

# Singular-Value-Decomposition Based Generalized Inverse Aided Method PAPR-Aware Linear Precoder Designing for MIMO Based OFDM System

Ch. Meeravali<sup>1</sup>, M. Kamal Iqbal<sup>2</sup>, K. Lakshmi prasanna<sup>3</sup>, K. Rajeswari<sup>4</sup>, B. Rajya Lakshmi<sup>5</sup>  
<sup>1,2,3,4,5</sup>BITS, Kurnool

**Abstract:** Here we propose a linear precoding scheme for a single user multiple-input–multiple-output orthogonal frequency division multiplexing (OFDM) system to minimize PAPR (peak to average power ratio) by using some redundant spatial resources at the transmitter through a singular-value-decomposition-based generalized inverse system. The proposed precoder based on the generalized inverse is made of two parts, one for minimizing PAPR and the other for obtaining the multiplexing gain. Moreover, this proposed precoder contains a scalar parameter  $\alpha$  that quantifies the received SNR (signal-to-noise) power ratio loss at the cost of PAPR reduction. Even in cases of small SNR loss, the proposed scheme will dramatically be reducing the PAPR. Furthermore, simulation results show that we can obtain a PAPR as close as 1 by using a few dozens of transmission antennas with small SNR loss.

**Keywords:** OFDM, MIMO, precoding, PAPR, PAR, convex optimization, SU-MIMO.

## I. INTRODUCTION

MIMO (Multiple input and multiple output) and orthogonal frequency division multiplexing (MIMO-OFDM) systems provide high spectral efficiency for wireless communication system. However, they have a major drawback of high PAPR (peak-to-average power ratio) which results in inefficient use of a power amplifier. Thus, many studies have sought to develop PAPR reduction methods. For SISO (single-input-single-output) OFDM, the authors of presented an efficient algorithm based on the iterative clipping and filtering (ICF) process, and an optimized ICF method effectively reduced PAPR. Although both schemes show good performance in SISO-OFDM, they cannot be readily extended to be applicable to spatial multiplexing MIMO-OFDM since precoded symbols are distorted in the ICF process.

For the MIMO-OFDM systems, even if a convex optimization based conventional method can be readily extended to a MIMO precoding system, the PAPR performance is degraded as the number of transmission antennas increases under these fixed number of data streams. Thus, we hereby aim to use the redundant spatial resources to minimize PAPR. With an identical concept, the authors of improved PAPR performance by using the null spaces of MIMO channels based on a convex optimization. Also, they showed a theoretical bound of the PAPR performance such that a near unity PAPR [such that  $\text{PAPR} \approx 1$ ] can be achieved when the number of transmission antennas is infinite. Although the presented scheme has effectively improved PAPR performance for large-scale MIMO systems, it hence shows the limited performance enhancement under the current practical MIMO configurations such as  $(4 \times 2)$  or  $(8 \times 2)$  or  $(64 \times 8)$ MIMO, which motivates us to rethink and propose a novel precoding method for further improvement of PAPR performance.

In this paper, we exploit a generalized inverse of the right singular matrix of the MIMO channel to use redundant spatial dimensions at the transmitter. The generalized inverse of a matrix here inherently includes an arbitrarily controllable matrix which is our main key design parameter to minimize the PAPR, and also has a fixed part that we use for obtaining the spatial multiplexing gain. We also introduce a constant parameter  $\alpha$  that quantifies the received SNR (signal-to-noise) power ratio loss at the cost of PAPR reduction. The constant  $\alpha$  also can show the tradeoff between PAPR reduction and the received SNR loss. Even in cases of small SNR loss, the proposed method significantly improves PAPR performance since the maximum amplitude of the time-domain signals is minimized while keeping the average transmission power at a certain level. The simulation results show that our proposed method outperforms the current methods and provides a PAPR close to 1 with small SNR loss in case if the number of transmission antennas is large enough.

## International Journal for Research in Applied Science & Engineering Technology (IJRASET)

In this particular work,  $(\bullet)^T, (\bullet)^H, (\bullet)^{-1}, (\bullet)^\dagger, (\bullet)^-, (\bullet)^\perp, \|\bullet\|_\infty, \|\bullet\|_F, \text{blkdiag}(\bullet), \text{tr}(\bullet)$  and  $(\bullet)_{k,l}^r$  respectively hereby indicate the transpose, conjugate transpose, inverse, pseudo-inverse, generalized inverse, orthogonal, projection, infinite norm, Frobenius norm, block diagonalization, trace and the  $l$ th row vector of a matrix for the  $k$ th subcarrier.

### II. BACKGROUNDS

#### A. System Model Description

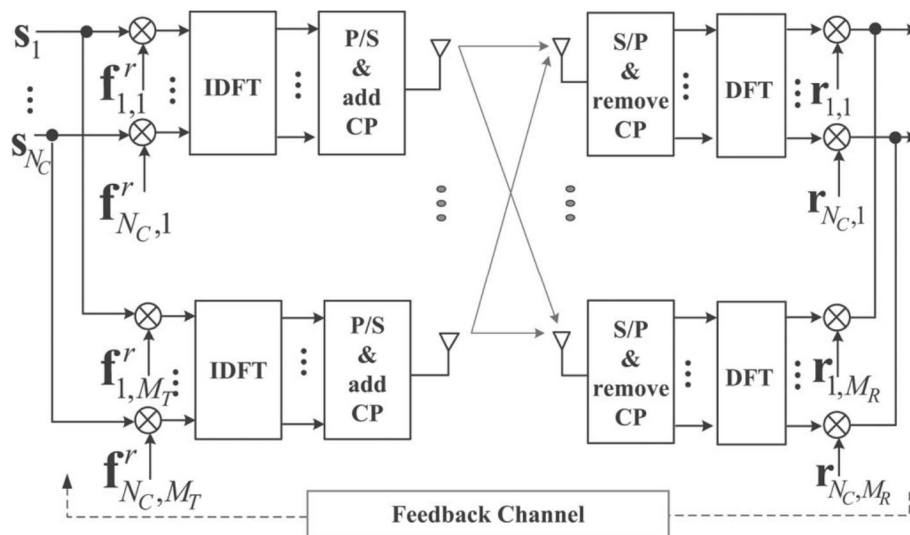
We consider a downlink single-user (SU) MIMO-OFDM system that consists of a transmitter equipped with  $M_T$  antennas and a receiver equipped with  $M_R$  antennas, where  $M_T > M_R \geq d_k$ . We assume the receiver perfectly reports channel information through an ideal feedback channel. The subscript  $k$  means the  $k^{\text{th}}$  subcarrier,  $\forall k \in \{1, \dots, N_C\}$ . The transmitter sends a  $d_k \times 1$  symbol vector  $s_k = [s_k, 1, \dots, s_k, d_k]^T$  satisfying  $E_{s_k}[s_k s_k^H] = (\mathcal{P}_s/d_k I)$ , then the received signal can be described as

$$y_k = R_k H_k F_k s_k + R_k n_k \quad (1)$$

where  $F_k$  denotes the transmission precoder for the  $k^{\text{th}}$  subcarrier satisfying  $E_{s_k}[F_k s_k s_k^H F_k^H] \leq \mathcal{P}_s$ .  $R_k$  is the receiver filter of the  $k^{\text{th}}$  subcarrier, and the complex Gaussian noise vector  $n_k$  satisfies  $\mathbf{E}[n_k n_k^H] = \sigma^2 \mathbf{I}$ .  $H_k$  is an  $M_R \times M_T$  Rayleigh fading MIMO channel, and the frequency selective fading MIMO-OFDM signaling is assumed as a series of narrow band frequency flat fading signals. For  $N_C$  subcarriers, the overall received signal can be denoted as

$$y = R H F s + R n \quad (2)$$

where  $n = [n_1^T, \dots, n_{N_C}^T]^T$ ,  $s = [s_1^T, \dots, s_{N_C}^T]^T$ ,  $F = \text{blkdiag}(F_1, \dots, F_{N_C})$ ,  $R = \text{blkdiag}(R_1, \dots, R_{N_C})$  and  $H = \text{blkdiag}(H_1, \dots, H_{N_C})$ .



#### B. Definition of Generalized Inverse and PAPR

Through a singular value decomposition (SVD),  $H_k$  can be divided into three parts such that  $H_k = U_k \Sigma_k V_k^H$ , where  $U_k$  denotes the unitary matrix  $M_R \times M_R$  and similarly  $V_k$  represents  $M_T \times M_T$  unitary matrix, and  $\Sigma_k$  is an  $M_R \times M_T$  diagonal matrix with singular values on the diagonal. For SU-MIMO data transmission, pre-coding matrix that minimizes VSER (vector symbol error rate) is defined as  $F_k = \bar{V}_k$  [7], where  $\bar{V}_k$  is composed of the first  $d_k$  columns of  $V_k$  and  $\text{tr}(\bar{V}_k^H \bar{V}_k) = d_k$ . A generalized inverse of  $\bar{V}_k^H$ , namely  $\bar{V}_k^{H-}$ , should satisfy a condition such that  $\bar{V}_k^H \bar{V}_k^{H-} \bar{V}_k^H = \bar{V}_k^H$  [8]

The matrix  $\bar{V}_k^{H-}$  can be represented as

$$\bar{V}_k^{H-} = \bar{V}_k^{H\dagger} + P_k^\perp T_k \quad (3)$$

Where  $P_k^\perp$  is an orthogonal projection matrix and is given by  $P_k^\perp = I - \bar{V}_k^{H\dagger} \bar{V}_k^H$

and  $T_k$  is an  $M_t \times d_k$  random matrix. In consideration of the inverse discrete Fourier transform (IDFT) at the transmitter, the PAPR of MIMO-OFDM system can be defined as

## International Journal for Research in Applied Science & Engineering Technology (IJRASET)

$$PAPR = \frac{j \in [1, l, N_C^{max}], i \in [1, M_T] |\tilde{x}_j^{(i)}|^2}{\frac{1}{l N_C M_T} \sum_{i=1}^{M_T} \sum_{j=1}^{l N_C} |\tilde{x}_j^{(i)}|^2} \quad (4)$$

where  $\tilde{x}_j^{(i)}$  is the  $j^{th}$  element of  $\tilde{x}^{(i)}$ . The vector  $\tilde{x}^{(i)}$  denotes the time-domain transmission signal at the  $i^{th}$  transmitter antenna, and  $l$  is an over-sampling factor.

### III. DOWNLINK MIMO PRECODER DESIGN

The ultimate objective is to design a MIMO precoder that minimizes the PAPR while producing a substantial data rate performance based on MIMO pre-coding gain. First, let us consider the PAPR minimization.

#### A. A Direct Approach

To minimize the maximum amplitude while maintaining the the average power consumption certain level, which can be described as

$$\underset{F_k \forall k}{\text{minimize}} \quad \max_{j, \forall i} |\tilde{x}_j^{(i)}|^2 \quad (5)$$

Subject to

$$\mathcal{P}_{avg}^- \leq E_{i,j} [|\tilde{x}_j^{(i)}|^2] \leq \mathcal{P}_{avg}^+ \quad (6)$$

where both  $\mathcal{P}_{avg}^-$  and  $\mathcal{P}_{avg}^+$  denote bounds on the average power satisfying  $\mathcal{P}_{avg}^- < \mathcal{P}_{avg}^+$  and  $\mathcal{P}_{avg}^- - \mathcal{P}_{avg}^+ \approx 0$

Unfortunately, the above problem is a non-convex due to the lower bound in (6), but we aim to minimize PAPR significantly by formulating a convex problem based on the direct approach.

#### B. Precoding Matrix Configuration

The key design factor is to use the null space of the right singular matrix of  $H_k$  that exists in the generalized inverse of  $V_k^H$ . Based on (3), we can represent the proposed precoding matrix as

$$F_k = \sqrt{\alpha} \cdot \bar{V}_k^{H\perp} + P_k^\perp T_k \quad (7)$$

where the constant  $\alpha$  is the received SNR loss factor satisfying  $0 < \alpha < 1$ . The SVD-based generalized inverse allows us to control the power consumption ratio by adjusting  $\alpha$  between the effective data transmission and the control signal  $P_k^\perp T_k$

This can be seen from the following equation

$$\text{tr}(F_k^H F_k) = \text{tr}(\bar{V}_k^H \bar{V}_k + T_k^H P_k^\perp T_k) + \sqrt{\alpha} \cdot \text{tr}(T_k^H P_k^\perp \bar{V}_k + \bar{V}_k^H P_k^\perp T_k) \quad (8)$$

where  $P_k^{H\perp} = P_k^\perp$  and  $P_k^\perp P_k^\perp = P_k^\perp$ .

With the precoder in (7) and the receiver  $R_k = U_k^H$ , the received signal can be easily calculated as

$$y_k = \sqrt{\alpha} \bar{\Sigma}_k s_k + U_k^H n_k \quad (9)$$

where  $\bar{\Sigma}_k$  indicates the first  $d_k$  column vectors of  $\Sigma_k$  and  $U_k^H H_k F_k = \sqrt{\alpha} \bar{\Sigma}_k I$ . It is observed that the second term in (7) has no effect on the received signal.

#### C. Precoder Design For Papr Minimization

In consideration of the design parameter  $T_k$  given in (7), we can reformulate the problem defined in (5) and (6) as follows

$$\underset{T_k \forall k}{\text{minimize}} \quad \max_{j, \forall i} |\tilde{x}_j^{(i)}| \quad (10)$$

$$\text{subject to} \quad \mathcal{P}_{avg}^- \leq E[s_k^H F_k^H F_k s_k] \leq \mathcal{P}_{avg}^+ \quad (11)$$

$$\text{where} \quad \tilde{x}^{(i)} = Q^{IDFT} [f_1^r, i s_1, \dots, f_{N_C}^r i s_{N_C}]^T \quad (12)$$

where  $Q^{IDFT}$  refers to the  $l N_C \times l N_C$  IDFT matrix. The precoding matrix  $F_k$  was not explicitly expressed as a function of the

## International Journal for Research in Applied Science & Engineering Technology (IJRASET)

$M_T \times 1$  target vector variable  $t_{k,l}$  but those are implicitly contained in the row vector of the precoding matrix  $\mathbf{F}k$ , which can be denoted as

$$f_{k,i}^r = \sqrt{\alpha} \bar{V}_{k,i}^r + [p_{k,i}^{\perp,r} t_{k,1}, \dots, p_{k,i}^{\perp,r} t_{k,d_k}] \quad (13)$$

By substituting  $\mathbf{F}k$  of (7) into (11), we can rewrite (11) as

$$\mathcal{P}_{avg}^- \leq \alpha \frac{\mathcal{P}_s}{d_k} \text{tr} \bar{V}_k^H \bar{V}_k + \frac{\mathcal{P}_s}{d_k} \sum_{i=1}^{d_k} t_{k,i}^H P_k^\perp t_{k,i} \leq \mathcal{P}_{avg}^+ \quad (14)$$

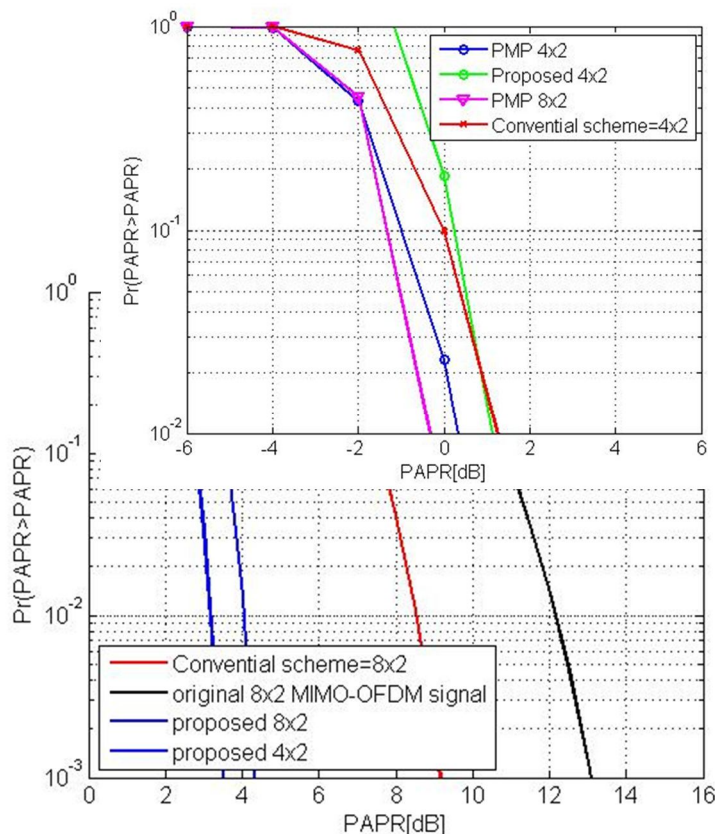
It can be seen that the average power consumption is always larger than  $\alpha \mathcal{P}_s$  because the average power is minimized when the design parameters  $t_{k,1}, \dots, t_{k,d_k} = 0$  due to the positive definite matrix  $P_k^\perp$ . We can assume that  $\mathcal{P}_{avg}^+ = \mathcal{P}_s$  and  $\mathcal{P}_{avg}^- = \alpha \mathcal{P}_s$

It can be observed that  $\alpha \mathcal{P}_s$  is the transmission power for carrying data signals, and  $(1 - \alpha) \mathcal{P}_s$  is the maximum power that can be utilized to decide the matrix  $P_k^\perp T_k$ . Consequently, even if the lower bound of (14) is neglected, the average power consumption is naturally maintained in between  $\alpha \mathcal{P}_s$  and  $\mathcal{P}_s$ . Thus, we can have an opportunity to minimize PAPR when  $\alpha$  is close to 1. Finally, by modifying (10) and (11), we formulate an infinite-norm minimization problem that is one of the second order cone programmings (SOCPs).

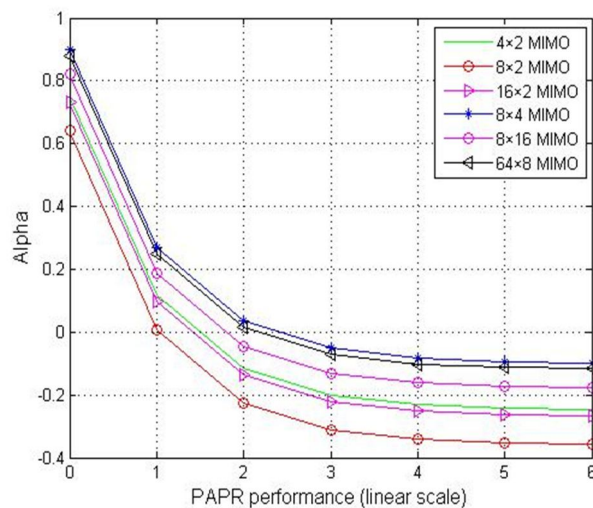
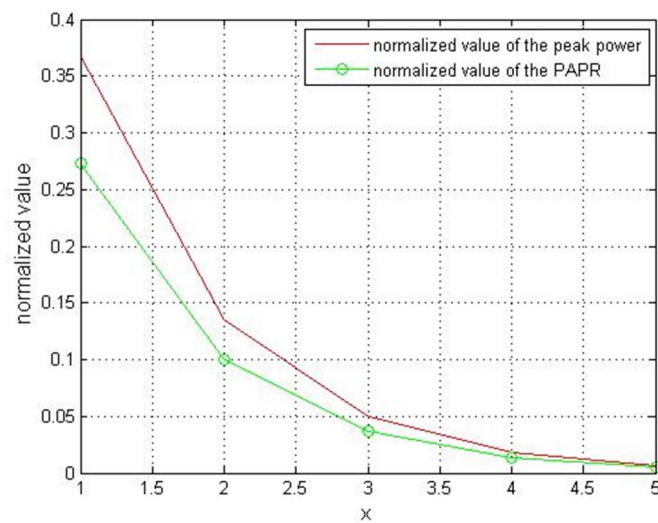
In between  $\alpha \mathcal{P}_s$  and  $\mathcal{P}_s$ , the solutions of the convex problem can be found through an interior point method (IPM) that based on the barrier method with an iterative process. Also, the solutions would be easily found by using a convex optimization package such as [9]. By introducing the generalized inverse, the proposed scheme utilizes  $d_k$  times more free variables compared to [5], [6], but it has a similar worst case computational complexity to [5], [6] since  $N_c$  is the dominant factor for this SOCP when  $N_c \gg M_T$ . Under a classical IPM approach such as primal-dual IPM [10], the computational complexity can be denoted as  $\mathcal{O}(nN_c^3)$  where  $n$  is the maximum number of iterations.

Thus, we can intuitively predict that the constant  $\alpha$  would be selected close to 1 when PAPR negligibly affects the error rate performance such that gain in the MIMO precoding stands more critical than PAPR performance. Albeit, it is noted that reducing the maximum amplitude at the expense of the received SNR loss may not assure the *peak-to-average power ratio* reduction due to the variation of the average transmission power. It implies that there is an  $\alpha$  which provides the best tradeoff between the PAPR performance and SNR loss given system parameters such as  $d_k$ , modulation order, dynamic range and  $M_T$ . However, we leave this problem as future work and we focus on the performance effect according to  $\alpha$  in the following section.

### IV. RESULTS



# International Journal for Research in Applied Science & Engineering Technology (IJRASET)



## V. CONCLUSION

We hereby have proposed a MIMO precoding scheme which consists of a sum of two matrices, one of which is made to MIMO precoding gain which is associated with the cost factor  $\alpha$ , and the other, of which it is designed to minimize PAPR by using redundant spatial dimensions at the transmitter point. When  $\alpha$  is close or approximately equal to 1, we then could guarantee the effective data rate as well as improving PAPR performance. It is to be expected that a decision criterion of  $\alpha$  can be made by finding

## International Journal for Research in Applied Science & Engineering Technology (IJRASET)

a connection point between our study on the project and the related theoretical work, which is hereby left in the future work. Also, the joint consideration of the transmitter and the receiver PAPR is worthy of further study.

### REFERENCES

- [1] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 89–101, Jan. 2002.
- [2] J. Armstrong, "Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering," *Electron. Lett.*, vol. 38, no. 5, pp. 246–247, Feb. 2002.
- [3] Y. C. Wang and Z. Q. Luo, "Optimized iterative clipping and filtering for PAPR reduction of OFDM signals," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 33–37, Jan. 2011.
- [4] A. Aggarwal and T. H. Meng, "Minimizing the peak-to-average power ratio of OFDM signals using convex optimization," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 3099–3110, Aug. 2006.
- [5] A. Aggarwal, E. Stauffer, and T. Meng, "Optimal peak-to-average power ratio reduction in MIMO-OFDM systems," in *Proc. IEEE ICC*, Jun. 2006, pp. 3094–3099.
- [6] C. Studer and E. G. Larsson, "PAR-aware large-scale multi-user MIMO-OFDM downlink," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 303–313, Feb. 2013.
- [7] D. J. Love and R. W. Heath, "Multimode precoding for MIMO wireless systems," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3674–3687, Oct. 2005.
- [8] A. Wiesel, Y. C. Eldar, and S. Shamai, "Zero-forcing precoding and generalized inverses," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4409–4418, Sep. 2008.
- [9] M. Grant and S. Boyd, *CVX: Matlab Software for Disciplined Convex Programming*, version 2.1, Mar. 2014. [Online]. Available: <http://cvxr.com/cvx>
- [10] S. J. Wright, *Primal-Dual Interior-Point Methods*. Philadelphia, PA, USA: SIAM, 1997.
- [11] W. Yin, C. Studer, and R. G. Baraniuk, "Signal representations with minimum l-norm," in *Proc. 50th Annu. Allerton Conf. Commun., Control, Comput.*, Oct. 2012, pp. 1270–1277.