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On Thermal Radiation Effect in the MHD Jeffery-Hamel Flow of a Second Grade Fluid

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Abstract*— This paper explores the thermal radiation effect in the two-dimensional and magnetohydrodynamic (MHD) flow of an incompressible non-Newtonian fluid in a convergent/divergent channel. The fluid is considered to be thermodynamically second grade. The mathematical formulation involves the conservation of laws of mass, linear momentum and energy with Rosseland's approximation. The resulting nonlinear analysis is computed with the implementation of differential transformation method (DTM). Solutions for velocity and temperature are derived. Computations for the skin friction coefficient and local Nusselt number are also established. Both cases of convergent and divergent channel are analyzed. Comparison of present results with the previous relevant approximate and numerical solutions is shown. It is observed that the temperature is decreasing function of thermal radiation.

***Keywords*—Jeffery-Hamel flow; thermal radiation; differential transformation method (DTM); second grade fluid**

Nomenclature

$\mathbf{A_1}$, $\mathbf{A_2}$	the first two Rivilin Erickson tensors	Re	Reynolds number
$\mathbf{B}$	total magnetic field	$\mathbf{T_1}$	Cauchy stress tensor
$\mathbf{B_0}$	an applied magnetic field	T	temperature
b	an induced magnetic field	u	radial velocity
c_f	skin friction coefficient	$\mathbf{V}$	flow velocity vector
c_p	specific heat at constant pressure	y	analytic function
De	Deborah number	Y	transformed function

Ec	Eckert number	***Greek***	
e	internal energy	α_1, α_2	material constants
F	dimensionless parameter	α	angle between two walls
G	transformed function	η	similarity variable
H	Hartmann number	μ	dynamic viscosity of fluid
$\mathbf{I}$	identify tensor	Θ	transformed function
$\mathbf{J}$	current density	θ	angular coordinate
k^*	mean absorption coefficient	ρ	fluid density
k_0	thermal conductivity (W/m. K)	σ	fluid electrical conductivity

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L	velocity vector gradient	σ^*	Stefan–Boltzmann constant
Nu	Nusselt number	τ	shear stress
p	pressure (Pa)	ζ	dimensionless parameter
Pr	Prandtl number	**Subscripts**	
Rd	Radiation parameter	*r*	radiation
r	Radial	*w*	wall
q	heat flux		

I. Introduction

Investigating of non-Newtonian fluids has been important topic over the last few decades owing to its numerous applications in engineering and industry. Such fluids cannot be analyzed using Newton's law of viscosity and thus well-known Navier-Stokes equations are inadequate for their flow description. The non-Newtonian fluids differ greatly from the viscous fluids in the sense that these cannot be examined by employing a single constitute equation. Hence several constitutive relationships for the non-Newtonian fluids have been suggested. Classification of non-Newtonian fluids in general is presented into these categories namely the differential, rate and integral. The governing equations in non-Newtonian fluids are more nonlinear and high order than the Navier-Stokes equations. In fact the rheological parameters in the constitutive equations of non-Newtonian fluids are the main culprit which make the resulting equations more tedious and highly nonlinear. The issue of boundary conditions as well as the nonlinearity of the involved equations limit the solutions of the flow for non-Newtonian fluids. Amongst the several subclasses of differential type fluids, the second grade model has received much attention. This is because of simplicity regarding its constitutive expression. More this subclass can predict the important phenomenon of normal stress effect. Good list of references on the flows of second grade fluid may be directed by the studies [1-10] and many attempts therein. In ref. [10] Hayat et al. discussed the MHD Jeffery-Hamel flow of second grade fluid when thermal radiation effect in the heat transfer has been ignored. Detailed review of Jeffery-Hamel problem under various aspects is given by Motsa et al. [11] and Makinde and Mhone [12].

Zhou [13] in 1986 firstly introduced the differential transformation method (DTM). This technique was utilized for the solution of linear and nonlinear initial value problems arising in electric circuit analysis. This technique has advantages in the sense that it can be applied directly to linear and nonlinear differential equations without requiring linearization, discretization or perturbation. Rashidi and Erfani [14] used DTM in order to find the fin efficiency of convective straight fins with temperature dependent thermal conductivity. They also compared their results with the HAM solutions. Hsiang Chang and Ling Chang [15, 16] used a new algorithm for computation of one and two-dimensional differential transform of nonlinear functions. The reduced differential transformation method for the solution of gas dynamic problem was employed by Keskin and Oturanç [17]. Chen and Ju [18] utilized the differential transformation method for the transient advective-dispersive transport equation. Linear and nonlinear initial value problems are also solved by Jang [19] using the projected differential transform method. This method can be easily applied to the initial value problem by less computational work. Hassan [20] used DTM for the solution of eigenvalue problems including those related to vibration. In fact, this method is used to solve a wide range of physical problems. This method provides a direct scheme for solving the linear and nonlinear deterministic and stochastic equations without linearization and yield rapidly convergent series solution.

In this article, we develop the analysis for thermal radiation effect in MHD Jeffery-Hamel flow of a second grade fluid. The fluid is electrically conducting in the presence of applied magnetic field only. The electric and induced magnetic fields are not accorded. The consideration of MHD flow in channel is quite significant in crystal growth, design of medical diagnostic devices, control of liquid metal flows, etc. Further several engineering processes, for example, fossil fuel combustion energy, astrophysical flows, gas turbines, solar power technology and many propulsion devices for aircrafts, satellites, missiles and space vehicle occur at high temperatures and hence thermal radiation effect becomes important. In particular thermal radiation has central role in engineering processes occurring at high temperature for the design of many advanced energy conversion systems and pertinent equipment. Approximate solutions of velocity and temperature are constructed using differential transformation

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method. The results of skin friction coefficient and Nusselt number are given proper attention. Plots are analyzed for the interesting parameters. It is found that skin friction coefficient in second grade fluid is similar to that of the viscous fluid. However, Nusselt number, temperature and velocity are strongly dependent upon parameter of second grade fluid.

II. BRIEF IDEA OF DIFFERENTIAL TRANSFORMATION METHOD

Consider the analytic function *y(t)* in a domain *D* where $t=t_i$ represent any point in it. The function *y(t)* is represented by a power series at the centre t_i. Taylor series expansion of *y(t)* can be written as [21]:

$$y(t)=\sum_{j=0}^{\infty}\frac{(t-t_i)^j}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=t_i} \qquad \forall t\in D$$

(1)

Maclaurin series of *y(t)* can be obtained from above equation when $t_i=0$. It is presented in the following form

$$y(t)=\sum_{j=0}^{\infty}\frac{t^j}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=0} \qquad \forall t\in D$$

(2)

The differential transformation of the function $y(t)$ is defined as:

$$Y(j)=\sum_{j=0}^{\infty}\frac{H_1^{\,j}}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=0},$$

(3)

In which *y(t)* is the original function and *Y(j)* is the transformed function. The differential spectrum of *Y(j)* is confined within the interval $t\in[0,H]$, where H_1 is a constant. The differential inverse transform of *Y(j)* is given by:

$$y(t)=\sum_{j=0}^{\infty}\left(\frac{t}{H_1}\right)^j Y(j)$$

(4)

Table I includes some of the original and transformed functions. It should be point out that concept of differential transformation is the Taylor series expansion. Clearly more terms in Eq. (4) lead to better accuracy of solution in Eq. (4).

TABLE I
THE FUNDAMENTAL OPERATIONS OF DIFFERENTIAL TRANSFORM METHOD

Original function	**Transformed function**
$f(x)=\alpha g(x)\pm\beta h(x)$	$F(k)=\alpha G(k)\pm\beta H(k)$
$f(x)=g(x)h(x)$	$F(k)=\sum_{i=0}^{k}G(i)H(k-i)$
$f(x)=g(x)^{(n)}$	$F(k)=(k+1)(k+2)\ldots(k+n)G(k+n)$
$f(x)=x^n$	$F(k)=\delta(k-n)=\begin{Bmatrix}1 & k=n\\ 0 & k\neq n\end{Bmatrix}$
$f(x)=\exp(\alpha x)$	$F(k)=\frac{\alpha^k}{k!}$
$f(x)=(1+x)^n$	$F(k)=\frac{k(k-1)\ldots(k-m-1)}{k!}$

III. DESCRIPTION OF THE PROBLEM

We consider a two-dimensional, steady and MHD of flow of second grade fluid at the intersection between two heated walls with *2α* angle. Fig. 1 presents the model and geometrical coordinate. We assume that the velocity is only along the radial direction and depends on *r* and *θ*, *V=V(u(r,θ),0)* [22, 23]. Further the flow in the channel subjected to the thermal radiation effect. The governing equations for MHD flow are:

$$div\,\mathbf{V}=0,$$

(5)

$$\rho\frac{d\mathbf{V}}{dt}=div\,\mathbf{T}_1+\mathbf{J}\times\mathbf{B},$$

(6)

$$\rho\frac{de}{dt}=\mathbf{T}_1.\mathbf{L}-div\,\mathbf{q}-\nabla\mathbf{q}_r$$

(7)

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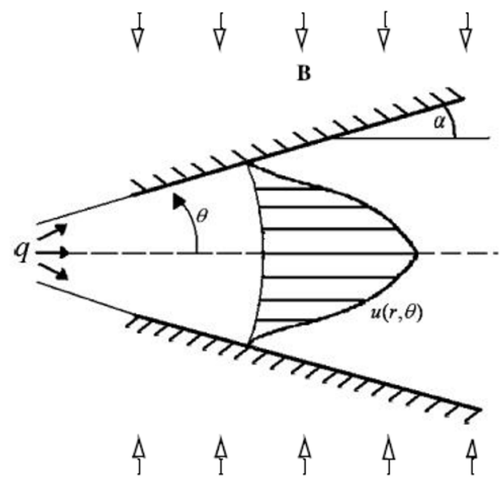

Fig. 1 Geometry of problem

In above equations $\mathbf{J}$ is the current density and $\mathbf{B} = \mathbf{B_0} + \mathbf{b}$ is total magnetic field, $\boldsymbol{\rho}$ is the fluid density, *d/dt* is the material derivative, $\mathbf{T_1}$ is the Cauchy stress tensor, *e* is the internal energy, $\mathbf{L}$ is the velocity gradient, $\mathbf{q}$ is the heat flux vector, $\mathbf{q}_r$ is the radiative heat flux and $\mathbf{B_0}$ is an applied magnetic field. The magnetic Reynolds number is taken small and hence the induced magnetic field is not accorded. The electrical field is neglected. In view of these assumptions, the Lorentz force $\mathbf{J}\times\mathbf{B}$ becomes

$$\mathbf{J}\times\mathbf{B} = -\sigma \mathbf{B}_0^2 \mathbf{V}.$$
(8)

where σ is electrical conductivity of fluid. Using Rosseland approximation [24] we have

$$\mathbf{q}_r = -\frac{4\sigma^*}{3k^*}\nabla T^4 \tag{9}$$

where σ^* is the Stefan–Boltzmann constant, k^* is the mean absorption coefficient and T is temperature. Employing Taylors' series for T^4 as out T_w, we obtain

$$T^4 \cong 4T_w^3 T - 3T_w^4$$
(10)

The Cauchy stress tensor T_1 in a second grade fluid is [25].

$$\mathbf{T}_1 = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2$$
(11)

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T$$
(12)

$$\mathbf{A}_2 = \frac{d\mathbf{V}}{dt} + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1$$
(13)

$$\mathbf{L} = \nabla\mathbf{V}$$
(14)

In which p is the pressure, $\mathbf{I}$ is an identity tensor, μ is the fluid dynamic viscosity, ∇ is the gradient operator, α_1 and α_2 are the material constants, *d/dt* is the material time differentiation, T in the superscript denotes the Matrix transpose and $\mathbf{A_1}$ and $\mathbf{A_2}$ are the first two Rivilin Erickson tensors. Furthermore, α_1 and α_2 satisfy the following constraints [26]

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0.$$
(15)

Using the definition of velocity in the present flow and substituting of Eqs, (12)-(14) into Eq. (11) we have

$$\begin{aligned} T_1 = {} & -p\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \mu\begin{bmatrix}2\frac{\partial u}{\partial r} & \frac{1}{r}\frac{\partial u}{\partial \theta}\\ \frac{1}{r}\frac{\partial u}{\partial \theta} & 2\frac{u}{r}\end{bmatrix} + \alpha_1\begin{bmatrix}2\left(\frac{\partial u}{\partial r}\right)^2 & \frac{\partial u}{\partial \theta}\left(\frac{2}{r}\frac{\partial u}{\partial r} + \frac{u}{r^2}\right)\\ \frac{1}{r}\frac{\partial u}{\partial \theta}\frac{\partial u}{\partial r} & \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 + 2\frac{u^2}{r^2}\end{bmatrix} \\ & + \alpha_1\begin{bmatrix}2\left(\frac{\partial u}{\partial r}\right)^2 & \frac{1}{r}\frac{\partial u}{\partial \theta}\frac{\partial u}{\partial r}\\ \frac{\partial u}{\partial \theta}\left(\frac{2}{r}\frac{\partial u}{\partial r} + \frac{u}{r^2}\right) & \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 + 2\frac{u^2}{r^2}\end{bmatrix} \\ & + \alpha_2\begin{bmatrix}4\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 & \frac{2}{r}\frac{\partial u}{\partial \theta}\frac{\partial u}{\partial r} + 2\frac{u}{r^2}\frac{\partial u}{\partial \theta}\\ \frac{\partial u}{\partial \theta}\left(\frac{2}{r}\frac{\partial u}{\partial r} + \frac{2u}{r^2}\right) & \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 + 4\frac{u^2}{r^2}\end{bmatrix} \end{aligned}$$
(16)

Now Eq. (5) takes the form

$$f(\theta) = ru(r,\theta)$$
(17)

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Invoking Eqs. (8)-(10) and Eqs. (15)-(17) into Eqs. (6) and (7) we arrive at

$$\mu\left(\frac{f'''}{r^3}+4\frac{f'}{r^3}\right)+2\rho\left(\frac{ff'}{r^3}\right)+\alpha_1\left(-4\frac{ff'''}{r^5}-16\frac{ff'}{r^5}\right)-\frac{\sigma B_0^2}{r^3}f'=0$$

(18)

$$\frac{f}{r}\frac{\partial T}{\partial r}=\frac{k_0}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right)+\frac{\mu}{\rho c_p r^4}\left(4f^2+(f')^2\right)-$$
$$\frac{2\alpha_1}{\rho c_p r^6}\left(f(f')^2+4f^3\right)+\frac{16\sigma^* T_w^3}{3k^* r^2 \rho c_p}\left(\frac{\partial^2 T}{\partial \theta^2}-\frac{\partial T}{\partial \theta}\right)$$

(19)

In which k_0 and c_p are the thermal conductivity and specific heat at constant pressure, respectively. The approximate boundary conditions in the present problem are:

- At the channel centerline:

$$\frac{\partial u(r,\theta)}{\partial \theta}=0,\quad \frac{\partial T}{\partial \theta}=0,\quad u(r,\theta)=U$$

(20)

- At the walls of channel:

$$u(r,\theta)=0,\quad T=T_w$$

(21)

Using the following dimensionless parameters

$$F(\eta)=\frac{f(\theta)}{f_{\max}},\quad f_{\max}=rU \quad \eta=\frac{\theta}{\alpha},\quad \xi(\eta)=\frac{T}{T_w}$$

(22)

Equations (18-21) are reduced into the following expressions

$$F'''(\eta)+2\alpha \operatorname{Re} F(\eta)F'(\eta)+(4-H)\alpha^2 F'(\eta)$$
$$+4De\left(F(\eta)F'''(\eta)+4\alpha^2 F(\eta)F'(\eta)\right)=0$$

(23)

$$\left(1+\frac{4}{3}Rd\right)\xi''(\eta)+\Pr Ec\left(4\alpha^2F^2(\eta)+\left(F'(\eta)\right)^2\right)+$$
$$2De\Pr Ec\left(4\alpha^2F^3(\eta)+F(\eta)\left(F'(\eta)\right)^2\right)-\frac{4}{3}Rd\,\alpha\xi'(\eta)=0$$

(24)

$$F(0)=0,\qquad F(1)=0,\qquad F'(0)=0,$$
$$\xi(1)=1,\qquad \xi'(0)=0,$$

(25)

With

$$\operatorname{Re}=\frac{rU\alpha}{\nu},\quad H=\sqrt{\frac{\sigma B_0^2}{\mu}},\quad De=\frac{\alpha_1 U}{r\mu},\quad Ec=\frac{\mu c_p}{k_0}$$
$$\Pr=\frac{U^2}{c_p T_w},\quad Rd=\frac{4\sigma^* T_w^3}{k^* k_0}$$

(26)

In above equations the Reynolds number, the Hartmann number, the Deborah number, the Eckert number, the Prandtl number radiation parameter are denoted by *Re*, *H*, *De*, *Ec*, *Pr*, *Rd*. Further for divergent channel $\alpha>0$, $U>0$ and convergent channel $\alpha<0$, $U<0$.

It should be pointed out that the problems of viscous flow can be recovered from Eqs. (23) and (24) when $De=0$ [23].

The skin friction coefficient is defined by:

$$c_f=\frac{\tau_w}{\rho U^2}$$

(27)

In which the surface shear stress in τ_w the second grade fluid is

$$\tau_w=\mu\left(\frac{1}{r}\frac{\partial u}{\partial \theta}\right)\Big|_{\theta=\alpha}+\alpha_1\left[\frac{-2}{r^2}u\frac{\partial u}{\partial \theta}+\frac{1}{r}u\frac{\partial^2 u}{\partial r\,\partial \theta}+\frac{1}{r}\frac{\partial u}{\partial r}\frac{\partial u}{\partial \theta}\right]\Big|_{\theta=\alpha},$$

(28)

Putting Eq. (28) into Eqs. (27) and then using Eq. (22) we finally have

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$$c_f = \frac{1}{\mathrm{Re}} F'(1)$$
(29)

The Nusselt number is given by:

$$Nu = \frac{r q_w \big|_{\theta=\alpha}}{k_0 T_w}, \qquad q_w = -(k_0 + \frac{16\sigma^* T_w^3}{3k^*})\nabla T$$
(30)

In view of Eq. (22) the above expression takes the following form

$$Nu = -\frac{1}{\alpha}(1+\frac{4}{3}Rd)\xi'(1)$$
(31)

In the next section we solve the problems consisting of Eqs. (23)-(25) by using DTM [13-21].

IV. SOLUTION BY DIFFERENTIAL TRANSFORMATION METHOD (DTM)

Using the standard DTM procedure we have from Eqs. (23)–(25) the following expressions

$$\begin{aligned}
&(k+1)(k+2)(k+3)G(k+3) + \\
&2\alpha \,\mathrm{Re}\sum_{i=0}^{k}(k-i+1)G(i)G(k-i+1) + (4-H)\alpha^2(k+1)G(k+1) \\
&+4De\left(\begin{aligned}&\sum_{i=0}^{k}(k-i+1)(k-i+2)(k-i+3)G(i)G(k-i+3) + \\ &4\alpha^2\sum_{i=0}^{k}(k-i+1)G(i)G(k-i+1)\end{aligned}\right) = 0
\end{aligned}$$
(32)

$$\begin{aligned}
&(1+\frac{4}{3}Rd)(k+1)(k+2)\Theta(k+2) + \\
&Ec\Pr\Bigg(4\alpha^2\sum_{i=0}^{k}G(i)G(k-i) + \\
&\sum_{i=0}^{k}(k-i+1)(i+1)G(i+1)G(k-i+1)\Bigg) + \\
&2De\Pr Ec\Bigg(4\alpha^2\sum_{i=0}^{k}\sum_{r=0}^{k-i}G(i)G(r)G(k-i-r) + \\
&\sum_{i=0}^{k}\sum_{r=0}^{k-i}(i+1)(k-i-r+1)G(r)G(i+1)G(k-i-r+1\Bigg) \\
&-\frac{4}{3}Rd\,\alpha(k+1)\Theta(k+1) = 0
\end{aligned}$$
(33)

$$G(0)=1,$$
(34)

$$G(1)=0, \quad \Theta(1)=0,$$
(35)

$$\sum_{i=0}^{\infty}G(i)=0,$$
(36)

$$\sum_{i=0}^{\infty}\Theta(i)=1,$$
(37)

In which denote the transformed functions of F and ξ, respectively. Letting $G(2)=\delta$ and $\Theta(0)=\beta$ and using Eqs. (32)- (35), another value of $G(i)$ and $\Theta(i)$ can be calculated. Thus we have

$$F(\eta) = \sum_{i=0}^{\infty} G(i)\eta^i$$
(38)

$$\xi(\eta) = \sum_{i=0}^{\infty} \Theta(i)\eta^i$$
(39)

Continuing this process and using above equations into Eq. (4) for H1=1, we obtain

$$\begin{aligned}
F(\eta) = {} & 1+\delta\eta^2 + \frac{\alpha\delta\left(-2\,\mathrm{Re}+(-4-16De+H)\alpha\right)}{12+48De}\eta^4 + \\
&\frac{\alpha\delta\left(\alpha\left(2\,\mathrm{Re}+(4+16De-H)\alpha\right)^2 - 12\left(\mathrm{Re}+2De\,H\,\alpha\right)\delta\right)}{360(1+4De)^2}\eta^6 + \ldots,
\end{aligned}$$
(40)

$$\begin{aligned}
\xi(\eta) = {} & \beta - \frac{6\alpha^2(1+2De)Ec\Pr}{3+4Rd}\eta^2 - \frac{8\alpha^3(1+2De)Ec\Pr Rd}{(3+4Rd)^2}\eta^3 - \\
&\frac{Ec\Pr\left(8\alpha^4(1+2De)Rd^2 + 2\alpha^2(1+3De)(3+4Rd)^2\delta + (1+2De)(3+4Rd)^2\delta^2\right)}{(3+4Rd)^3}\eta^4 \\
&-\frac{4\alpha Ec\Pr Rd\left(8\alpha^4(1+2De)Rd^2 + 2\alpha^2(1+3De)(3+4Rd)^2\delta + (1+2De)(3+4Rd)^2\delta^2\right)}{5(3+4Rd)^4}\eta^5 + \ldots,
\end{aligned}$$
(41)

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In order to determine the values of and we use the boundary conditions Eq. (25). Hence we have

$$F(1)=1+\delta+\frac{\alpha\delta\left(-2\,\mathrm{Re}+\left(-4-16De+H\right)\alpha\right)}{12+48De}+\frac{\alpha\delta\left(\alpha\left(2\,\mathrm{Re}+\left(4+16De-H\right)\alpha\right)^{2}-12\left(\mathrm{Re}+2De\,H\,\alpha\right)\delta\right)}{360\left(1+4De\right)^{2}}+\ldots=0 \quad (42)$$

$$\xi(1)=\beta-\frac{6\alpha^{2}(1+2De)Ec\,\mathrm{Pr}}{3+4Rd}-\frac{8\alpha^{3}(1+2De)Ec\,\mathrm{Pr}\,Rd}{\left(3+4Rd\right)^{2}}-\frac{Ec\,\mathrm{Pr}\left(8\alpha^{4}\left(1+2De\right)Rd^{2}+2\alpha^{2}\left(1+3De\right)\left(3+4Rd\right)^{2}\delta+(1+2De)\left(3+4Rd\right)^{2}\delta^{2}\right)}{\left(3+4Rd\right)^{3}}-\frac{4\alpha Ec\,\mathrm{Pr}\,Rd\left(8\alpha^{4}\left(1+2De\right)Rd^{2}+2\alpha^{2}\left(1+3De\right)\left(3+4Rd\right)^{2}\delta+(1+2De)\left(3+4Rd\right)^{2}\delta^{2}\right)}{5\left(3+4Rd\right)^{4}}+\ldots=1 \quad (44)$$

The solutions of above equations through MATHEMATICA gives δ and β.

V. RESULTS AND DISCUSSION

The objective of this section is to examine the influence of different emerging parameters on the dimensionless velocity, temperature, the skin friction coefficient and local Nusselt number. Both cases of diverging and converging channels are accorded. Effects of the Reynolds number *Re*, Hartmann number *H*, Deborah number *De* and angle are given due attention. Figs. 2-8 illustrate the dimensionless velocity for both divergent and convergent channels. Effects of the Reynolds number *Re*, Deborah number *De*, Prandtl number *Pr*, Eckert number *Ec*, radiation parameter *Rd* and angle α on the dimensionless temperature in divergent channel are plot in the Figs. 9- 14, respectively. Influence of the Hartman number H and angle α on the dimensionless temperature in convergent channel is shown in the Figs. 15 and 16 respectively. Fig. 2 shows that with the dimensionless velocity in divergent channel decreases when Reynolds number is increased. It is noticed from Fig. 3 that results for convergent channel are reverse.

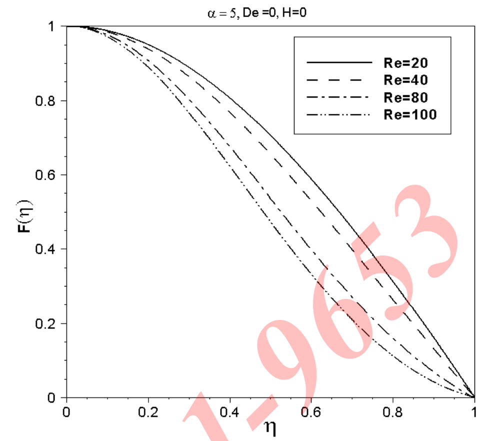

Fig. 2 The effect of the Reynolds number on the dimensionless velocity in case of divergent channel

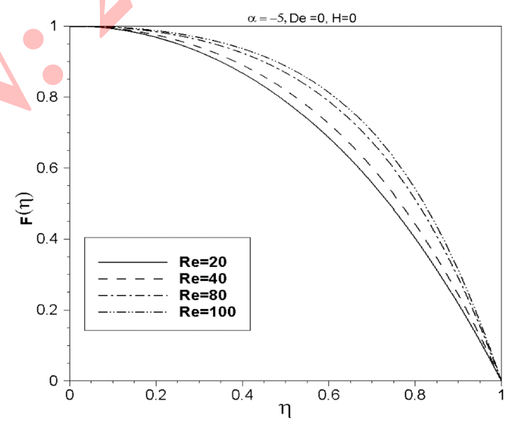

Fig. 3 The effect of the Reynolds number on the dimensionless velocity in case of convergent channel

Plots of velocity in divergent and convergent channels for Hartman number *H* are shown in the Figs. 4 and 5. Here in both cases, the dimensionless velocity increases when *H* increases.

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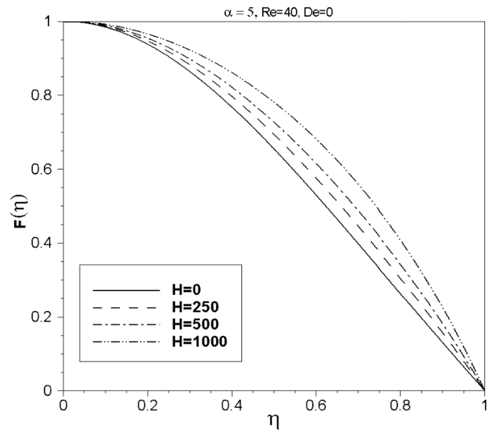

Fig. 4 The effect of the Hartmann number on the dimensionless velocity in case of divergent channel

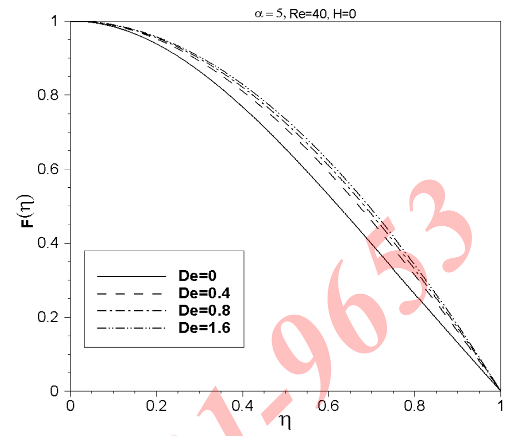

Fig. 6 The effect of the Deborah number on the dimensionless velocity in case of divergent channel

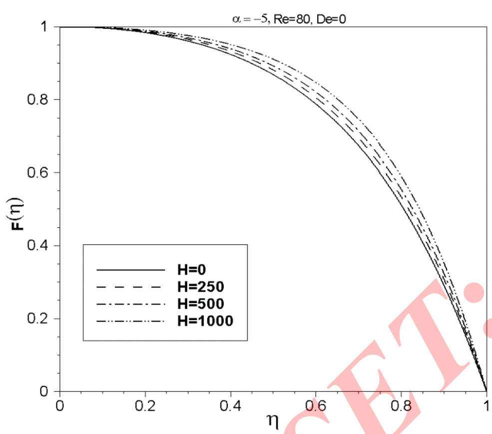

Fig. 5 The effect of the Hartmann number on the dimensionless velocity in case of convergent channel

Variation of Deborah number on the dimensional velocity in divergent channel is given in Fig. 6. Fig. 7 represents the dimensionless velocity via Deborah number in convergent channel. It is found that velocity increases in divergent case and it decreases in the convergent case when Deborah number increases.

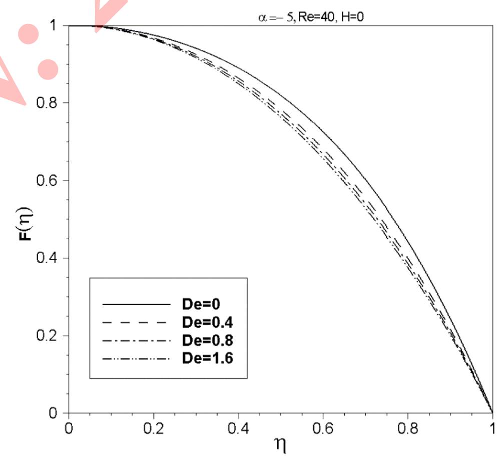

Fig. 7 The effect of the Deborah number on the dimensionless velocity in case of convergent channel

Fig. 8 depicts that how velocity is affected by angle α. It is revealed that in divergent channel, the velocity decreases, when α increases. However such results for convergent channel are opposite. From the Figs. 9-15 are evident that the dimensionless temperature in divergent channel increases when the parameters *Re*, *De*, *Pr*, *Ec*, *H* and α is increased. Fig. 13 shows that the dimensionless temperature is decreasing function of *Rd*. Fig. 16 illustrates that the dimensionless temperature in convergent channel decreases when the angle increases.

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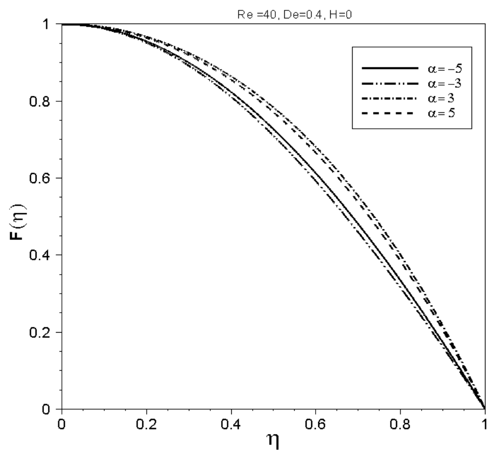

Fig. 8 The effect of the angle on the dimensionless velocity

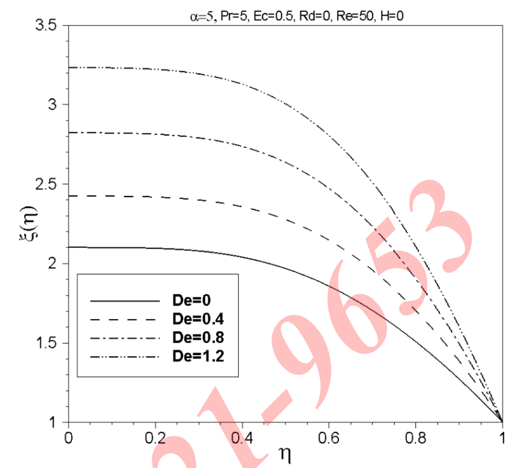

Fig. 10 The effect of the Deborah number on the dimensionless temperature for in case of divergent channel

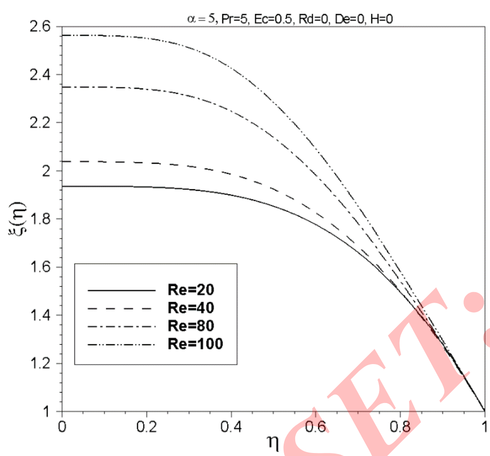

Fig. 9 The effect of the Reynolds number on the dimensionless temperature for in case of divergent channel

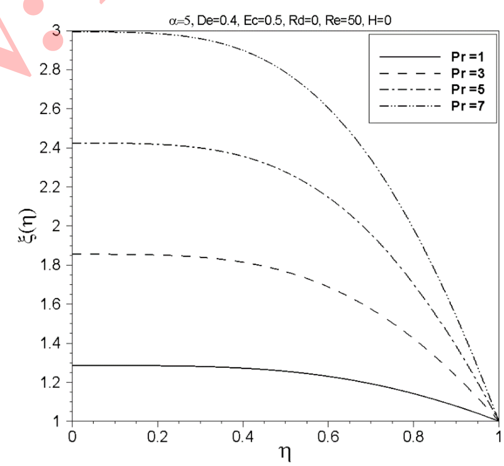

Fig. 11 The effect of the Prandtl number on the dimensionless temperature for in case of divergent channel

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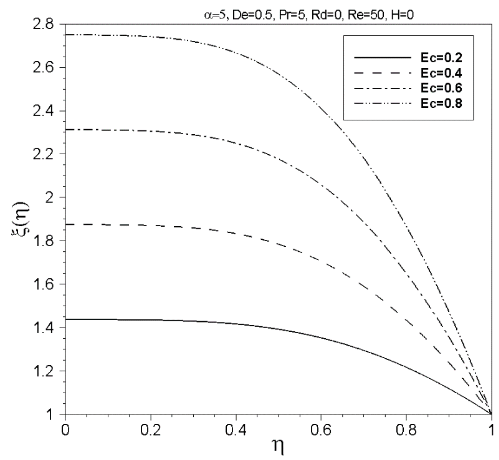

Fig. 12 The effect of the Eckert number on the dimensionless temperature for in case of divergent channel

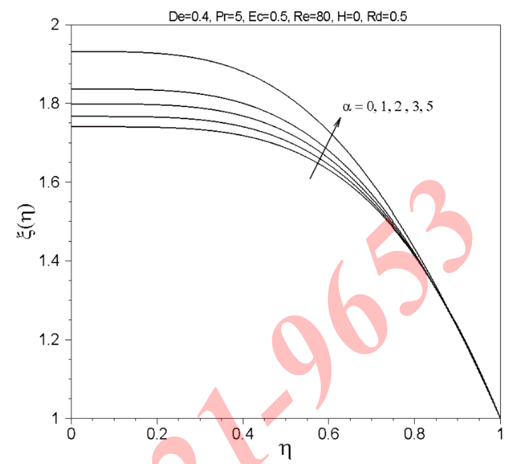

Fig. 14 The effect of the angle on the dimensionless temperature for in case of divergent channel

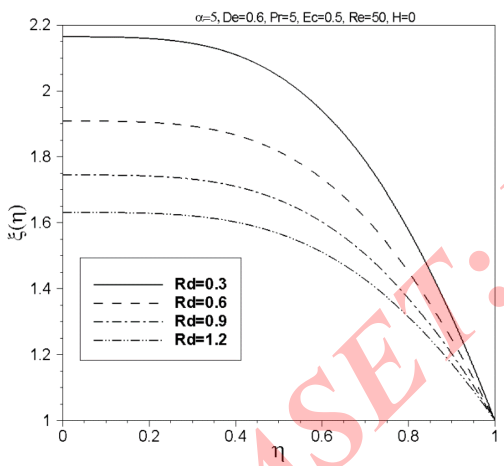

Fig. 13 The effect of radiation on the dimensionless temperature for in case of divergent channel

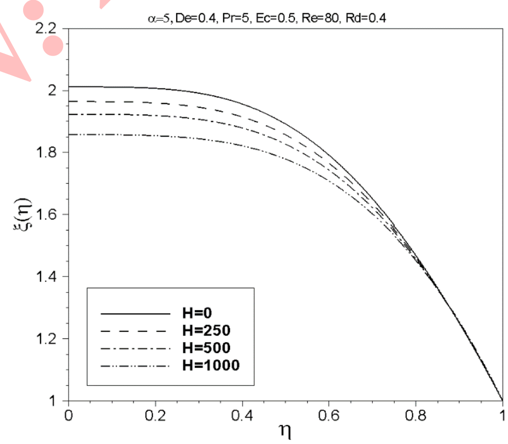

Fig. 15 The effect of the Hartmann number on the dimensionless temperature for in case of divergent channel

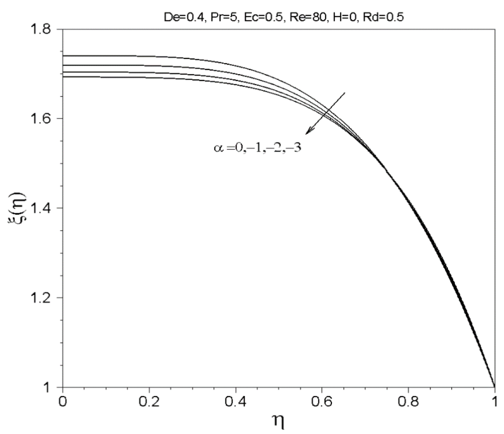

Fig. 16 The effect of the angle on the dimensionless temperature for in case of convergent channel

Tables II-V provide a comparative study of present results with the already reported results by other methods. Such Tables in fact confirm the validity of DTM. In particular, the comparison amongst results by DTM, homotopy perturbation method (HPM) [27] and numerical technique [27] is shown in Table II when for viscous fluid. Table III gives the comparison amongst DTM, OHAM [28], and SHAM [11] results for Newtonian fluid in the case of divergent channel. Excellent agreement with DTM solutions is noticed.

TABLE II
COMPARISON BETWEEN PRESENT, HPM AND NUMERICAL RESULTS FOR MHD VISCOUS FLOW ($De = 0$) IN DIVERGENT CHANNEL WHEN: $\alpha = 7.5^{\circ}$ AND $\mathrm{Re} = 50$.

***H*=0**			
η	DTM	HPM [27]	NS [27]
0	1	1	1
0.1	0.9771426	0.9770711	0.9771426
0.2	0.9114793	0.9112020	0.9114792
0.3	0.8110055	0.8104115	0.8110052
0.4	0.6869152	0.6859230	0.6869148
0.5	0.5512889	0.5498427	0.5512883
0.6	0.4151097	0.4131698	0.4151089
0.7	0.2870556	0.2846024	0.2870546
0.8	0.1731234	0.1702791	0.1731221
0.9	0.0768731	0.0744232	0.0768716
1	0.0000018	0	0
***H*=250**			
η	DTM	HPM [27]	NS [27]
0	1	1	1
0.1	0.9837368	0.9837340	0.9837368
0.2	0.9363462	0.9363350	0.9363460
0.3	0.8617138	0.8616894	0.8617134
0.4	0.7653821	0.7653405	0.7653815
0.5	0.6534583	0.6533961	0.6534573
0.6	0.531548	0.5314621	0.5315467
0.7	0.4039245	0.4038130	0.4039228
0.8	0.2730003	0.2728708	0.2729980
0.9	0.1390445	0.1389433	0.1390416
1	0.0000037	0	0
***H*=500**			
η	DTM	HPM [27]	NS [27]
0	1	1	1
0.1	0.9883197	0.9883197	0.9883197
0.2	0.9537953	0.9537955	0.9537953
0.3	0.8978512	0.8978515	0.8978511

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0.4	0.8224693	0.8224699	0.8224691
0.5	0.7296808	0.7296817	0.7296804
0.6	0.6209735	0.6209748	0.6209729
0.7	0.4966627	0.4966644	0.4966618
0.8	0.3552091	0.3552115	0.3552079
0.9	0.1923791	0.1923821	0.1923775
1	0.0000021	0	0

TABLE III
COMPARISON BETWEEN DTM, OHAM AND SHAM RESULTS FOR NEWTONIAN FLUID ($De=0$) WHEN: Re=50 AND $\alpha=5^{\circ}$

Re=50, $\alpha=5^{\circ}$			
η	DTM	OHAM [28]	SHAM [11]
0	1	1	1
0.1	0.982431	0.98251808	0.982431
0.2	0.931226	0.93156588	0.931226
0.3	0.850611	0.8513815	0.850611
0.4	0.746791	0.74826039	0.746791
0.5	0.626948	0.62953865	0.626848
0.6	0.498234	0.50242894	0.498234
0.7	0.366966	0.37293383	0.366966
0.8	0.238124	0.24508197	0.238124
0.9	0.115152	0.1207156	0.115152
1	0.00000021	0.000000001	0

ABLE IV
COMPARISON BETWEEN DTM AND HAM RESULTS FOR SKIN FRICTION COEFFICIENT FOR DIFFERENT VALUES OF Re AND De IN DIVERGENT AND CONVERGENT CHANNEL WHEN: $H=0$.

Re	De	α	$Re\, c_f$ present	$Re\, c_f$ [10]	α	$Re\, c_f$ present	$Re\, c_f$ [10]
40	0.8	2^o	-1.90718	-1.90964	-2^o	-2.08997	-2.09020
80			-1.81398	-1.81347		-2.17952	-2.17999
120			-1.71957	-1.71881		-2.26786	-2.26856
160			-1.62399	-1.62295		-2.35499	-2.35591
100	0	2^o	-1.291	-1.28910	-2^o	-2.67623	-2.67796
	0.4		-1.65163	-1.65067		-2.33228	-2.33315
	0.8		-1.76693	-1.76629		-2.22384	-2.24443
	1.2		-1.82427	-1.77509		-2.16943	-2.11700
100	0.8	0^o	-2.00000	-2.00000	0^o	-2.00000	-2.00000
		2^o	-1.76693	-1.76629	-2^o	-2.22384	-2.24443
		4^o	-1.52462	-1.42330	-4^o	-2.43867	-2.43997
		6^o	-1.27345	-1.27140	-6^o	-2.64482	-2.64644

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TABLE V
COMPARISON BETWEEN DTM AND HAM RESULTS FOR NUSSELT NUMBER FOR DIFFERENT VALUES OF Re, De, Pr AND Ec IN DIVERGENT CHANNEL WHEN: $Rd = 0,\ H = 0$

Re	***De***	***α***	***Pr***	***Ec***	αN_u **present**	αN_u **[10]**
40	0.8	5^o	5	0.5	5.36605	5.34102
80					5.21292	5.19476
120					5.10229	5.09998
160					5.06877	5.06849
100	0	5^o	5	0.5	2.96071	2.96815
	0.25				3.54499	3.54436
	0.5				4.2758	4.26952
	0.75				5.00702	5.03598
100	0.8	5^o	1	0.5	1.03089	1.02804
			3		3.09267	3.08413
			5		5.15445	5.14021
			7		7.21623	7.19630
100	0.8	5^o	5	0.3	3.09267	3.08413
				0.5	5.15445	5.14021
				0.7	7.21623	7.19630
				0.7	9.27801	9.25239
100	0.8	0^o	5	0.5	5.46667	5.46667
		2^o			5.29137	5.28043
		4^o			5.18203	5.16667
		6^o			5.13829	5.13443

The skin friction coefficient and local Nusselt number for different values of emerging parameters are compared with the HAM results by Hayat et al. [10]. Tables IV and V provide such comparisons. It is noted from Table IV that skin friction coefficient in divergent channel case decreases when the Reynolds number *Re* and angle α increase. Also the skin friction coefficient increases when the Deborah number *De* increases. The results of skin friction coefficient in a convergent channel are opposite to that of divergent channel. Table V describes that with increasing parameters *De*, *Pr*, *Ec* and angle α, the Nusslt number increases as well. While the Nusselt number in the case of divergent channel decreases when the Reynolds number increases. Table IV also illustrates that the skin friction coefficient for Newtonian fluid is higher than the second grade fluid in the case of convergent channel. The effect of radiation on the Nusselt number is given in Table VI. Here we can see that when the radiation parameter in the case of divergent channel increases, the Nusselt number increases as well. It is evident from Tables IV and V that there is a good agreement with DTM and HAM results for the skin friction coefficient and Nusselt number.

TABLE VI
EFFECT OF RADIATION ON THE NUSSELT NUMBER IN THE DIVERGENT CHANNEL WHEN: RE=50, DE=0.8, H=0, PR=5 AND EC=0.5

	α=1º	***α=2º***	***α=4º***
Rd	αN_u	αN_u	αN_u
0	5.41975	5.38208	5.33408
0.3	5.42802	5.39896	5.36956
0.6	5.43263	5.40837	5.38941
0.9	5.43556	5.41437	5.4021
1.2	5.43759	5.41853	5.4109

VI. CONCLUSIONS

In this study, we investigate the DTM results for MHD flow of a second grade fluid in divergent and convergent channel with thermal radiation effect. The following observations can be made from presented DTM solutions:

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- Variation of Hartmann number on velocity in the cases of divergent and convergent channel are qualitative similar.
- Behavior of Reynolds and Deborah numbers on the velocity are similar in the both cases of convergent and divergent channels.
- Results of Hartmann and Deborah numbers on velocity in divergent channel are opposite
- Effects of α, Prandtl, Eckert, Deborah and Reynolds numbers on the temperature in convergent and divergent channel cases are opposite in qualitative manner.
- Skin friction coefficient in divergent channel is different for Reynolds and Deborah numbers.
- In divergent channel, the effect of α, Prandtl and Eckert numbers on the Nusselt number are similar.
- Skin friction coefficient in second grade fluid is not significant when compared with the viscous fluid.

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