Six Component Force Analysis Using Torque Sensors in Gough-Stewart Platform Manipulators with Interval Analysis

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Abstract: The six-degrees-of-freedom Gough-Stewart platform manipulator is a promising alternate architecture for the mechanical design of a six component force-torque sensor. Two basic configurations of the Gough-Stewart platforms are used for the design of six component force-torque sensor. In an isotropic configuration, equal sensitivity for all the six components of the force and torque being measured can be obtained. In a singular configuration, large mechanical magnification can be obtained for certain selected components of the force and torque, and, as a consequence, very small forces and/or torque along the selected directions can be measured. In this paper, we revisit the use of the Gough-Stewart platform manipulator as a sensor. Algorithms to determine the isotropic and singular configuration of the Gough-Stewart platform manipulator are presented. Two specific configurations of the Gough-Stewart platform with enhanced sensitivity to selected components of external forces and torque are taken up for analysis, design, and fabrication and testing. Experimental results show that the prototype six component force-torque sensors can measure the external forces and torques as designed.

Keywords: Stewart Platform, Force-Torque Sensor, Isotropic and Singular Configurations, Directionally Sensitive.

I. INTRODUCTION
The Gough-Stewart platform is a six-degree-of-freedom parallel manipulator. It was rest proposed for the testing of wear in tires by Gough and later by Stewart as a flight simulator. The Gough-Stewart platform manipulator consists of a moving platform connected to a fixed base by means of six extendable legs. Controlled extension and/or retraction of the legs results in six-degree-of-freedom motion of the moving platform. The main advantages of a Gough-Stewart platform manipulator is its increased load carrying capability since the load acting on the moving platform is ‘shared’ by the six legs. The Gough-Stewart platform in an isotropic configuration, as the name implies, has equal sensitivity for all components of force and torque. Whereas a Gough-Stewart platform in a near-singular configuration will measure the six components of force and torque with enhanced sensitivity along selected directions. Enhanced sensitivity is useful in many applications where it is known to be a priority that some components of force or moment are much larger (in robotic assembly and manufacturing, the force in the normal direction is known to be 5 to 10 times larger than in the tangential directions. In aero-dynamics, it is known that the drag forces, pitching and other moments are typically 10 to 20 times smaller than the lift forces in a wing. In this paper, algorithms developed to obtain isotropic and singular configurations are presented. Design, analysis and testing of two near-singular configuration six component force-torque sensors are also presented.

II. KINEMATICS AND STATICS OF GOUGH-STEWART PLATFORM
Figure 1 shows a schematic of the Gough-Stewart platform manipulator. It consists of six extensible legs, denoted by $B_i - P_i$, $i = 1, 2, 6$, with prismatic (P) joints in each leg. One end of a leg, $P_i$, is connected to the moving platform with spherical (S) joint and the other end, $B_i$, is connected to the axed base with Hooke or universal (U) joint. The Gough-Stewart platform manipulator, in a general configuration, has six degrees-of-freedom one can obtain arbitrary desired translator and rotary motion along and about the X, Y and Z co-ordinate axis by appropriately actuating the six prismatic joints.

The direct kinematics problem for a Gough-Stewart platform manipulator may be stated as follows: given the displacements at the six prismatic joints, obtain the position and orientation of the moving platform. The inverse kinematics problem is ‘opposite’ given the position and orientation of the moving platform, obtain the displacements of the prismatic joints. The direct kinematics problem is known to be one of the hardest problems in robot kinematics.
III. ISOTROPY IN GOUGH- STEWART PLATFORM

The term ‘isotropy’ implies same or identical ‘properties’ in all directions. A large amount of literature exists on kinematic isotropy where the goal to design Gough-Stewart platforms which can move equally well in all spatial directions [18, 19, 20, 21]. In the context of force-moment isotropy in Gough-Stewart platform, it implies the ability of platform to resist external forces and moments equally well in all spatial directions. Intuitively, it would be desirable to have a isotropic configuration for a Gough-Stewart platform based six component force-torque sensor since the measurements would be equally sensitive or accurate for all the six components of the externally acting force and moment.

Mathematically, isotropy implies that the eigenvalues (or singular values) of the $[H]$ matrix are all identical. Unfortunately, this does not have much physical significance since the elements of the top and bottom $3 \times 6$ sub-matrices of $[H]$ have different units { $s$, has no units whereas $b_i \times s_i$ has units of length, and the elements of the bottom $3 \times 6$ sub-matrix The moving platform and the axed base can be circum-scribed in two circles of radii $r_t$ and $r_b$ centered at $P_0$ and $B_0$, respectively. Moreover there is a 3-way symmetry in an SRPSM. The angle subtended by the arcs $P_1 - P_2$, $P_3 - P_4$ and $P_5 - P_6$ at the centre $P_0$ are equal and denoted by $\gamma_t$. Likewise the angle subtended by the arcs $B_1 - B_2$ and $B_3 - B_4$ and $B_5 - B_6$ at centre $B_0$ are also equal and denoted by $\gamma_b$ (see figure 1). As shown in reference [22], with the moving platform kept horizontal at a height of $z = 0.4627$ and rotated about Z axis by an angle $\pi/18$, the SRPSM will exhibit combined force-moment isotropy when $r_b = 1$, $\gamma_b = \pi/5$, $\gamma_t = \pi/10$ and $r_t = 0.3789$. The in the combined isotropic configuration is shown in figure 3.
Figure 3: Combined force-moment isotropic configuration for an SRSPM from IV. SINGULARITY IN GOUGH-STEWART PLATFORM

The singular configuration for a Gough-Stewart plat-form manipulator is obtained when $|H| = 0$. From equation (5), we can write and if $|H| \to 0$, a finite $(F; M)^T$ will give rise to in finite forces in one or more legs, and some components of the external force-moment $(F; M)$ can-not be supported by the leg forces. The Eigenvectors corresponding to the zero Eigen values of $|H|$ when mapped to $(F; M)^T$ give the singular directions and Gough-Stewart platform cannot withstand any force/moment applied along the singular directions. If the Gough-Stewart platform is in a near-singular configuration with $|H|$ small, then a small force/moment along the singular direction will give rise to large axial forces along the legs. This mechanical magnification can be used to design sensitive six component force-torque sensors where small forces/moments along certain directions can be more easily measured. This key concept was used in reference to design a force-torque sensor with enhanced sensitivity to forces along X and Y directions and to moments about Z direction. The algorithm to determine the singular direction for a Gough-Stewart platform and some of the key results from reference are described next.

V. TWO SENSORS BASED ON NEAR-SINGULAR GOUGH-STEWART PLAT-FORMS

Table 1: Nominal geometry of Configuration 1 sensor

<table>
<thead>
<tr>
<th>Base coordinates</th>
<th>Platform coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point No.</td>
<td>x (mm)</td>
</tr>
<tr>
<td>B₁</td>
<td>43.30</td>
</tr>
<tr>
<td>B₂</td>
<td>0</td>
</tr>
<tr>
<td>B₃</td>
<td>-43.30</td>
</tr>
<tr>
<td>B₄</td>
<td>-43.30</td>
</tr>
<tr>
<td>B₅</td>
<td>0</td>
</tr>
<tr>
<td>B₆</td>
<td>43.3</td>
</tr>
</tbody>
</table>

Length units chosen to be millimeters. This indicates that there will be significant amplification. Based on this analysis, the nominal coordinates of B₁ and P₁ points are given in Table 1. It may be noted that centre of the moving platform is chosen to be 100mm above the centre of the axed platform.

Each leg is modeled as a thin rod with a ring shaped sensing element mounted with strain gauges and edible hinges. The leg is made of titanium for its high strength, low weight and its ability to undergo large strain without failure. A finite element model of the sensor was made and analyzed in. For a typical external loading of $F_x = F_y = F_z = 0.98$ N, $M_x = M_y = M_z = 49.05$ N-mm, the maximum deformation obtained from the finite element analysis was found to be 0.5 mm.

A prototype of the designed force-torque sensor was made and is shown in the prototype sensor was loaded externally, in a specially designed fixture, by means of standard dead weights. The loading and unloading was done in steps and limited to 0.98 N and 49.05 N-mm, respectively. The strain values were measured for each leg. Using a calibration done for each leg the strain measurements were converted to axial forces.
Form based sensor sensitive to $M_x$, $M_y$ and $M_z$, extensive numerical simulations and additional requirements of maximum size and load carrying capability were taken into account. The nominal dimension for a near-singular configuration is as given in Table 2. It may be noted that the centre of the top platform is at a height of 37 mm from the centre of the bottom platform. The condition number of is about 707 which gives rise to fairly good mechanical amplification.

Due to the large load handling requirements, the joint in a leg of this sensor is redesigned to make it stronger.
A CAD model of the sensor was made and as in the Configuration 1 sensor, extensive finite element analysis was performed to test the integrity of the sensor. The prototype sensor is shown in figure 8.

Experimental tests are being conducted to obtain the performance and accuracy of the sensor in measuring all six components of force and moment. Preliminary tests show that the sensor is, as expected, sensitive to all the three components of moments and the sensor is quite accurate. A sample plot of the applied moment, \( M_y \), versus the strain measured in leg 1 is shown in figure 9. As it can be seen that the deviation from a linear fit is quite small. More experiments are being planned to collect enough data so that the \([H] \) matrix can be estimated accurately and the sensor is made capable of actual measurements of all the six components of applied force and torque.

![Figure 9: Applied moment versus strain in leg](image)

**VI. CONCLUSIONS**

This paper deals with the use of Gough-Stewart platform configuration for six component force-torque sensors. The Gough-Stewart platform based sensors can be either in isotropic or near-singular configurations, each with its advantages and disadvantages. In this paper, an analytic formulation to obtain the isotropic and singular configurations of \( 6-6 \) Gough-Stewart platforms are presented. Two near-singular configuration six component force-torque sensors were designed, fabricated and tested. The experimental results demonstrate that Gough-Stewart platform based force-torque sensors, especially the ones in a near-singular configuration, have a lot of promise.

**REFERENCES**

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