Volume: 5 Issue: V Month of publication: May 2017

DOI:

www.ijraset.com
IC Value: 45.98
Volume 5 Issue V, May 2017
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Application of Homotopy Analysis Method for Solution of Instability Phenomenon Arising in Fluid Flow through Porous Media with Inclination and Gravitational Effect

V. P. Gohil [1], Dr. G. A. Ranabhatt[2]
[1,2]*Assistant professor, Department of Mathematics, Government Engineering College, Bhavnagar*

Abstract: In this paper, instability phenomenon in homogeneous porous media with inclination effect is discussed. Homotopy analysis method is used here to study the saturation rate for homogeneous porous media. A detailed discussion of saturation rate at various inclination angle has been done with dimensionless time and a comparison study with its physical interpretation has been done by using mathematican software
Keywords: Instability, imbibition, gravitation, homotopy analysis method, porous media.

I. INTRODUCTION

Imbibitions phenomenon occurs due to difference in wetting abilities of the two immiscible fluids flowing in the medium. Imbibitions means that a wetting fluid displaces the non-wetting one, while the opposite case is called drainage

The instability phenomenon has much significance in secondary oil recovery process of petroleum technology. The instability or fingering phenomenon in porous media may arise when two phases of different viscosities and densities are separated by an initially sharp interface that becomes unstable under certain conditions. Figure-1 represents the diagram of instabilities in cylindrical piece of porous media. When native phase (oil) is filled in pores of the porous media and another phase (water) is injected, then instead of regular displacement of whole front at common interface protuberances take place which shoot through the porous media at relatively very high speed.

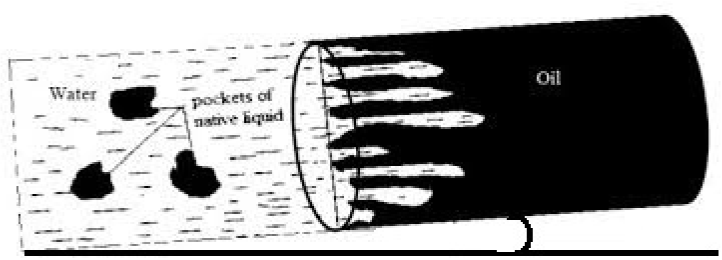

Figure-1 The Instability Phenomenon in Vertical Cross Section of Cylindrical Porous Media

II. MATHEMATICAL FORMATION

From Darcy's law the seepage velocity of water V_w and oil V_o can be written

$$V_w = -\frac{K_w}{\mu_w} K \left(\frac{\partial P_w}{\partial x} + \rho_w \, g \, \sin\alpha \right) \tag{1}$$

$$V_o = -\frac{K_o}{\mu_o} K \left(\frac{\partial P_o}{\partial x} + \rho_w \, g \, \sin\alpha \right) \tag{2}$$

where " K " is permeability of homogeneous medium, K_o and K_w are relative permeability of oil and water which are function

Volume 5 Issue V, May 2017
ISSN: 2321-9653

of S_o and $S_w \cdot \rho_w$ and ρ_w are constant densities of water and oil respectively, while μ_W and μ_W are constant kinematic viscosity of the phases in homogeneous porous media, α is the inclination of the bed, g is acceleration due to gravity.

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3}$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \tag{4}$$

Where "ϕ is the porosity of medium. From the definition of phase saturation, we have

$$S_w + S_o = 1 \tag{5}$$

The capillary pressure, which is defined as the pressure discontinuity of the following phases across the common interface is written as

$$P_c = P_o - P_w \tag{6}$$

Implies

$$\frac{\partial P_w}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \tag{7}$$

The equation of motion for saturation can be obtained by substituting of the values of V_w and V_o from equation (1) and (2) in equation (3) and (4) respectively. Thus, we have

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(\left(\frac{K_w}{\mu_w} K\right)\left(\frac{\partial P_w}{\partial x} + \rho_w\, g\, \sin\alpha\right)\right) \tag{8}$$

$$\phi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left(\left(\frac{K_o}{\mu_o} K\right)\left(\frac{\partial P_o}{\partial x} + \rho_o\, g\, \sin\alpha\right)\right) \tag{9}$$

Equation (7) and Equation (8) together

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(\left(\frac{K_w}{\mu_w} K\right)\left(\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} + \rho_w\, g\, \sin\alpha\right)\right) \tag{10}$$

Now by considering Equation (5), (9) and (10), it becomes

$$\left[\frac{\partial P_o}{\partial x} K\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right) - \frac{K_w}{\mu_w} k \frac{\partial P_c}{\partial x} + g\, \sin\alpha K\left(\frac{K_o}{\mu_o}\rho_o + \frac{K_w}{\mu_w}\rho_w\right)\right] = -q \tag{11}$$

Where q is constant of integration.

Equation (11) Implies

$$\frac{\partial P_o}{\partial x} = \left[\frac{-q + \frac{K_w}{\mu_w} k \frac{\partial P_c}{\partial x}}{K\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)} - \frac{g\, \sin\alpha\, K\left(\frac{K_o}{\mu_o}\rho_o + \frac{K_w}{\mu_w}\rho_w\right)}{K\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)}\right] \tag{12}$$

Equation (10) and (12) together gives

$$\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\frac{K \cdot K_o}{\mu_o}\left(\frac{\partial P_c}{\partial x} + g\, \sin\alpha\,(\rho_o - \rho_w)\right)}{\left(1 + \frac{K_o \cdot \mu_w}{K_w \cdot \mu_o}\right)} + \frac{q}{\left(1 + \frac{K_o \cdot \mu_w}{K_w \cdot \mu_o}\right)}\right] = 0 \tag{13}$$

Volume 5 Issue V, May 2017
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

The value of the pressure of oil (P_o) can be written as

$$P_o = \bar{P} + \frac{1}{2}P_c \Rightarrow \frac{\partial P_o}{\partial x} = \frac{1}{2}\frac{\partial P_c}{\partial x} \tag{14}$$

where $\bar{P}$ is the mean pressure.
Equation (14), (11) together with equation (13) gives,

$$\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left(\left(\frac{K_w}{\mu_w}K\right)\left(\frac{1}{2}\frac{\partial P_c}{\partial x} - g\rho_w \sin\alpha\right)\right) = 0 \tag{15}$$

on using equation (10) in equation (15), it reduces to

$$\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x}\left(\left(\frac{S_w}{\mu_w}K\right)\left(\frac{\beta}{2}\frac{\partial S_w}{\partial x} + g\rho_w \sin\alpha\right)\right) = 0 \tag{16}$$

Equation (16) describe the equation of motion for saturation of wetting phase in fingering imbibition in fluid flow through fracture porous media with inclination and gravitational effect.

Using the dimensionless variables, Equation (16) reduces to, $X = \frac{x}{L_c}$, $T = \left(\frac{gK\rho_w}{\mu_w \phi L_c}\right)t$

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left(S_w \frac{\partial S_w}{\partial X}\right) + \sin\alpha \frac{\partial S_w}{\partial X} = 0 \tag{17}$$

We choose appropriate initial and Dirichlets boundary condition due to the behaviour of saturation of displaced water at the interface in instability phenomena; that is, instability of oil and water zone at the interface is high, and it becomes stable as it becomes away from the interface and by [11] as,

$$\begin{aligned} S_w(X, 0) &= e^{=X}, \\ S_w(0, T) &= f_1(T), \\ S_w(1, T) &= f_2(T) \end{aligned} \tag{18}$$

Where f_1 and f_2 are the saturation of water at common interface $X = 0$ and saturation of water at end of matrix of length $X = 1$ (i.e. $x = L$). It is necessary to discuss the behaviour of saturation of displace water by solving (17) together with (18). Equation (17) is the desired nonlinear partial differential equation with_suitable initial and boundary conditions which describes the saturation of displaced water in instability phenomena arising during oil recovery process.

III. RESULTS AND DISCUSSION

After applying homotpy analysis method on the governing equation (17), we get analytical solution $S_w(X, T)$. Using Mathematica software; all numerical values have been derived.
From figure-2 to 4 and table-I to III, it is apparent that Saturation of water increases as time increases at a fixed point $X = 0.2$, $X = 0.5$ and $X = 0.8$. For inclination at $\alpha = 0^0$, $\alpha = 5^0$, $\alpha = 10^0$, $\alpha = 15^0$ checked the value of saturation rate.

Table –I Saturation Vs Time with inclination effect $[X = 0.2]$

T	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.824093	0.823246	0.822456	0.821778
0.4	0.829456	0.827761	0.826181	0.824824
0.6	0.834818	0.832276	0.829906	0.827871
0.8	0.840181	0.836791	0.833631	0.830918

Table –II Saturation Vs Time with inclination effect $[X = 0.5]$

T	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.609474	0.608846	0.608261	0.607758
0.4	0.612417	0.611161	0.609991	0.608986
0.6	0.615360	0.613476	0.611721	0.610213
0.8	0.618363	0.615791	0.613451	0.611441

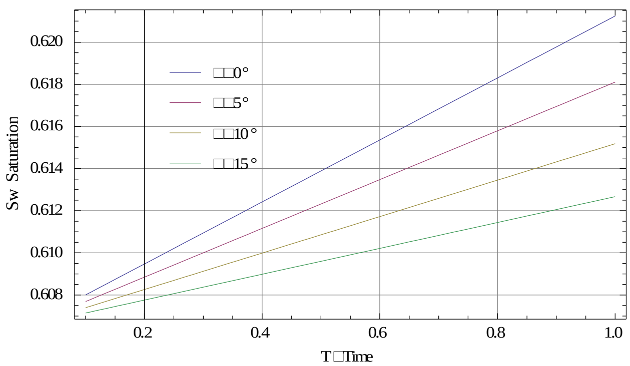

Figure -2 Saturation Vs Time (X=0.5)

Table –III Saturation Vs Time with inclination effect $[X = 0.8]$

T	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.450944	0.450479	0.450045	0.449673
0.4	0.452559	0.451629	0.450762	0.450018
0.6	0.454174	0.452779	0.451479	0.450362
0.8	0.455790	0.453929	0.452195	0.450705

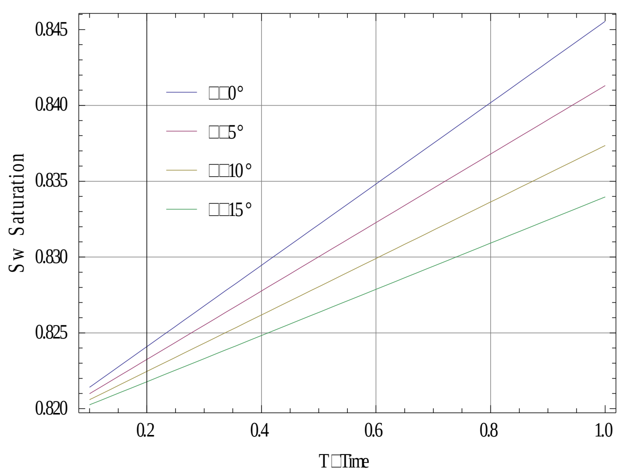

Figure -3 Saturation Vs Time (X=0.2)

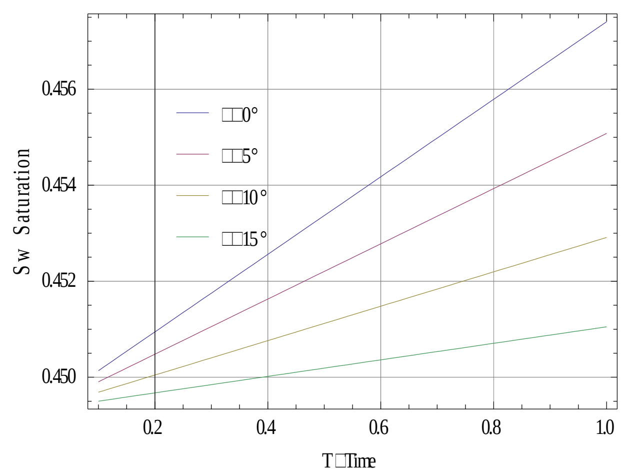

Figure- 4 Saturation Vs Time (X=0.8)

IV. CONCLUSION

In the instability phenomenon, Homotopy analysis method has been applied success- fully using freedom of choosing parameter. Equation (17) represents the saturation of wetting phase for instability phenomena with inclination and dimensionless time and distance. Studied the variation of saturation of water in and direction for particular parametric values of the parameter. Tables and graphs shows that the saturation of wetting phase is maximum for zero inclination which leads increase the rate of saturation.

REFERENCES

[1] A. E. Scheidegger, The Physics of Flow through Porous Media, University of Toronto Press,(1960)

[2] A. K. Parikh, M. N. Mehta and V. H. Pradhan, "Generalized separable solution of counter-current imbibition phenomenon in homogeneous porous medium in horizontal direction", The International Journal of Engineering and Science, 2 , no. 1, 220-226 ,2013.

[3] A. P. Verma, "On stabilization of fingers in a slightly cracked heterogeneous porous medium", Developments in Soil Science, 2 , 221-228, 1972.

[4] A. P.Verma, "Imbibition in a cracked porous medium", Canadian Journal of Physics, 47 , no. 22, 2519-2524, 1969.

[5] H. S. Patel and R. Meher, "Approximate Analytical Study of Counter-Current Imbibition Phenomenon in a Heterogeneous Porous Media", Applied Mathematical Sciences, 10 , no. 14, 673 681,2016.

[6] J. Bear, Dynamic of fluid in Porous Media, American Elsevier Publishing Company,Inc, 1972.

[7] K. R. Patel, M. N. Maheta, T. R.. Patel, "A mathematical model of imbibition phenomena in heterogeneous porous media during secondary oil recovery process", Applied Mathematical Modelling 37,2933-2942, 2013.

[8] R. Meher, M. N. Mehta, S. K. meher, "Exponential Self Similar Solutions Technique for Imbibition Phenomenon Arising in Double Phase Flow Through Porous Medium with Capillary Pressure", Int,J.ofAppl.Math and Mech.7,no.8,2940, 2011.

[9] R. Meher, M. N. Mehta and S. K. Meher, "Instability phenomenon arising in double phase flow through porous medium with capillary pressure, International Journal of Applied Mathematics and Mechanics", 7 , no. 15, 97-112, 2011

[10] R. K. Meher and V. P. Gohil, "Application of homotopy analysis method for the solution of Cubic Boussinesq equation and Boussinesq-Burger equation", Communications in Applied Analysis 20 , 379-396, 2016

[11] V.H. Pradhan and A.P. Verma, "On imbibition equation in the theory of fluid flow through porous media, A numerical approach", J. Indian Acad. Math., 19 , no. 1, 19-29 ,1977

[12] V. P. Gohil, R. K. Meher, "Homotopy analysis method for solving Counter current imbibition phenomena of the time fractional type arising in the heterogeneous porous media", International Journal of Mathematics and Computation, Vol. 28, Issue No.-2, 2017