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Statistical Investigation of Various Methods to Find the Initial Basic Feasible Solution of a Transportation Problem

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Abstract: The first step to solve a Transportation Problem (TP) is finding an initial basic feasible solution (IBFS). The most commonly used methods to find the IBFS are North-West Corner Method (NWCM), Least Cost method (LCM), Vogel's Approximation Method (VAM). Along with these three popular methods, other methods like Modified VAM, Maximum Difference Method (MDM), Extreme Difference Method (EDM) etc., also exist in literature.
This study presents a statistical investigation of the recent solution methods and their efficiency is tested using Analysis of Variance (ANOVA).
Keywords: Transportation problem, Initial Basic Feasible Solution, Analysis of Variance.

I. INTRODUCTION

Transportation Problem (TP) is a special case of Linear Programming Problem which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a way that the total transportation cost is minimum. Transportation problem was first presented by F.L.Hitchcock [1], in his paper "The distribution of a product from several sources to numerous localities" in the year 1941. Later in the year 1947, T.C.Koopmans [2], presented his historic paper "Optimum utilization of the transportation system". These two papers are the milestones in the development of the various methods to solve a transportation problem. A transportation problem when expressed in terms of an LP model can also be solved by simplex method given by G B Dantzig in the year 1951 but it involves a large number of variables and constraints, solving it using simplex methods takes a long time. Several researchers have developed alternative methods for finding an initial basic feasible solution which takes costs into account.

The solution of a Transportation problem consists of two stages; namely i) finding an initial basic feasible solution and ii) optimal solution.

A set of non-negative allocations $x_{ij} \geq 0$, which satisfy the row and column restriction is known as a feasible solution. A feasible solution is basic if the no. of positive allocations is m+n-1. If the no. of positive allocations is less than m+n-1, then it is called degenerate feasible solution. A feasible solution is called an optimal solution if it minimizes the total transportation cost.

There are three well-known methods namely, north West Corner Method (NWCM), Least cost Method (LCM), Vogel's Approximation Method (VAM) to find the Initial Basic Feasible Solution (IBFS) of a Transportation Problem. In the recent time, some new heuristic methods have been developed such as Maximum Difference Method (MDM), Extremum Difference Method (EDM), and Modified Vogel's Approximation Method (MVAM) and so on.

II. MATHEMATICAL REPRESENTATION OF TP

The following notations are used for the mathematical representation of a Transportation Problem.

Let 'i' denote an origin point out of 'm' number of origins.

Let 'j' denotes a destination point out of 'n' number of destinations.

Let a_i denotes the available number of units at 'i'.

Let bj denotes the required number of units at 'j'.

Let c_{ij} denotes the transportation cost of a unit from 'i'th origin to 'j'th destination.

Let x_{ij} be the number of units to be transported from 'i'th origin to 'j'th destination.

Using these notations, the transportation problem can be put in the form mathematically as: finding a set of decision variables x_{ij}'s, for i=1, 2… m; j = 1, 2… n to

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i; i = 1,2,..., m$

$\sum_{i=1}^{m} x_{ij} = b_j;\ j = 1,2,..., n$

$x_{ij} \geq 0$.

Two transportation algorithms, namely Stepping Stone Method and the MODI (modified distribution) Method have been developed for finding the optimal solution of a transportation problem [3].
The allocation rule followed in all the algorithms is minimum of the supply and demand of the corresponding cell.

III. ALGORITHMS OF VARIOUS INITIAL BASIC FEASIBLE SOLUTION METHODS

A. North-West Corner Method (NWCM)

Step 1: Select the North-West corner cell of the transportation table and allocate the amount of transportation as the minimum of the supply and the demand.
Step 2: If the demand for the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.
Step 3: If the supply for the first row is exhausted, then move down to the first cell in the second row.
Step 4: Continue the process until all the supply and demand are exhausted.

B. Least Cost Method (LCM)

Step 1: Choose the cell with minimum cost and allocate as much as possible according the Allocation Rule. If such a cell is not unique, choose any one arbitrarily.
Step 2: Cross out the satisfied row or column. Consider the remaining part of the table and continue the process until all the supply and demand are satisfied.

C. Vogel's Approximation Method (VAM)

Step 1: Compute the 'penalty' of each row and column. Here penalty is defined as the difference between the least and the next to the least costs.
Step 2: Identify the row or column with largest penalty and assign to the least cost cell in that row or column. If there is a tie, choose one arbitrarily.
Step 3: Continue this process, until all the supply and demand are satisfied

D. _Algorithm of Maximum Difference Method(MDM)

1) Identify the cells having maximum and next to maximum transportation cost in each row and write the difference along the side of the table against the corresponding row.
2) Identify the cells having maximum and next to maximum transportation cost in each column and write the difference along the side of the table against the corresponding column.
3) Identify the maximum penalty. If it is along the side of the table make maximum allotment to the cell having minimum cost of transportation in that row. If it is below the table make maximum allotment to the cell having minimum cost of transportation in that column.
4) If the penalty corresponding to two or more rows or columns are equal, select the topmost row and the extreme left corner.
5) No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two and the remaining row or column is assigned zero supply or demand.
6) Calculate the fresh penalties for the remaining sub matrix as in step 1 and step 2 and allocate in the same manner. Continue the process until all rows and columns are satisfied.
7) Finally the total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply or demand.

E. Algorithm of Extremum Difference Method (EDM)

1) Identify the cells having maximum and minimum transportation cost in each row and write the difference along the side of the table against the corresponding row.
2) Identify the cells having maximum and minimum transportation cost in each column and write the difference along the side of the table against the corresponding column.
3) Steps 3 to 7 the same as in MDM method

F. Algorithm of Modified VAM's Method (M VAM)

1) Identify the cells having maximum and next to maximum transportation cost in each row and write the difference along the side of the table against the corresponding row.
2) Identify the cells having minimum and next to minimum transportation cost in each column and write the difference along the side of the table against the corresponding column.
3) Steps 3 to 7 the same as in MDM method.

IV. PROBLEMS AND DISCUSSIONS

A. Problem 1

Consider the following transportation table.

Allocation	Destination						Supply
	1	2	3	4	5	6	
1	1	2	1	4	5	2	30
2	3	3	2	1	4	3	50
3	4	2	5	9	6	2	75
4	3	1	7	3	4	6	20
Demand	20	40	30	10	50	25	175

B. Problem 2

A company has four factories manufacturing the same commodity, which is required to be transported to meet the demands in four warehouses. The supplies and demands as also the cost of transportation from factory to warehouse in rupees per unit of the product are given in table as follow:

Factories	Warehouses				Supply Units
	W	X	Y	Z	
A	25	55	40	60	60
B	35	30	50	40	80
C	36	45	26	66	160
D	35	30	41	150	150
DEMAND Units	90	100	120	140	450

C. Problem 3 : Solve the following transportation problem.

Source	Destination			Supply
	1	2	3	
1	6	10	14	50
2	12	19	21	50

3	15	14	17	50
Demand	30	40	55	

V. RESULTS

The initial basic feasible solution of the above listed problems 1, 2 and 3 are tabulated below.

Methods	Problem 1	Problem 2	Problem 3
NWCM	740	30570	1815
LCM	470	17510	1885
VAM	450	16130	1745
MDM	450	15910	1650
EDM	450	15910	1695
MVAM	450	15910	1695

A. The ANOVA Calculations

Let the null hypothesis be there is no significant difference among the various solution methods to find the initial basic feasible solution of a TP

Step-1. Calculation of averages and variances of the problems.

Method / Values	NWCM	LCM	VAM	MDM	EDM	MVAM
Average	11041.7	6621.7	6108.3	6003.3	6018.3	6018.3
Variance	16920.6	9456.1	8703.1	8600.3	8589.0	8589.0

Step-2. Calculation of analysis of variance.

Source	d.f	SS	MS	F	P-value
Treatments	5	60564140.3	12112828.1	0.108	0.9884
Error	12	1345953850.0	112162820.8		
Total	17	1406517990.3			

V1. CONCLUSION

The probability of the null hypothesis, using ANOVA, is 0.988. It indicates that the methods taken do not show any significance difference in the objective function values. Moreover, it shows a high degree of acceptance of the null hypothesis.

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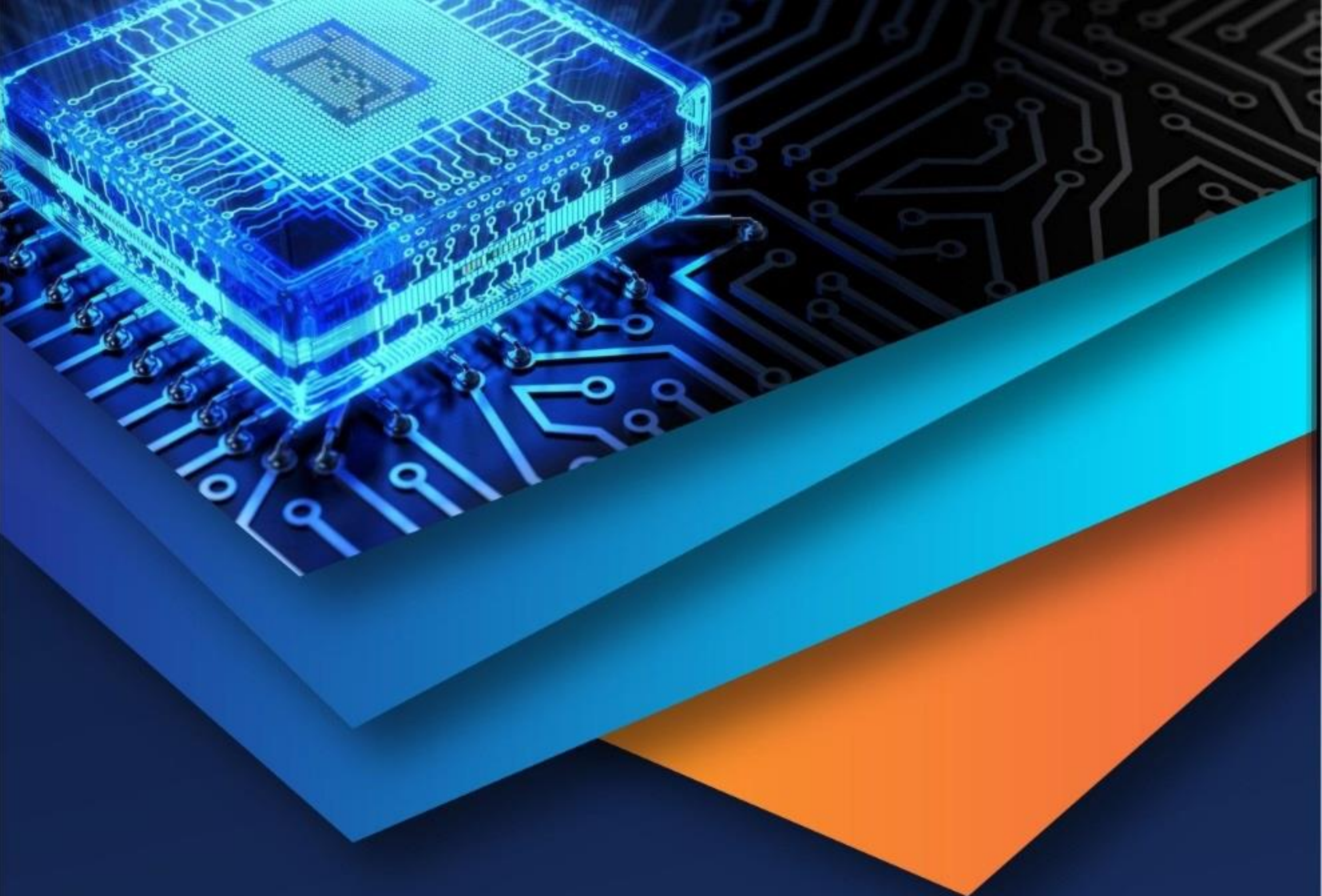

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