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Fixed Point Theorem on Dislocated B-Metric Spaces

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Abstract: In this paper, we prove Fixed point theorem for cyclic contractions in dislocated b-metric spaces.This paper generalized many result in the current literature. Mathematics Subject Classification: 47H05; 47H10; 47J25
Keywords: Fixed point, Contraction mapping, Dislocated b-metric.

I. INTRODUCTION AND PRELIMINARIES

Fixed point theory plays one of the important roles in Mathematical Analysis. Many authors [1-6]presented fixed point theorem in different ways. In Banach contraction principle was introduced in 1922 by Banach [7] as follows:

Let (X, d) be a metric space and $T : X \to X$.Then T is called a banach contraction mapping if there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$.

The concept of kannan mapping was introduced in 1969 by kannan[8] as follows: (ii) T is called a kannan mapping if there exists $r \in [0, \frac{1}{2})$ such that

$d(Tx, Ty) \leq rd(x, Tx) + rd(y, Ty)$ for all $x, y \in X$

Now,we recall the definition of cyclic map.Let A and B be non-empty subsets of a metric space (X, d) and $T : A \cup B \to A \cup B$. T is called a cyclic map iff $T(A) \subseteq B$ and $T(B) \subseteq A$.

In 2003, kirk etal.[9] introduced cyclic contraction as follows:

(iii) A cyclic map $T : A \cup B \to A \cup B$ is said to be cyclic contraction if there exists $\alpha \in [0, 1)$ such that

$d(Tx, Ty) \leq \alpha d(x, y)$ for all $x \in A$ and $y \in B$.

In 2010, Karapinar and Erhan[10] introduced kannan type cyclic contraction as follows:

(iv) A cyclic map $T : A \cup B \to A \cup B$ is called a kannan type cyclic contraction if there exists $b \in [0, \frac{1}{2})$ such that

$d(Tx, Ty) \leq bd(x, Tx) + bd(y, Ty)$ for all $x \in A$ and $y \in B$.

If (X, d) is a complete metric space, at least one of (i),(ii),(iii) and (iv) holds, then it has a unique fixed point[7-10].Next, we discuss the development of space.

Definition 1.1. [7] Let X be a nonempty set. Suppose that the mapping $d : X \times X \to [0, \infty)$ satisfies the following conditions:

(d_1) $d(x, x) = 0$ for all $x \in X$

(d_2) $d(x, y) = d(y, x) = 0$ implies $x = y$ for all $x, y \in X$

(d_3) $d(x, y) = d(y, x)$implies for all $x, y \in X$

(d_4) $d(x, y) \leq [d(x, z) + d(z, y)]$ for all $x, y, z \in X$

If d satisfies conditions (d_1),(d_2) and (d_4),then d is a called quasi-metric on X. If d satisfies conditions (d_2),(d_3) and (d_4),then d is a called dislocated metric on X.If d satisfies conditions (d_1),(d_3) and (d_4),then d is a called a metric on X.

In 2005 the concept of dislocated quasi-metric,which is a new generalization of quasi-b-metric spaces and dislocated b-metric space,was introduced.By Definition 1.1,if setting conditions (d_2), and (d_4) holds true,then d is called a dislocated quasi- metric on X.

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A. Remark

It is obvious that metric spaces are quasi-metric spaces and dislocated metric spaces,but the converse is not true. Definition 1.2. [11] Let X be a non-empty set. Suppose that the mapping $b : X \times X \to [0,\infty)$ such that the constant $s \geq 1$ satisfies the following conditions:

(b_1) $b(x, y) = b(y, x) = 0 \Leftrightarrow x = y$ for all $x, y \in X$

(b_2) $b(x, y) = b(y, x) = 0$ for all $x, y \in X$

(b_3) $b(x, y) \leq s[b(x, z) + b(z, y)]$ for all $x, y, z \in X$

The pair (X, d) is then called a b-metric space.

II. MAIN RESULTS

A. The Following Theorem Generalizes Theorem

Theorem 2.1. Let (X, d) be a complete dislocated b-metric with $s \geq 1$.Let A and B be a non-empty closed subset of X. Let $T : A \cup B \to A \cup B$ be a self map such that $d(Tx, Ty) \leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx)$ where $a_i \geq 0$, $i = 1, 2, 3, 4, 5$ and $a_1 + a_2 + a_3 + 2sa_4 + 2sa_5 <$ Then T has a unique fixed point in $A \cap B$.

Proof. Let $T^n \subseteq X, \{T^{2n}\} \subseteq A$ and $\{T^{2n+1}\} \subseteq B$. Fix $x \in A$.

$$
\begin{aligned}
d(T^2x, Tx) &= d(T(Tx), Tx) \\
&\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + a_4 d(Tx, Tx) + a_5 d(x, T^2x) \\
&\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + sa_4 [d(Tx, x) + d(x, Tx)] \\
&+ sa_5 [d(x, Tx) + d(Tx, T^2x)] \\
d(T^2x, Tx) &\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + 2sa_4 d(Tx, x) \\
&+ sa_5 d(x, Tx) + sa_5 d(Tx, T^2x) \\
&= (a_1 + a_3 + 2sa_4 + sa_5)d(Tx, x) + (a_2 + sa_5)d(T^2x, Tx) \\
&\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(Tx, x) \\
d(T^2x, Tx) &\leq kd(Tx, x), \text{ where } k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)}
\end{aligned}
$$

Now,

$$
\begin{aligned}
d(T^3x, T^2x) &= d(T(T^2x), T(Tx)) \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + a_4 d(T^2x, T^2x) \\
&+ a_5 d(Tx, T^3x) \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + sa_4 [d(T^2x, Tx) + d(Tx, T^2x)] \\
&+ sa_5 [d(Tx, T^2x) + d(T^2x, T^3x)] \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) \\
&+ 2sa_4 d(T^2x, Tx) + sa_5 d(Tx, T^2x) + a_5 d(T^2x, T^3x) \\
d(T^3x, T^2x) &\leq (a_1 + a_3 + 2sa_4 + sa_5)d(T^2x, Tx) + (a_2 + sa_5)d(T^2x, T^3x) \\
&\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(T^2x, Tx)
\end{aligned}
$$

$$d(T^3x, T^2x) \leq kd(T^2x, Tx), \text{ where } k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1-(a_2 + sa_5)}$$

B. Fixed Point Theorem on Dislocated b-Metric Spaces

$$d(T^3x, T^2x) \leq k[kd(Tx, x)]$$

$$d(T^3x, T^2x) \leq k^2 d(Tx, x)] \text{ induction,}$$

$$d(T^{n+1}x, T^n x) \leq k^n d(Tx, x)$$

In general $n, m \in N$ where $m > n$

$$d(T^n x, T^{n+m} \leq sd(T^n x, T^{n+1}x) + s^2 d(T^{n+1}x, T^{n+2}x) + \dots + s^m d(T^{n+m-1}$$

$$\leq sk^n d(x, Tx) + s^2 k^{n+1} d(x, Tx) + \dots + s^m k^{n+m-1} d(x, Tx)$$

$$\leq sk^n [1 + sk + \dots + (sk)^{m-1}]d(x, Tx)$$

$$d(T^n x, T^{n+m} \leq sk^n \frac{1}{1-sk} d(x, Tx)$$

$$d(T^n x, T^{n+m}x) \to 0 \text{ as } n \to \infty$$

Hence $\{T^n\}$ is a Cauchy sequence.

Since (X, d) is complete, Then $\{T^n\}$ coverages to some point $x \in X$. Since $\{T^{2n}\} \subseteq A$ and $\{T^{2n+1}\} \subseteq B$.

Thus $x \in A \cap B$

We show that $Tx = x$. Now,

$$d(Tx, x) = d(Tx, T^{2n}x)$$

$$\leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(T^{2n}x, x) + a_4 d(T^{2n}x, x) + a_5 d(x, Tx) \text{ Letting } n \to \infty$$

$$d(Tx, x) \leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(x, x) + a_4 d(x, x) + a_5 d(x, Tx)$$

$$\leq sa_1 [d(x, Tx) + d(Tx, x)] + a_2 d(Tx, x) + sa_3 [d(x, Tx) + d(Tx, x)]$$

$$+ a_4 [d(x, Tx) + d(Tx, x)] + a_5 d(x, Tx)$$

$$\leq 2sa_1 d(x, Tx) + a_2 d(Tx, x) + 2sa_3 d(x, Tx) + 2sa_4 d(x, Tx) + a_5 d(x, Tx)$$

$$\leq (2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)d(x, Tx)$$

$$[1-(2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)]d(x, Tx) \leq 0$$

Hence $d(Tx, x) = 0$.

This implies $Tx = x$. Therefore,T has a fixed point.

Uniqueness: Let x and y be two fixed points of T, that is $Tx = x$ and $Ty = y$

$$(x, y) = d(Tx, Ty)$$

$$\leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx)$$

$$\leq a_1 d(x, y) + a_2 d(x, x) + a_3 d(y, y) + a_4 d(y, x) + a_5 d(y, x)$$

$$\leq a_1 d(x, y) + a_2 s[d(x, y) + d(y, x)] + sa_3 [d(y, x) + d(x, y)] + a_4 d(y, x) + a_5 d(y, x)$$

$$d(x, y) \leq (a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)d(y, x)$$

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$$[1-(a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)]d(x, y) \leq 0$$

Hence $d(x, y) = 0$.

Therefore,$x = y$.

Hence,T has a unique fixed point.

Example 2.2. Let $X = R, A = [-2, 0]; B = [0, 2]$. Define $d(a, b) = \frac{1}{2}[|a-b| + |a|$

$+ |b|]$

Then d is the dislocated metric us define $T : A \cup B \rightarrow A \cup B$ by $Ta = -a$

Then T is a cyclic mapping.Here $^{0}0^{0}$ is the unique fixed point.

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