Performance Analysis of Two-Way Relaying Transceiver Hardware Impairments over Nakagami Fading Channel

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Abstract: Hardware impairments in physical transceivers are acknowledged to have a lethal consequence on communication systems; Even though very scarce contributions have examined their influence on relaying. This paper on performance of two way amplify-and-forward configuration of transceiver impairments is being analyzed over nakagami channel and is compared with outcomes of rayleigh channel. More precisely the real signal to noise and distortion ratios at both transmitter nodes are attained. These are used to calculate the outage probabilities, as well as submissive expressions for SER i.e. symbol error rates precisely. It is revealed from simulated outcomes that SER and OP have better response over nakagami channel than that of rayleigh channel.

Keywords: Two-way relay transceiver, Hardware Impairment, Symbol error rate, Outage probability, Amplify and forward (AF) Protocol, Nakagami channel.

I. INTRODUCTION

Relays can bring substantial performance gains to wireless networks in a cost-effective way; for eg. uniform quality of service, spatial diversity gains, and coverage gains. [1]. In the standard half duplex approach, the transmission among a source and a destination inhabits two time slots, so the throughput of an effective system in bits per channel use is condensed by a factor of two [2] – [4]. Two-way relaying permits two nodes to communicate in two time slots with the assistance of a relay node and can be used to resolve this problem. The two nodes transmit information concurrently in the 1st time slot to the relay, and the relay directs the information to the selected destinations in the 2nd time slot. Most research aids in the field of relaying suppose that the transceiver hardware of the relay node is ideal. Even though in practice, the transceiver hardware of wireless equipments are always impacted by impairments; for eg. phase noise, amplifier amplitude-amplitude non-linearity’s, and IQ imbalance [5] – [7]. Impairments forms an essential capacity upper limit that cannot be overcome by increasing the transmit power; therefore, they have a very substantial influence particularly in high rate systems [8]. Meanwhile relays are required to be of low cost equipment their transceiver hardware are more prone to impairments as they are of lower quality.

In spite of the significance of impairments for relaying, there are very rare pertinent works and these only examine their impact on one-way relaying. In this framework [9], [10] and references there in examined how transceiver impairments distress SER i.e. symbol error rate and OP i.e. outage probability respectively in one-way relaying. Inspired by the above conversation hereafter methodically evaluate the influence of relay transceiver impairments in a two-way relaying configuration, by taking the amplify and forward AF protocol. More precisely, expressions for the signal to noise and distortion ratio i.e. SNDR on both transmitting nodes, as well as closed form expressions for the precise and asymptotic OP/SER. This allows an accurate classification of the influence of transceiver hardware impairments on both metrics. Our asymptotic examination delivers engineering visions on how the best
communication performance differs with the level of impairments. This paper gives the review on the impact of hardware impairments in a two-way relaying configuration [5], [14].

II. IMPLEMENTATION

A two-way AF relaying configuration involving two transmitting nodes (T1 and T2) and a relay node R. Communication takes place in two time slots, where in the first-time slot $T_1$ & $T_2$ transmit the information symbols $s_1$ & $s_2$, respectively, to R. The relay receives a superimposition of the symbols and broadcasts an amplified version of it to $T_1$ & $T_2$ in the second-time slot. A block diagram is specified in Fig. 1. For briefness, the projected receiver of $s_i$ will be denoted as $T_{ri}$, where $r_i \triangleq \frac{i}{2}$ for $i = 1, 2$. The subscripts 1, 2, 3 to states to terms related with $T_1$, $T_2$, & R respectively. The signal received at R in the first-time slot is given by [14], [18]

$$y_3 = h_1 s_1 + h_2 s_2 + \eta_{3r} + \nu_3$$

(1)

where $s_i \sim \mathcal{CN}(0, P_i)$, for $i = 1, 2$, is a symbol information from a zero-mean gaussian circularly-symmetric complex distribution with power $P_i$. In accumulating, $\nu_i \sim \mathcal{CN}(0, N_i)$ signifies the additive gaussian complex noise at $T_1, T_2$, and R for $i = 1, 2, 3$. The channel coefficient for the link $T_i \rightarrow R$ or vice-versa $R \rightarrow T_i$ is represented by $h_i$ for $i = 1, 2$. Each of them is modelled as independent Rayleigh fading distributed with average gain $\Omega_i = \mathbb{E}[|h_i|^2]$, which means that $h_i \sim \mathcal{CN}(0, \Omega_i)$. As such, the probability density function i.e. PDF and cumulative density function i.e. CDF of the channel gains, $\rho_i \triangleq |h_i|^2$, are respectively given by [14], [18]

$$f_{\rho_i}(x) = \frac{1}{\Omega_i} e^{-\frac{x}{\Omega_i}}, \quad F_i(x) = 1 - e^{-\frac{x}{\Omega_i}}, \quad x \geq 0$$

(2)

In the second timeslot, the transmitted signal, $s_3$, by R is basically an augmented version of its received signal $y_3$, or $s_3 = G y_3$. Suppose that all nodes have perfect instantaneous knowledge of the fading channels $h_1, h_2$. Thus, R can apply variable gain relaying, [14], [18]

$$G \triangleq \sqrt{\frac{P_3}{\rho_1 P_1 + \rho_2 P_2 (1 + \kappa_{3r}) + N_3}}$$

(3)

where $P_3$ is the avg. transmit power of the relay node. Note that the level of impairments $\kappa_{3r}$ in eq. (3) may not be seamlessly identified. Such a potential mismatch will reduce the system performance and can be easily merged in the consequent analysis. Hence, can direct the signals received at $T_1 & T_2$ as [14], [18]

$$y_i = h_i (G y_3 + \eta_{3i}) + \nu_i = G h_1 h_2 s_{ri} + G h_2 s_i + G h_i (\eta_{3i} + y_3) + h_i \eta_{3i} + \nu_i$$

(4)

for $i = 1, 2$, where $\eta_{3i} \sim \mathcal{CN}(0, \kappa_{3i}^2 P_i)$ model distortion noise in the transmitting hardware of the relay. Note that (4) simplifies $y_i = G h_1 h_2 s_{ri} + G h_2 s_i + G h_i y_3 + \nu_i$ for ideal hardware; this special case was considered in eq. (1) & (2). The node $T_i$ needs to citation $s_{ri}$ from $y_i$. As it identifies its own transmitted symbol $s_i$, it can effortlessly eradicate the consistent self-interference term $G h_i^2 s_i$. Then, the effective SNDR at Ti for recognition of the symbol is $s_{ri}$ is [14], [18]

$$\text{SNDR}_i = \frac{\rho_i \rho_2 P_i}{\rho_i (N_3 + \kappa_{3i}^2 (\rho_1 P_1 + \rho_2 P_2)) + \frac{P_3 \kappa_{3i}^2 P_2 + N_3}{g^2}}$$

(5)

By replacing (3) into (5), eq. obtain [14], [18]

$$\text{SNDR}_i = \frac{P_i P_2}{P_i P_2 P_i + P_i b_1 + P_i (a_i + \frac{P_1}{P_{ri}} h_i) N_3 P_i P_3}$$

(6)

where $a_i \triangleq \frac{N_3}{P_{ri}} (1 + \kappa_{3i}^2), \quad b_i \triangleq \frac{N_3}{P_3} (1 + \kappa_{3i}^2)$, and $c \triangleq \kappa_{3i}^2 + \kappa_{3i}^2 + \kappa_{3i}^2 \kappa_{3i}^2$ for $i = 1, 2$.

A. Performance Analysis

B. Exact Outage Probability Analysis
The O/P at \( T_i \) is symbolized by \( P_{out,i}(x) \) and is the probability of falling \( SNDR_i \) under a specific threshold due to channel fading \( x \), of satisfactory communication quality; i.e., [14], [18]

\[
P_{out,i}(x) = P_r\{SNDR_i \leq x\}
\]

Result 1: The outage probability at \( T_i \) when attaining \( s_{ri} \) is given by [14], [18] \( P_{out,i}(x) = 1 \) for \( x \geq \frac{1}{c} \) and

\[
P_{out,i}(x) = 1 - e^{-\left( \frac{x}{1-cx}\frac{a_i b_i}{\Omega_i} + \frac{x^2}{1-cx^2\Omega_i P_{ri}} \right)} \times 2^{\sqrt{\pi}n\left(\Omega_i - \Omega_i^0\right)} \frac{\left(1 - e^{-\frac{cx}{\Omega_i P_{ri}}}\right)}{1 - e^{-\frac{cx}{\Omega_i P_{ri}}}}
\]

(8)

for \( 0 \leq x < \frac{1}{c} \), where \( K_i(\cdot) \) represents the second kind first order modified Bessel function [14], [18].

C. Asymptotic Outage Probability Analysis

In order to gain some engineering visions into an essential effect of impairments, we now intricate on the high-power regime. In this case, we suppose, without important loss of generality, that \( P_1 = P_2 = \tau P_3 \) raise large (with \( \tau > 0 \)), which in turns relaying gain \( G \) in (3) converges to [14], [18]

\[
G_{\infty} = \frac{1}{\sqrt{\pi(\rho_1 + \rho_2)(1 + k_3^2)}}
\]

and remains positive and finite. It is easy to see that the \( SNDR \) in (5) becomes equivalent [14], [18] to

\[
SNDR_{i\infty} = \frac{\rho_i P_2}{\rho_i c + \rho_1 \rho_2 c} = \frac{\rho_r}{(\rho_1 + \rho_2)c}
\]

(10)

Result 1: In the high power regime where \( P_1 = P_2 = \tau P_3 \to \infty \) and \( \tau > 0 \), the outage probability at \( T_i \) turn out to be [14], [18]

\[
P_{out,i}(x) = \begin{cases} \frac{\Omega_i c x}{\Omega_i + \frac{\Omega_i}{1 - \Omega_i}} & \text{if } x < \frac{1}{c} \\ 1 & \text{if } x \geq \frac{1}{c} \end{cases},
\]

(11)

D. Exact and Asymptotic Symbol Error Rate Analysis

We now turn our courtesy to the \( SER \). To this end, we first raise that for many modulation formats like M-ary PAM, BFSK, BPSK with orthogonal signaling, the avg. \( SER \) at \( T_i \) can be signified by the generic formula [14], [18]

\[
SER_i = \mathbb{E}[SNDR_i \{\alpha Q(\sqrt{2\beta}SNDR_i)\}] \quad i = 1, 2
\]

(12)

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}dt \) is the Gaussian \( Q-function \) and \( \alpha, \beta \) are modulation precise constants. Using integration by parts, (12) can be retransformed into the statistically more suitable form [14], [18]

\[
SER_i = \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-\beta x}}{\sqrt{x}} FSND \{SNDR_i(x)\} dx
\]
where the CDF of SNDR\textsubscript{\text{\text{\text{\text{\text{\text{\text{L}}}}}}}} is FSNDR\textsubscript{\text{\text{\text{\text{\text{\text{\text{L}}}}}}}(x) = P_{\text{out},t}(x)$ by classification. Combining Proposal 1 and (13), it does not seems that the resultant integral can be assessed in closed-form; even though, the SER can be attained from (1) by arithmetical integration which is much more effective than Monte Carlo outcomes. We hereafter take the high-power regime as well-defined in above Section and attain the following outcome\cite{14}, \cite{18}.

Result 2: Assume the high-power regime where $P_1 = P_2 = \tau P_3 \rightarrow \infty$. For alike avg. channel gains $\Omega_1 = \Omega_2$, the SERs at $T_1$ and $T_2$ are identical and equal to\cite{14}, \cite{18}:

$$\text{SER}_1^\infty = \text{SER}_2^\infty = \frac{ac}{2\beta \sqrt{\pi}} y \left( \frac{3\beta}{2c} \right) + \frac{a}{2} erf \left( \frac{\beta}{\sqrt{c}} \right)$$

(14)

Where lower partial gamma function $y(p,x) = \int_0^x t^{p-1} e^{-t} dt$ and complementary error function $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$.

E. Nakagami fading Channel

The Nakagami fading model was mainly proposed as it matches the experimental results for short wave ionospheric propagation \cite{15}. The Nakagami distribution is associated to the gamma distribution. It has two constraints a controlling spread $\Omega$ and shape parameter $m$. By a simple scaling transformation on a Chi-distributed random variable $Y \sim \chi(2m)$, \cite{16} Nakagami random variable $X$ is generated as below \cite{17}:

$$X = \sqrt{(\Omega/2m)}Y$$

(15)

III. SIMULATED RESULTS

In this chapter, numerical results have been presented a set to authenticate our previous theoretical outcomes. A situation is being considered with noise powers and symmetric signal $N_i = 1$ for $i = 1,2,3$ & $P_1 = P_2 = 2P_3$ transmitted over a Nakagami channel. In Fig. 2 compares the simulated OP at transmitter 1 $T_1$ against the particular expression of Proposition 1 and the asymptotic bound of Result 1 over Rayleigh channel. A high rate system is being considered with $x = 2^5 - 1$ which means 5 bits / channel used and different heights of impairments $k_{3l} = k_{3r} = k$.

It can be observed that how drastically an influence of impairments can be in high rate systems. At high level of impairments i.e. at $k = 0.2$, the system is continually in outage and no communication can be supported irrespective of the channel it may be Rayleigh or Nakagami fading channel and transmit power control level. At reasonable level of impairments, the OP approaches a non-zero saturation value in the high-power command, that is precisely foreseen in Result 1. This stances in important contrast in case of ideal hardware $k = 0$, where the OP goes sharply to zero with both channels Rayleigh as well as Nakagami as shown in simulated Fig. 6 and Fig. 7 \cite{14}, \cite{18}.

![Fig. 2: Outage probability (OP) at node $T_1$ against the transmit power $P_1$. Simulation parameters: $x = 2^5 - 1$, $\Omega_1 = 2$, $\Omega_2 = 1$, and $P_1 = P_2 = 2P_3$.](image)

In Fig. 3 demonstrates OP for transmitter antenna 1 i.e. $T_1$ with transmitting power $P_1$. A high rate system is being considered with $x = 2^5 - 1$ which means 5 bits / channel used and different heights of impairments $k_{3l} = k_{3r} = k$. Simulated graph displays that...
outcomes sustain same for high level of impairments i.e. at \( k = 0.2 \), the system is continually in outage and no communication can be supported irrespective of the channels Rayleigh or Nakagami. Similarly, for low impairments reaches to zero reflects OP also reaches to zero.

Nakagami channel has two constraints shape factor denoted by \( m \) and spread factor denoted by \( \Omega \). When shape factor \( m = 1 \) nakagami channel behaves exactly as Rayleigh channel. Simulated results been analysed by considering three different cases by varying value of shape factor \( m \) and spread factor \( \Omega \).

Case 1: \( m = 0.5 \) and \( \Omega = 2 \)

But as shown in Fig. 3 in when shape factor \( m = 0.5 \) and spread factor \( \Omega = 2 \). At low transmitting powers nakagami channel response superimpose on Rayleigh channel response.

At higher transmitting power, i.e. above 15 dB, OP response in case nakagami channel is better than Rayleigh channel for each case of impairment i.e. at \( k = 0.2, k = 0.1, k = 0.05 \) and \( k = 0 \).

Fig. 3: Outage Probability Nakagami Channel Shape factor, \( m = 0.5 \), Omega (\( \Omega \)) = 2.

Case 2: \( m = 1 \) and \( \Omega = 2 \)

Fig. 4 shape factor \( m = 1 \) and spread factor \( \Omega = 2 \) is being considered. At low transmitting powers nakagami channel response superimpose on Rayleigh channel response. At impairment quality \( k = 0.1 \) transmitting power from \( P_1 = 25 \) to \( 35 \) dB, OP for nakagami channel is lesser than on Rayleigh channel. But when transmitting power exceeds 35 dB, OP increases when compared with Rayleigh channel response. But when OP is evaluated at impairment quality \( k = 0.05 \) and \( k = 0 \), after transmitting power 15 dB OP in case of nakagami channel is far better than that on Rayleigh channel.

Fig. 4: OP over Nakagami Channel Shape factor, \( m = 1 \), Omega (\( \Omega \)) = 2.

Case 3: \( m = 2 \) and \( \Omega = 2 \)
Fig. 5 shape factor $m = 2$ and spread factor $\Omega = 2$ is being considered. At low transmitting powers nakagami channel response superimpose on Rayleigh channel response. At impairment quality $k = 0.1$ transmitting power from $P_1 = 25$ to $30$ dB, OP for nakagami channel is lesser than on Rayleigh channel. But when transmitting power exceeds $30$ dB, OP increases when compared with Rayleigh channel response while OP is evaluated at impairment quality $k = 0.05$ and $k = 0$, after transmitting power $15$ dB OP in case of nakagami channel is outperforms i.e. very less than that on Rayleigh channel.

![Fig. 5: OP over Nakagami Channel Shape factor, $m = 2$, Omega $\Omega = 2$](image1)

In Fig. 6, SER with BPSK modulation is being considered i.e. at $\alpha = \beta = 1$ and examine different impairment combinations for which $k_{3t} + k_{3r} = 0.2$ is fixed to be remain constant. The exact curves are attained by numerical evaluation, while the high power SER bounds stem from Result 2. As expected, the best optimal for minimizing the SER is to have the same hardware quality ($k_{3t} = k_{3r} = 0.1$) at the transmit and receive side of the relay. Such a optimal asymptotically lessens the SER by a factor of 2, when compared with case where $k_{3t} = 0, k_{3r} = 0.2$. Usually, in other words when a relay node with low quality hardware on transmitter side and a high quality hardware on the receiver side should be evaded [14], [18].

![Fig. 6: Symbol error rate (SER) at node $T_1$ against the transmit power $P_1$. Simulation parameters: BPSK modulation, $\Omega_1 = \Omega_2 = 1$, and $P_1 = P_2 = 2P_3$.](image2)
In Fig. 11 SER with BPSK modulation is being considered i.e. at $\alpha = \beta = 1$ and examined over nakagami channel with shape factor $m = 0.5$ and spread factor $\Omega = 2$ and is compared with rayleigh channel by making 03 different impairment combinations by such that $k_{3_t} + k_{3_r} = 0.2$ remains constant.

Case 1: $k_{3_t} = 0, k_{3_r} = 0.2$ at $m = 0.5$ and $\Omega = 2$

When $k_{3_t} = 0, k_{3_r} = 0.2$ i.e. hardware impairment at one end is of worst quality and at other end it is of superior quality then over nakagami channel results follows the same way as that of over Rayleigh channel but SER is slightly better in case of nakagami channel upto 30 dB transmitting power after that SER is same as that of over Rayleigh channel.

Case 2: $k_{3_t} = 0.05, k_{3_r} = 0.15$ at $m = 0.5$ and $\Omega = 2$

In this case, SER is slightly better than previous case as hardware impairment quality is improved at reciever end while at transmitting end hardware used is of little bit of lower quality than that was used in previous case. When it is compared on channel wise nakagami channel is still responded slightly better than rayleigh channel.

Case 3: $k_{3_t} = 0.1, k_{3_r} = 0.1$ at $m = 0.5$ and $\Omega = 2$

When hardware used at both ends are of same quality results are superior of all of 03 cases over both rayleigh as well as nakagami channel. But SER in case of nakagami channel is slightly better than over rayleigh channel upto transmitting power $P^1 = 30$ dB as it exceeds beyond it SER is same for both channels.

In Fig. 8SER with BPSK modulation is being considered i.e. at $\alpha = \beta = 1$ and examined over nakagami channel with shape factor $m = 1$ and spread factor $\Omega = 2$ and is compared with rayleigh channel by making 03 different impairment combinations by such that $k_{3_t} + k_{3_r} = 0.2$ remains constant.

Case 1: $k_{3_t} = 0, k_{3_r} = 0.2$ at $m = 1$ and $\Omega = 2$

When $k_{3_t} = 0, k_{3_r} = 0.2$ i.e. hardware impairment at one end is of worst quality and at other end it is of superior quality then over nakagami channel results follows the same way as that of over Rayleigh channel but SER is much better in case of nakagami channel.

Case 2: $k_{3_t} = 0.05, k_{3_r} = 0.15$ at $m = 1$ and $\Omega = 2$

In this case, SER is slightly better than previous case as hardware impairment quality is improved at reciever end while at transmitting end hardware used is of little bit of lower quality than that was used in previous case. When it is compared over channel wise nakagami channel is still responded much better than rayleigh channel.
Fig. 8: SER Over Nakagami Channel Shape factor, $m = 1$, Omega $\Omega = 2$

Case 3: $k_{3t} = 0.1, k_{3r} = 0.1$ at $m = 1$ and $\Omega = 2$
When hardware used at both ends are of same quality results are superior of all of 03 cases over both rayleigh as well as nakagami channel. But SER in case of nakagami channel is much better than over rayleigh channel.

In Fig. 9SER with BPSK modulation is being considered i.e. at $\alpha = \beta = 1$ and examined over nakagami channel with shape factor $m = 2$ and spread factor $\Omega = 2$ and is compared with rayleigh channel by making 03 different impairment combinations by such that $k_{3t} + k_{3r} = 0.2$ remains constant.

Case 1: $k_{3t} = 0, k_{3r} = 0.2$ at $m = 1$ and $\Omega = 2$
When $k_{3t} = 0, k_{3r} = 0.2$ i.e. hardware impairment at one end is of worst quality and at other end it is of superior quality then over nakagami channel results follows the same way as that of over rayleigh channel but SER is far better in case of nakagami channel.

Case 2: $k_{3t} = 0.05, k_{3r} = 0.15$ at $m = 1$ and $\Omega = 2$
In this case, SER is slightly better than previous case as hardware impairment quality is improved at reciever end while at transmitting end hardware used is of little bit of lower quality than that was used in previous case. When it is compared over channel wise nakagami channel is still responded far better than rayleigh channel.

Case 3: $k_{3t} = 0.1, k_{3r} = 0.1$ at $m = 1$ and $\Omega = 2$
When hardware used at both ends are of same quality results are superior of all of 03 cases over both rayleigh as well as nakagami channel. But SER in case of nakagami channel is far better than over rayleigh channel.
IV. CONCLUSION

All in this research work, performance of transceiver hardware impairments on two-way relaying systems over rayleigh channel and nakagami channel have been analysed analytically and were compared by taking parameters like outage probability OP and symbol error rate SER. Simulated outcomes concluded that the transmit and receive hardware of the relay node must be of the same quality to minimalize both SER and OP. when comparison is taken in between rayleigh and nakagami channel, SER and OP have better response over nakagami channel than that of rayleigh channel.

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