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Fixed Point Theorem on Dislocated B-Metric Spaces

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***Abstract:** In this paper, we prove Fixed point theorem for cyclic contractions in dislocated b-metric spaces.This paper generalized many result in the current literature. **Mathematics Subject Classification:** 47H05; 47H10; 47J25*
***Keywords:** Fixed point, Contraction mapping, Dislocated b-metric.*

I. INTRODUCTION AND PRELIMINARIES

Fixed point theory plays one of the important roles in Mathematical Analysis. Many authors [1-6]presented fixed point theorem in different ways. In Banach contraction principle was introduced in 1922 by Banach [7] as follows:

Let (X, d) be a metric space and $T : X \to X$.Then T is called a banach contraction mapping if there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$.

The concept of kannan mapping was introduced in 1969 by kannan[8] as follows: (ii) T is called a kannan mapping if there exists $r \in [0, \frac{1}{2})$ such that
$d(Tx, Ty) \leq rd(x, Tx) + rd(y, Ty)$ for all $x, y \in X$

Now,we recall the definition of cyclic map.Let A and B be non-empty subsets of a metric space (X, d) and $T : A \cup B \to A \cup B$. T is called a cyclic map iff $T(A) \subseteq B$ and $T(B) \subseteq A$.

In 2003, kirk etal.[9] introduced cyclic contraction as follows:

(iii) A cyclic map $T : A \cup B \to A \cup B$ is said to be cyclic contraction if there exists $\alpha \in [0, 1)$ such that
$d(Tx, Ty) \leq \alpha d(x, y)$ for all $x \in A$ and $y \in B$.

In 2010, Karapinar and Erhan[10] introduced kannan type cyclic contraction as follows:

(iv) A cyclic map $T : A \cup B \to A \cup B$ is called a kannan type cyclic contraction if there exists $b \in [0, \frac{1}{2})$ such that
$d(Tx, Ty) \leq bd(x, Tx) + bd(y, Ty)$ for all $x \in A$ and $y \in B$.

If (X, d) is a complete metric space, at least one of (i),(ii),(iii) and (iv) holds, then it has a unique fixed point[7-10].Next, we discuss the development of space.

Definition 1.1. [7] Let X be a nonempty set. Suppose that the mapping $d : X \times X \to [0, \infty)$ satisfies the following conditions:

(d_1) $d(x, x) = 0$ for all $x \in X$
(d_2) $d(x, y) = d(y, x) = 0$ implies $x = y$ for all $x, y \in X$
(d_3) $d(x, y) = d(y, x)$implies for all $x, y \in X$
(d_4) $d(x, y) \leq [d(x, z) + d(z, y)]$ for all $x, y, z \in X$

If d satisfies conditions (d_1),(d_2) and (d_4),then d is a called quasi-metric on X. If d satisfies conditions (d_2),(d_3) and (d_4),then d is a called dislocated metric on X.If d satisfies conditions (d_1),(d_3) and (d_4),then d is a called a metric on X.

In 2005 the concept of dislocated quasi-metric,which is a new generalization of quasi-b-metric spaces and dislocated b-metric space,was introduced.By Definition 1.1,if setting conditions (d_2), and (d_4) holds true,then d is called a dislocated quasi- metric on X.

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A. Remark

It is obvious that metric spaces are quasi-metric spaces and dislocated metric spaces,but the converse is not true. Definition 1.2. [11] Let X be a non-empty set. Suppose that the mapping $b : X \times X \to [0, \infty)$ such that the constant $s \geq 1$ satisfies the following conditions:

(b_1) $b(x, y) = b(y, x) = 0 \Leftrightarrow x = y$ for all $x, y \in X$

(b_2) $b(x, y) = b(y, x) = 0$ for all $x, y \in X$

(b_3) $b(x, y) \leq s[b(x, z) + b(z, y)]$ for all $x, y, z \in X$

The pair (X, d) is then called a b-metric space.

II. MAIN RESULTS

A. The Following Theorem Generalizes Theorem

Theorem 2.1. Let (X, d) be a complete dislocated b-metric with $s \geq 1$.Let A and B be a non-empty closed subset of X. Let $T : A \cup B \to A \cup B$ be a self map such that $d(Tx, Ty) \leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx)$ where $a_i \geq 0$, $i = 1, 2, 3, 4, 5$ and $a_1 + a_2 + a_3 + 2sa_4 + 2sa_5 <$ Then T has a unique fixed point in $A \cap B$.

Proof. Let $T^n \subseteq X, \{T^{2n}\} \subseteq A$ and $\{T^{2n+1}\} \subseteq B$. Fix $x \in A$.

$$\begin{aligned}
d(T^2x, Tx) &= d(T(Tx), Tx) \\
&\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + a_4 d(Tx, Tx) + a_5 d(x, T^2x) \\
&\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + sa_4 [d(Tx, x) + d(x, Tx)] \\
&+ sa_5 [d(x, Tx) + d(Tx, T^2x)] \\
d(T^2x, Tx) &\leq a_1 d(Tx, x) + a_2 d(T^2x, Tx) + a_3 d(Tx, x) + 2sa_4 d(Tx, x) \\
&+ sa_5 d(x, Tx) + sa_5 d(Tx, T^2x) \\
&= (a_1 + a_3 + 2sa_4 + sa_5) d(Tx, x) + (a_2 + sa_5) d(T^2x, Tx) \\
&\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(Tx, x) \\
d(T^2x, Tx) &\leq kd(Tx, x), \text{ where } k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)}
\end{aligned}$$

Now,

$$\begin{aligned}
d(T^3x, T^2x) &= d(T(T^2x), T(Tx)) \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + a_4 d(T^2x, T^2x) \\
&+ a_5 d(Tx, T^3x) \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + sa_4 [d(T^2x, Tx) + d(Tx, T^2x)] \\
&+ sa_5 [d(Tx, T^2x) + d(T^2x, T^3x)] \\
&\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) \\
&+ 2sa_4 d(T^2x, Tx) + sa_5 d(Tx, T^2x) + a_5 d(T^2x, T^3x) \\
d(T^3x, T^2x) &\leq (a_1 + a_3 + 2sa_4 + sa_5) d(T^2x, Tx) + (a_2 + sa_5) d(T^2x, T^3x) \\
&\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(T^2x, Tx)
\end{aligned}$$

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$d(T^3x, T^2x) \leq kd(T^2x, Tx)$, where $k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1-(a_2 + sa_5)}$

B. Fixed Point Theorem on Dislocated b-Metric Spaces

$d(T^3x, T^2x) \leq k[kd(Tx, x)]$

$d(T^3x, T^2x) \leq k^2 d(Tx, x)]$ induction,

$d(T^{n+1}x, T^nx) \leq k^n d(Tx, x)$

In general n, m ∈ N where m > n

$d(T^nx, T^{n+m} \leq sd(T^nx, T^{n+1}x) + s^2 d(T^{n+1}x, T^{n+2}x) + ... + s^m d(T^{n+m-1}$

$\leq sk^n d(x, Tx) + s^2 k^{n+1} d(x, Tx) + ... + s^m k^{n+m-1} d(x, Tx)$

$\leq sk^n [1 + sk + ... + (sk)^{m-1}]d(x, Tx)$

$d(T^nx, T^{n+m} \leq sk^n \frac{1}{} d(x, Tx)$

$d(T^nx, T^{n+m}x) \to 0$ as $n \to \infty$

Hence $\{T^n\}$ is a Cauchy sequence.

Since (X, d) is complete, Then $\{T^n\}$ coverages to some point $x \in X$. Since $\{T^{2n}\} \subseteq A$ and $\{T^{2n+1}\} \subseteq B$.

Thus $x \in A \cap B$

We show that $Tx = x$. Now,

$d(Tx, x) = d(Tx, T^{2n}x)$

$\leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(T^{2n}x, x) + a_4 d(T^{2n}x, x) + a_5 d(x, Tx)$ Letting $n \to \infty$

$d(Tx, x) \leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(x, x) + a_4 d(x, x) + a_5 d(x, Tx)$

$\leq sa_1 [d(x, Tx) + d(Tx, x)] + a_2 d(Tx, x) + sa_3 [d(x, Tx) + d(Tx, x)]$

$+ a_4 [d(x, Tx) + d(Tx, x)] + a_5 d(x, Tx)$

$\leq 2sa_1 d(x, Tx) + a_2 d(Tx, x) + 2sa_3 d(x, Tx) + 2sa_4 d(x, Tx) + a_5 d(x, Tx)$

$\leq (2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)d(x, Tx)$

$[1-(2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)]d(x, Tx) \leq 0$

Hence $d(Tx, x) = 0$.

This implies $Tx = x$. Therefore,T has a fixed point.

Uniqueness: Let x and y be two fixed points of T, that is $Tx = x$ and $Ty = y$

$(x, y) = d(Tx, Ty)$

$\leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx)$

$\leq a_1 d(x, y) + a_2 d(x, x) + a_3 d(y, y) + a_4 d(y, x) + a_5 d(y, x)$

$\leq a_1 d(x, y) + a_2 s[d(x, y) + d(y, x)] + sa_3 [d(y, x) + d(x, y)] + a_4 d(y, x) + a_5 d(y, x)$

$d(x, y) \leq (a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)d(y, x)$

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$[1-(a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)]d(x, y) \leq 0$

Hence $d(x, y) = 0$.

Therefore,$x = y$.

Hence,T has a unique fixed point.

Example 2.2. Let $X = R, A = [-2, 0]; B = [0, 2]$. Define $d(a, b) = \frac{1}{2}[|a-b| + |a|$

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$+|b|]$

Then d is the dislocated metric us define $T : A \cup B \rightarrow A \cup B$ by $Ta = -a$

Then T is a cyclic mapping.Here $^{0}0^{0}$ is the unique fixed point.

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REFERENCES

[1] Basha,ss.Veeramani,p: Best proximity pair theorems for multifunction with open fi- bres.J.Approx.theory 103,119-129(2000).
[2] Enjouji,Y.Naknishi,M.Suzuki,T:A generalization of kannan's fixed point Theory Appl. 2009,Article ID 192872(2009).
[3] Kikkawa,M.suzuki,T: Some Similarity between Contractions and Kannan mapping s.Fixed point Theory Appl.2008,Article ID 649749(2008).
[4] Nakanishi,M.Suzuki,T:An observation on kannan mappings.Cent.Eur.J.math.8,170-178(2010).
[5] Reich.S:Kannan fixed point theorem.Boll.Unione Mat.ltal.4,1-11(1971)
[6] Shioji,N,Suzuki,T,Takahashi,W: Contraction mapping,Kannan mappings and metric completeness.proc.Am.Math.Soc.126,3117-3124(1998).doi:10.1090/S0002-9939-98-04605-X
[7] Banch,S:Surles Operations dansles ensembles abstraits et leur application aux equation integrales.Fundam.Math.3,133-181(1922).
[8] Kanna,R:Some results on fixed Points-II.Am.Math.Mon.76,405-408(1969).
[9] Kirk,WA,Srivasan,PS,Veeramani,P:Fixed points for mapping satisfying cyclic contractive conditions.fixed point theory 4,79-89(2003).
[10] Karapinar,E,Erhan,IM:Best proximity on different type contractions.Appl.Math.inf.sci.5,558-569(2010).
[11] Bakhtin,IA:The contraction principle in quasi metric spaces.ln:Factional Analysis,Vol.30,pp26-37(1989)

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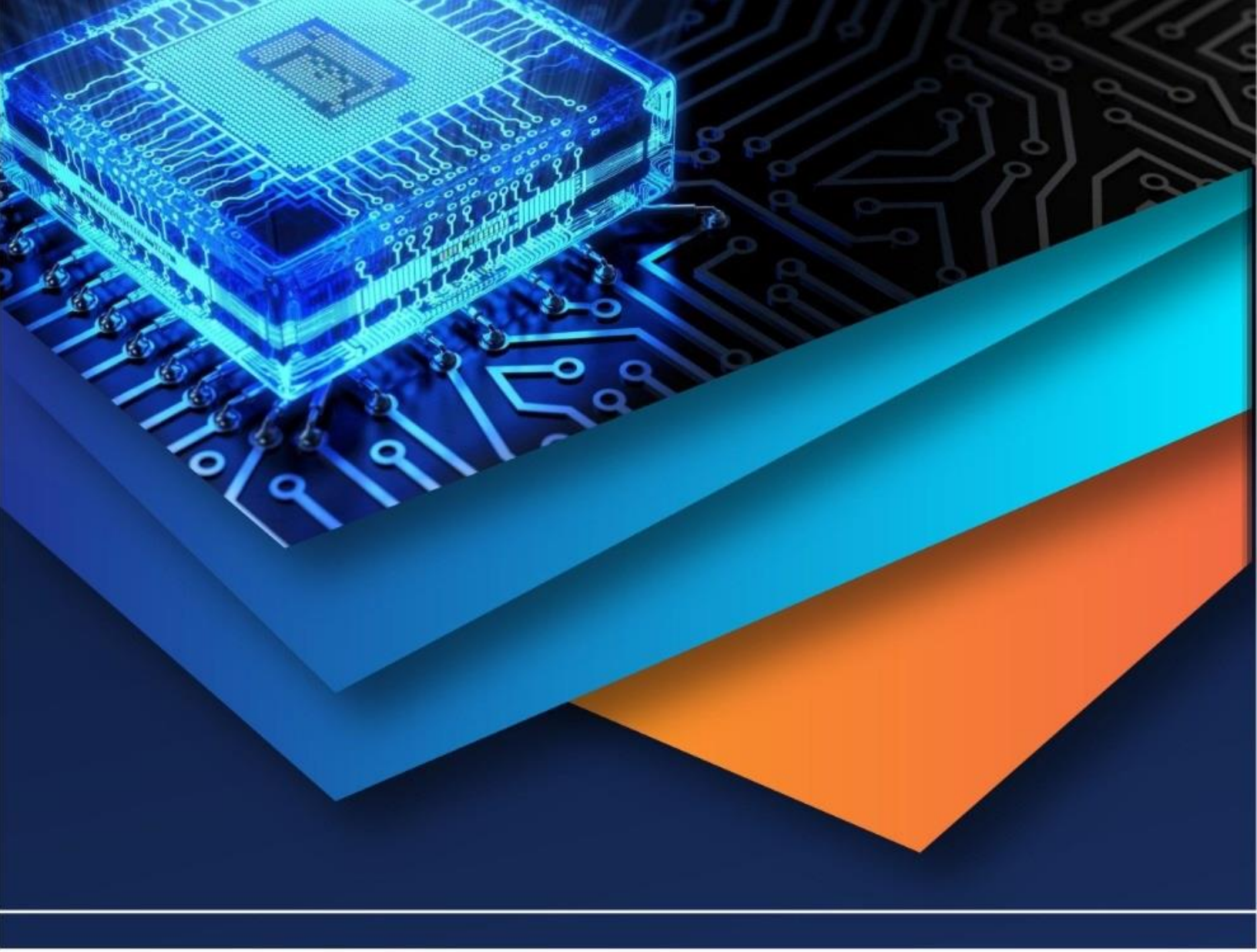

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