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Five-Dimensional Cosmological Model with Variable G and Λ

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Abstract: ***In this paper, we have considered the Einstein's Field equations with variable G and Λ and we have obtained several sets of explicit solution in the five dimensional Kaluza-klein type cosmological model.***
Keywords: Kaluza Klein cosmology, cosmological constant

I. INTRODUCTION

There are two parameters, the cosmological constant Λ and the gravitational constant G, present in Einstein's field equations. The Newtonian constant of gravitation G plays the role of coupling constant between geometry and matter in the Einstein field equations. In several papers it has observed that both parameters vary together in a way that leaves Einstein's equations formally unchanged. With this motivation it is interesting to study five dimensional Kaluza-Klein type metric with perfect fluid in the presence of variable G and Λ. Kaluza - Klein achievement is shown such that five dimensional general relativity contains both Einstein four dimensional theory of gravity and Maxwell's theory of electromagnetism. In this chapter we study five dimensional Kaluza-Klein type of metric with perfect fluid and varying G and Λ. The Physical variable such as expansion and shear are expressed in terms of higher dimensional metric. This work is the work obtained earlier by Baysal and Yilmaz (2007) in ve dimensional space - time. In connection with this work we have also obtained the solution for the case n = 1 which was ignored by Baysal and Yilmaz (2007) and pointed out by Arbab (2008). Some physical properties of model are also examined.

II. EINSTEIN FIELD EQUATIONS

We consider the Kaluza Klein type metric as

$$ds^2 = -dt^2 - R^2(t)\left(dx^2 + dy^2 + dz^2\right) + A^2(t)\,dm^2 \qquad (1)$$

where R and A are functions of t.

The expression for energy-momentum tensor for perfect fluid is given by

$$T_{\mu\nu} = (\rho + p)\,\mu_a \mu_b - p g_{ab}, \qquad (2)$$

where ϑ_μ is five velocity, ρ and p are the matter's density and isotropic pressure. we use commoving coordinate system $\vartheta_\mu = \delta_0^\mu$, and the Einstein field equations can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\vartheta} R = T_{\mu\nu} + \Lambda\, g_{\mu\nu}, \qquad (3)$$

where $R_{\mu\nu}$ is the Ricci tensor.

The Einstein's field equations (3) for the line element (1) with the help of equation (2) can be expressed as

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}\dot{A}}{RA} = 8\pi G\rho + \Lambda \qquad (4)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = -8\pi G\rho + \Lambda \qquad (5)$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} = -8\pi G\rho + \Lambda \qquad (6)$$

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Here the dot over the variables represents derivatives with respect to time.
The physical variable, namely the expansion and sheer scalar, have the following expressions for the above metric:

$$\theta = \mu^{a}_{:a} = 3\frac{\dot{R}}{R} + \frac{\dot{A}}{A}, \tag{7}$$

$$\sigma^{2} = \frac{3}{8}\left(\frac{\dot{R}}{R} - \frac{\dot{A}}{A}\right)^{2}. \tag{8}$$

We note that Equations (4)-(5) supply only three independent equations in six unknown parameters ρ , p, R, A, G, and Λ . Therefore to obtain an exact solution of the field equations we need three more relations connecting these variables. We assume that the polynomial relation between the metric coefficients reeds

$$A = \mu R^{n}, \tag{9}$$

and the equation of state

$$p = \omega \rho \tag{10}$$

where μ , n and ω are constants.

The law of conservation of energy $\left(T^{ab}_{:b} = 0\right)$ gives

$$\dot{\rho} + \left(3\frac{\dot{R}}{R} - \frac{\dot{A}}{A}\right)(\rho + p) = 0, \tag{11}$$

From equations (4)-(6), (10) and (11) we obtain,

$$\dot{G} = -\frac{\Lambda}{8\pi\rho} \tag{12}$$

Equation (12) represents the variation of G and Λ with time. Substituting Equation (7) into Eq. (6) and using equation (9), yields

$$(n-1)\left(\frac{\ddot{R}}{R} + (n+2)\frac{\dot{R}^{2}}{R^{2}}\right) = 0, \tag{13}$$

The solution of equation (13) is either

$$\left(\frac{\ddot{R}}{R} + (n+2)\frac{\dot{R}^{2}}{R^{2}}\right) = 0, \tag{14}$$

or

$$n = 1, \tag{15}$$

Case I: In this case we obtained the solution of the differential equation (14). After integrating equation (14), we obtain

$$R = (\alpha t + \beta)^{\frac{1}{n+3}}, n \neq -3, \tag{16}$$

where α and β are the constants of integration.

From Equations (10) and (16), we have

$$A = \mu(\alpha t + \beta)^{\frac{n}{n+3}}, \tag{17}$$

Substituting Equations (10) and (16) in equation (11), we have

$$\dot{\rho} + \frac{(1+\omega)\alpha}{(\alpha t + \beta)}\rho = 0, \tag{18}$$

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Integrating Equation (18), we obtain

$$\rho = m(\alpha t+\beta)^{-(1+\omega)}, \tag{19}$$

where m is the positive integration constant. Substituting equation and (19) in equations (4) (5) and solving this equations we obtain

$$G=\frac{k\alpha^2}{2\pi m(\omega+1)(\alpha t+\beta)^{-\omega}}, \tag{20}$$

$$\Lambda=\frac{2k\alpha^2(\omega-1)}{(\omega+1)(\alpha t+\beta)^2}, \tag{21}$$

where $k=3(n+1)/[2(n+1)^2]>0.$ Here G, Λ and ρ satisfy equation (13).

By using values from (10) and (17) in Equation (8) and (9) we obtain the following expression for kinematical quantities:

$$\theta=(n+3)\frac{\dot{R}}{R}=\frac{\alpha}{\alpha t+\beta}, \tag{22}$$

$$\sigma^2=\frac{3}{8}\left(\frac{\dot{R}^2}{R^2}-2n\frac{\dot{R}^2}{R^2}+n^2\frac{\dot{R}^2}{R^2}\right),\sigma^2=\frac{3}{8}\frac{\dot{R}^2}{R^2}\left(n^2-2n+1\right),$$

$$\sigma^2=\frac{3}{8}\left(\frac{\dot{R}}{R}-\frac{\dot{A}}{A}\right)^2,\sigma^2=\frac{3}{8}\left(\frac{\dot{R}^2}{R^2}-2\frac{\dot{R}}{R}\frac{\dot{A}}{A}+\frac{\dot{A}^2}{A^2}\right),\qquad \sigma^2=\frac{3}{8}\frac{\dot{R}^2}{R^2}(n-1)^2,$$

$$\sigma^2=\frac{3}{8}\frac{\alpha^2(n-1)^2}{(n+3)^2(\alpha t+\beta)^2}, \tag{23}$$

If we analyze our solutions for ω we can find the following situations:

Case (i) For ω = 0 (Dust phase), from Equations (19)-(21) we obtain

$$\rho=\frac{m}{\alpha t+\beta}, p=0, \tag{24}$$

$$G=\frac{k\alpha^2}{2\pi m(\alpha t+\beta)} \tag{25}$$

$$\Lambda=\frac{-2k\alpha^2}{(\alpha t+\beta)^2}, \tag{26}$$

Case (ii) For $\omega=-1$ (Dark energy phase), from Equations (19)-(21) we obtain

$\rho = m,\ p=-\rho$ and Λ diverges. (27)

Case (iii) For $\omega=1$ (Stiff matter), from Equations (19)-(21) we obtain

$$\rho=\frac{m}{(\alpha t+\beta)^2}, p=\rho, \tag{28}$$

$$G=\frac{k\alpha^2}{4\pi m} \tag{29}$$

$$\Lambda=0, \tag{30}$$

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Case (iv) For $\omega = \frac{1}{3}$ (Radiation dominated phase) from Eq, (19) and (21) we get,

$$\rho = \frac{m}{(\alpha t + \beta)^{\frac{4}{3}}}, p = \frac{1}{3}\rho, \tag{31}$$

$$G = \frac{3k\alpha^2}{8\pi m(\alpha t + \beta)^{2/3}}, \tag{32}$$

$$\Lambda = \frac{-k\alpha^2}{(\alpha t + \beta)^2}, \tag{33}$$

Case 2: Here we obtained the solution in particular for n=1. So from Eq. (9) we get,

$$\frac{\dot{R}}{R} = \frac{\dot{A}}{A} \tag{34}$$

Then Eq (11) with the equation of state $p = \omega\rho$, implies

$$\dot{\rho} + 4\frac{\dot{R}}{R}(1+\omega)\rho = 0, \tag{35}$$

which has the solution of the form

$$\rho = CR^{-4(1+\omega)}, \text{ C=constant.} \tag{36}$$

Subtracting Eq. (4) from (5) we get

$$3\left(\frac{\dot{R}^2}{R^2} - \frac{\ddot{R}}{R}\right) = 8\pi G(1+\omega)\rho \tag{37}$$

and by using Eq. (36) in Eq. (37) we get,

$$R(t) = B(1+\omega)t^{\frac{1}{2(1+\omega)}}, \omega \neq -1 \tag{38}$$

where B = constant involving G and Λ, and we have set the integration constant to be zero.

We study the case $\omega = -1$ and obtain an inflationary solution, as is evident from equation (36) above. Hence, R = e^{Nt} and A = e^{Nt}, and ρ = constant, N = constant, μ = constant. This is the familiar de-Sitter solution. We remark here that the shear scalar vanishes, i.e. $\sigma = 0$ and the universe most probably isotropizes during this period.

Another solution which is not studied in the letter of Bysal and Yilmaz (2007) in the case n = -3. This is obtained from Eq. (14) above, we get

$$\frac{\dot{R}}{R} = -\frac{1}{3}\frac{\dot{A}}{A} = F, \text{ F=constant.} \tag{39}$$

This solution does not require the equation of state $p = -\rho$ or $\omega = -1$ in comparison with the above inflationary solution. This clearly shows that the extra dimension decreases exponentially with time when the universe inflated. It be-comes exceedingly compactified, $R \prec \exp(-3Ft)$. This is interesting as it shows why today the extra dimension has no significant effect on the evolution of the universe. Apart from these new solutions we presented here, we remark that the sign of cosmological constant must be negative for Eq. (12) to be hold. Otherwise Eq. (12) cannot be satisfied by two decreasing variables.

III. CONCLUSION

In this study, we have obtained several sets of explicit solutions in the five dimensional Kaluza - Klein type cosmological models with variable G and Λ. In our solutions we have obtained the following properties.

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For the five dimensional models, from Eq. (16) we note that the universe starts at an initial epoch $t=-\frac{\beta}{\alpha}$.From Eq. (19) one can see that $\rho\geq 0,$ would hold only for m > 0. If -3 < n < 0, R(t) increases while A (t) decreases. Thus extra dimension becomes insignificant as the time proceeds after the creation and we are left with the real four - dimensional world. At $t=-\frac{\beta}{\alpha}$, the physical parameters θ and σ^2 diverge.

At time increases the rate of expansions (θ) and shear σ^2 slows down ad for $t\to\infty$ the shear dies out and the expansion stops.

It is easy to see that $\frac{\sigma}{\theta}\approx 0.61$, for these models. The present upper limit of $\frac{\sigma}{\theta}$ is 10^{5} obtained from direct arguments concerning the isotropy of the premordial black body radiation (Collins et al. (1980)). The $\frac{\sigma}{\theta}$ for our models is considerably greater than the present value. This fact indicates that our solutions represent the early stages of evolution of the universe.

In case (i), Λ varies inversely with the square of time while energy density ρ and G are varying inversely with the time. ρ, G, Λ are constants for $t\to 0$; ρ, G and Λ approaches zero for $t\to\infty$. In case (ii) we obtain the static universe with dark energy. In this case Λ and G diverge.

In case (iii) G is always constant and Λ is zero while energy density varies inversely within the square of time; ρ is constant for $t\to 0$ and approaches zero for $t\to\infty$.

In case (iv), ρ, G and Λ vary inversely with the power of time. ρ, G and Λ are constants for $t\to 0$. For $t\to\infty$, G approaches zero.

From the case (i) and (iv) we have found that the cosmological parameter Λ varies inversely with the square of time, which matches with its natural units. This supports the views in favor of the dependence $\Lambda\approx t^{-2}$ first expressed by Bertolai (1986) and later observed by authors (Dolgov et al., (1990); Dolgov (1997); Sahni and Starobinsky (2000); Padmanabhan (2003); Peebles (2003); Gasperini (1987); (1988); Freese et al., (1987); Ozer and Taha (1987). Now the case n = 1, which was not mentioned by Baysal and Yilmaz (2007) and pointed out by Arbab (200), it is observed that from Equation (35) is positive and the scale factor R is the increasing function of t. Similarly for the dimensional solutions there is no mechanism for the compactification of higher dimension. Therefore for further studies it will be interesting to find solutions where compactification of higher dimension is possible.

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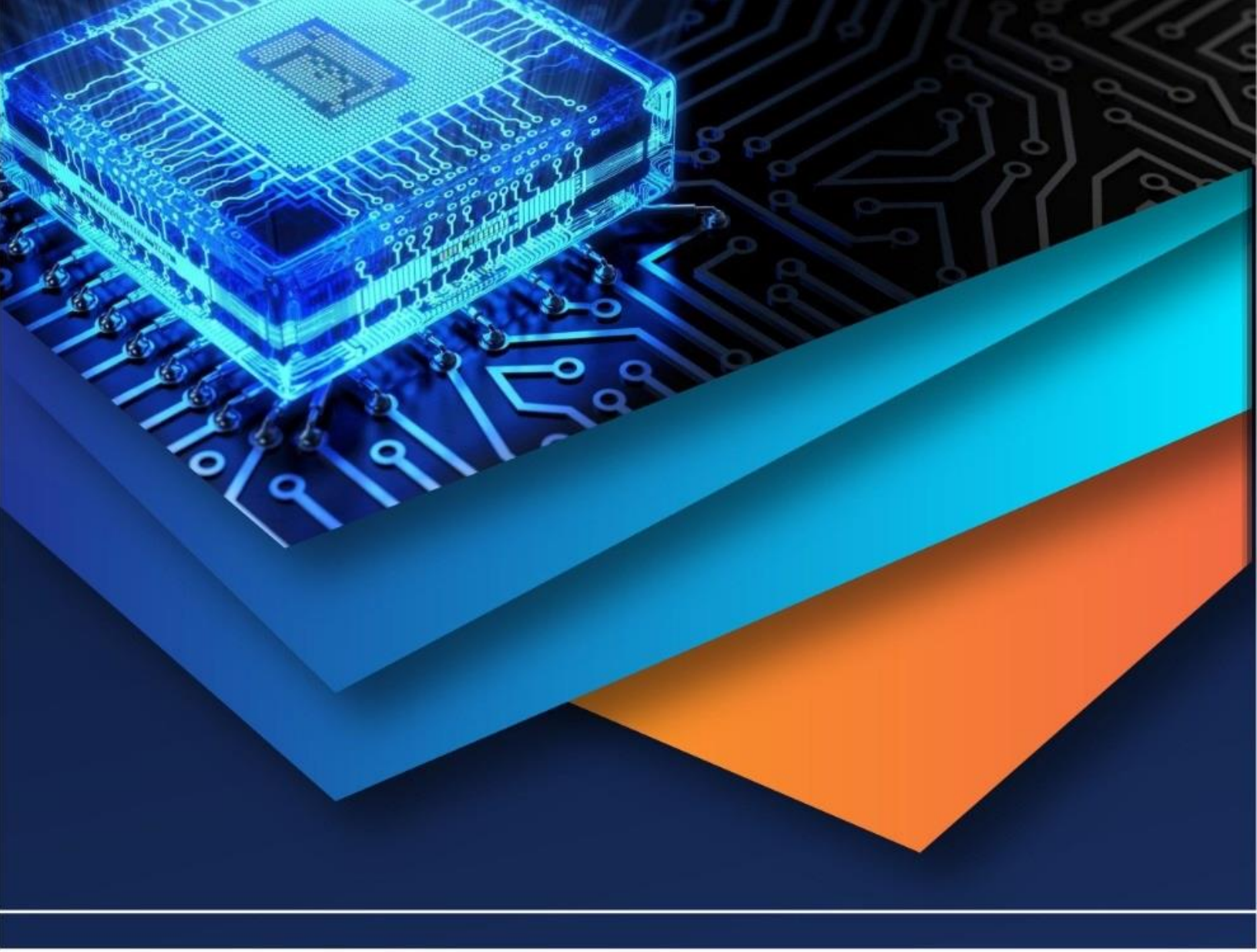

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