y} + L_{t_0} e^{-m_3 y} + (L_{t_0} + L_{t_2}) e^{-m_1 y}) \]
\[ + e^{nt} (1 + L_{t_2} e^{-m_2 y} + L_{t_1} e^{-m_3 y} + (L_{t_1} + L_{t_3}) e^{-m_2 y} + L_{t_3} e^{-m_2 y} + L_{t_2} e^{-m_1 y}) \]
\[ \theta(y, t) = e^{-m_1 y} + e^{nt} (L_{t_0} e^{-m_3 y} + L_{t_0} e^{-m_3 y}) \]
\[ \phi(y, t) = [(1 - L_{t_2}) e^{-m_2 y} + L_{t_2} e^{-m_2 y}] + e^{nt} (L_{t_3} e^{-m_2 y} + L_{t_2} e^{-m_2 y} + L_{t_4} e^{-m_2 y} + L_{t_4} e^{-m_2 y}) \] (36)

A. Skin friction
The non-dimensional skin friction at the plate is given by
\[ C = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left( \frac{\partial u_0}{\partial y} + e^{nt} \frac{\partial u_1}{\partial y} \right)_{y=0} \]
\[ = (-m_3 L_{t_0} + m_4 L_{t_0} + m_1 (L_{t_1} + L_{t_2})) \]
\[ - e^{nt} (m_3 L_{t_2} + m_5 L_{t_2} + m_4 (L_{t_1} + L_{t_2} + m_3 L_{t_2} + m_1 (L_{t_2} + L_{t_2})) \] (36)

B. Nusselt number
The non-dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by
\[ N_u = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = - \left( \frac{\partial \theta_0}{\partial y} + e^{nt} \frac{\partial \theta_1}{\partial y} \right)_{y=0} = m_4 + e^{nt} (m_3 L + m_1 L_1) \] (37)

C. Shearwood Number
The non-dimensional form of the rate of mass transfer in terms of Shearwood number at the plate is given by
\[ S_h = - \left( \frac{\partial \phi}{\partial y} \right)_{y=0} = - \left( \frac{\partial \phi_0}{\partial y} + e^{nt} \frac{\partial \phi_1}{\partial y} \right)_{y=0} \]
\[ = -[(1 - L_{t_2}) m_3 + L_4 m_1] + e^{nt} (m_4 L_{t_2} + m_3 L_3 + m_2 L_2 + m_4 (L_{t_1} + L_{t_1} + L_{t_2} + L_{t_2})) \] (38)
Figure 1: Velocity profile for different values of M.

Figure 2: Velocity profile for different values of R.

Figure 3: Velocity profile for different values of Q.

Figure 4: Velocity profile for different values of Sc.

Figure 5: Velocity profile for different values of Sr.

Figure 6: Velocity profile for different values of K.
Figure 7: Velocity profile for different values of $G_m$.

Figure 8: Velocity profile for different values of $G_r$.

Figure 9: Velocity profile for different values of $\alpha$.

Figure 10: Temperature profile for different values of $R$.

Figure 11: Temperature profile for different values of $Q$.

Figure 12: Temperature profile for different values of $Pr$. 
IV. RESULTS AND DISCUSSION

In order to get physical insight into the problem, we have calculated the velocity field, temperature field, concentration field, coefficient of skin-friction $C_f$ at the plate, the rate of heat transfer in terms of Nusselt number $Nu$ and the rate of mass transfer in terms of Sherwood number $Sh$ by assigning specific values to the different values to the parameters involved in the problem, viz., Radiative parameter $R$, Prandtl number $Pr$, Suction parameter $S$, Magnetic field parameter $M$, Grashof number for heat transfer $Gr$, Grashof number for mass transfer $Gm$, Soret effect $Sr$, Schmidt number $Sc$, and time $t$. Thoroughout our investigation the value of Prandtl number $Pr = 0.71$, $t = 1$, $n = 0.1$, $A = 1$, $Sc = 0.6$, $Gm = 4$, $Gr = 6$, $h = 0.3$, $Q = 1$, $K = 1$, $R = 2$, $ε = 0.2$, $α = 1$, $M = 3$ are kept constant. All graphs therefore correspond to these unless specifically indicated on the appropriate graph. The numerical results are demonstrated through different graphs and table and their results are interpreted physically.

Fig(1) depicts the dimensionless velocity $u$ profiles for the different values of the magnetic parameter. It is noticed that an increase in magnetic field results in increase in the velocity profile. This conclusion agrees with the fact that the magnetic field exerts retarding force on the free-convection flow. Fig (2) Illustrates the effect of radiation on the dimensionless velocity $u$. It shows that velocity component increases with an increases in the radiation parameter. Fig(3) indicates the fact that an increase in Schmidt number $Sc$ accelerates the fluid flow. The mass diffusivity causes the fluid velocity to increases. Fig 5: Illustrates the effect of Soret on the dimensionless velocity $u$. It shows that velocity component decreases with an increases in the soret effect parameter. This Fig (6) shows that the fluid motion is retarded on account of chemical reaction. This shows that the consumption of chemical species leads to fall in the concentration field which in turn diminishes the buoyancy effects due to concentration gradients, Consequently the flow field is decelerated. It is observed from the Fig (7) an increase in Grashof number for mass transfer leads to a rise in the values of velocity $u$ due to enhancement in buoyancy force. The plot of velocity profile for different values of Grashof number for the thermal transfer is given in Fig(8). It is observed that the velocity increases for the increasing values of $Gr$. Fig(9) it depicted the change in velocity profile due to different permeability of porous medium. Here, it is seen that due to increase of porosity of the
medium fluid motion is accelerated.

It is observed from the Fig (10) that the temperature \( \theta \) decreases as the radiation parameter \( R \) increases. This result qualitatively agrees with expectation, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid. Fig(11) indicates the fact that an increase in \( Q \) heat source parameter decelerates fluid flow in the temperature profile. Fig(12) indicates the fact that an increase in \( Pr \) Prandtl number decelerates fluid flow in the temperature profile.

Fig(13) The concentration level of the fluid drops due to increasing Schmidt number indicating the fact that the mass diffusivity raises the concentration level steadily. The variation of species concentration against the span-wise coordinate \( y \) under the influence of Chemical reaction parameter \( k \) and Schmidt number \( Sc \). Fig(14), displays the effects of the chemical reaction parameter \( K \), on concentration profiles. We observe that concentration profiles decreasing with increasing \( K \). For values of Soret parameter \( Sr \), the concentration profile is plotted in Fig(15). Clearly as \( Sr \) increases, the dimensionless concentration decreases.

V. CONCLUSION

The present work is concerned with the study of Thermophoresis effect on MHD mass transfer flow past a vertical porous plate embedded in a porous medium in a Slip Flow Regime with Thermal Radiation and Chemical Reaction and investigated the effect of various parameters. The governing non-linear partial differential equations are first transformed into a dimensionless form and thus resulting non-similar set of equations has been solved using the perturbation technique. Results are presented graphically with help of MATLAB and discussed quantitatively for parameter values of practical interest from physical point of view. We conclude from these results that

An increase in \( M, R, Q, Gr, Gm, \alpha \) increases the velocity field, while an increase in \( Pr, Sr, K \) and \( Sc \) decreases the velocity field. An increase in \( R, Q \), and \( Pr \) decreases the temperature distribution.

An increase in \( Sr \) increases the concentration distribution, while an increase in \( K, Sc \) decreases the concentration distribution.

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