Stress and Buckling Based Design for a Hemispherical Dome

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Abstract: This paper addresses the stress and buckling based design for a hemispherical dome. Stiffeners are also added to this hemispherical dome to see its effect under applied pressure. Buckling load factor is obtained from eigen-buckling analysis in ANSYS software. These domes are used as heads and bottoms in cylindrical tanks and vessels. Reinforced concrete and structural steel domes of buildings, air-supported rubber fabric shells and under-water pressure vessels are also made in the form of hemispherical shells. The analysis has been carried out using ANSYS finite element software. The finite element analysis has been carried out for an isotropic outside stiffened hemispherical dome by considering the master element as a shell 3-d 4node 181element and beam 2 node 188 as a stiffener. Basic results of design and theoretical calculations of stress state and buckling are compared. Effect of stiffeners on response of dome is investigated.

Keywords: hemispherical dome, stiffener, buckling analysis, finite element analysis and ANSYS software.

I. INTRODUCTION

The hemispherical domed pressure vessels are designed for nuclear reactors because they are capable of storing liquefied gases with all safety. They are also used in air crafts, submarines, space ships, hot water storage tanks, diving cylinders, heat exchangers, transportable containers. The design analysis is done with steel material properties because steel is strong and can resist impacts. Also the hemispherical domes roofs are widely used on large diameter tanks and silos, as they provide high strength using very limited amount of material. To increase the stability of the domes and to withstand the stress and deformations developed it is not possible to increase the thickness of the domes. Instead of increasing the thickness of domes and thereby increasing the cost of materials, stiffeners are provided. These stiffener elements presenting relatively small part of the weight of the structure substantially influence the strength of the structure under different loading conditions. The stiffened hemispherical domes are used as heads and bottoms for pressure vessels which are widely used in industries for processing and storing fluids which are at different temperature and pressure in analogous to ambient.

Stiffeners in a stiffened dome make it possible to sustain highly directional loads, and introduce multiple load paths which may provide protection against damage and crack growth under the compressive and tensile loads. The biggest advantage of the stiffeners is the increased bending stiffness of the structure with a minimum of additional material, which makes these structures highly desirable for loads and destabilizing compressive loads. In addition to the advantages already in using them, there should be no doubt that stiffened plates designed with different techniques bring many benefits like reduction in material usage, cost, better performance, etc.

Finite element software ANSYS is used to carry out the analysis in this work. Basic ideas of finite element method originated from advances in aircraft structural analysis. Finite element analysis was applied to non-linear problems and large deformations. Today, the development in main frame computers and availability of powerful microcomputers has brought this method within reach of students and engineers working in small industries.

II. REVIEW OF LITERATURE

J. Michael Rotter, Grieg Machenzei and Martin Lee (2016) [1] worked on spherical dome buckling with edge ring support. Their paper presents the first thorough study of the influence of realistic boundary conditions, in the form of a ring at the eaves, on the linear bifurcation of these domes. They produce a clear image of the variation of the linear bifurcation resistance of spherical dome shells with a ring resistance at the eaves, exploring a range of geometric parameters including the radius, thickness, subtended angle, and size of the eaves ring. From their work, they are expecting that small rings will lead to buckling pressures that are far below those with larger rings. Linear Eigen value calculations were performed on very many dome geometries with a very wide range of
The purpose of these calculations was to obtain the basis for new shell design process termed Reference Resistance Design (RRD). Here, the applied pressure in each case was the classical value for complete perfect sphere buckling. 

N.Ganesan and Ravi Kiran Kadoli (2003) [2] studied on linear thermo elastic buckling and free vibration analysis of geometrically perfect hemispherical shells with cutout subjected to axisymmetric temperature variation. The variations of various field variables are assumed in the circumferential direction and the finite element matrices used in the numerical studies are based on the semi analytical method. Thermal buckling temperatures are evaluated for deep shells having a cut out at the apex. Hemispherical shells are used with a/h ratios of 100 and 500 and each with cutout angle at apex equal to 7°, 30°, 45°. Boundary conditions considered are clamped-clamped and clamped free.

J. Blachut (2016) [3] worked on buckling of composite domes with localized imperfections and subjected to external pressure. He showed that Force Induced Dimple (FID), initial geometric imperfection leads to a far worse deterioration of buckling strength (far up to four times) than currently used modulated Eigen shape(s) or lower bound increased radius shape deviations from perfect geometry. Here, the FID approach provides safer estimates of buckling resistance and it is efficient in terms of computing time for a range of shell geometries.

Jeevan and Divya (2013) [4] studied on finite element modeling for the stress, buckling and modal analysis of a cylindrical pressure vessel with tori spherical enclosure. They investigated the effect of cylindrical pressure vessel with tori spherical enclosure on the stress, buckling and vibrational characteristics subjected to an internal pressure by using finite element method (FEM). The 2-d static stress analysis is performed for vessel thickness to analyze the stress and deflections in the vessel walls due to the internal pressure. 

Value buckling analysis was performed for different vessel thickness and different knuckle radius to determine critical buckling pressure and modal analysis is performed to determine vibrational characteristics of a vessel. FEA is performed using software program ANSYS. Based on results obtained from static stress analysis, they found that thickness of vessel plays an important role in withstanding the applied internal pressure and the buckling pressure is more sensitive to the thickness of the vessel. 

J. Blachut (2009) [5] investigated on buckling of multilayered metal domes. His paper provides results of a numerical and experimental investigation into static stability of externally pressurized hemispherical and torispherical domes. The hybrid wall of considered domes includes steel-aluminum, titanium-aluminum, and copper-steel-copper configurations. Buckling tests were conducted on domes manufactured from copper-steel-copper layered material. This hemispherical head is manufactured from flat sheet using spinning. Details are provided of manufacture of domes, pre-test measurements, testing, and the finite element analysis of measured geometries of dome. Quasi-static incremental load was applied in all cases. The end of load carrying capacity was sudden and well defined. Values of experimental buckling pressure varied from 1.7 to 10 MPA. The radius to wall thickness (R/T), was in the range from 40 to 200.

III. PROBLEM FORMULATION

In literature, for dome with stiffener there are no formulae available. By using Euler- Bernoulli’s beam theory and Timoshenko theory of shells we can find the plate and beam deflection values individually. So, that these theoretical results will be compared with the numerical results from ANSYS software. If the theoretical and numerical results were same then we can extend our work to dome with stiffeners numerically.

<table>
<thead>
<tr>
<th>Section/Parameter</th>
<th>Young’s Modulus(N/m²)</th>
<th>Density(kg/m³)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>2*10¹¹</td>
<td>7850</td>
<td>0.3</td>
</tr>
<tr>
<td>Beam</td>
<td>2*10¹¹</td>
<td>7850</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3.2. Geometrical properties of dome

<table>
<thead>
<tr>
<th>Section/Parameter</th>
<th>Diameter(m)</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dome</td>
<td>1.414</td>
<td>0.003</td>
</tr>
</tbody>
</table>

IV. THEORETICAL APPROACH

BEAM: Beam is a structural element which is capable of sustaining load by bending. The bending phenomenon is included into the material of beam as a result of external loads. Beams are usually used for buildings or civil engineering structures but smaller frames like truck or automobile. Machine and mechanical systems also contain beam structures. A thin walled beam is a very important type of beam. The cross section of thin walled beams is made up from thin panels connected among together to form...
closed or open cross sections of a beam. Typical closed sections include round, square, and rectangular tubes. Open sections include I-beams, T-beams, L-beams, etc. Thin walled beams preferably exist because their bending stiffness per unit cross sectional area is higher than that for solid cross sections such a rod or bar. In this way, stiffness in beams can be achieved with minimum weight.

A. Euler-Bernoulli’s Beam Theory

Moment of inertia equation in bending moment,

\[ M = EI \frac{d^2 w(x)}{dx^2} \]

The maximum deflection occurs at the center of the beam and its value is given by,

\[ y_{max} = \frac{w l^4}{384EI} \]

Where, \( w = \) load per unit length; \( l = \) length of the beam; \( EI = \) flexural rigidity; \( E = \) modulus of elasticity; \( I = \) area moment of inertia of the cross-section about the neutral axis.

1) Dome: Shells that have the form of surfaces of revolution find extensive application in various kinds of containers, tanks and domes. A surface of revolution is obtained by rotating a plane curve about an axis lying in the plane of the curve. This curve is called meridian, and its plane is a meridian plane. Stephen P. Timoshenko and S. Woinowsky-Krieger theory of plates and shells: Displacements in symmetrically loaded shells having the form of a surface of revolution. The hemispherical dome in this analysis is symmetric about an axis, so the deformation \( \theta \) in the direction of the tangent to the meridian is zero. The deformation \( \omega \) in the direction of the normal to the middle surface can be calculated directly from the equation in shell theory.

\[ \epsilon_\theta = \frac{\theta}{r^2} \cot \phi - \frac{\omega}{r^2} \]

From the above equation we can write that \( \omega = f(t, \epsilon_\theta, \phi) \)

Since \( \theta = 0 \), \( \omega = r \epsilon_\theta \).

From Hooke’s law \( \epsilon_\theta = \frac{1}{E} \left( 1 - \cos \phi - \omega \sin \phi \right) \)

For dome \( \sigma_m = \sigma_\theta \)

\[ \epsilon_\theta = \frac{1}{E} \left( \sigma_\theta - m \sigma_m \right) \]

\[ = \frac{Pr}{2Et} (1 - m) \]

\[ \epsilon_\theta = \frac{\omega}{r} \]

\[ \omega = \frac{Pr}{2Et} (1 - m) \]

\( P = \) external pressure applied on dome surface

\( r = \) radius of dome

\( E = \) modulus of elasticity

\( t = \) thickness of shell

\( m = \) poiseuille’s ratio

B. Stress Analysis

For a hemispherical dome, the hoop and meridional orientations are same and hence, hoop and meridional radius of curvature and stresses are same. For this case, \( \sigma_m = \sigma_\theta = \frac{Pr}{2t} \). The maximum radial stress occurs at the inner radius of the shell and is given as:

\( \sigma_r = -p \).

The vonmises stress is maximum at the inner radius of the shell and is given as: \( \sigma_{equ} = \sigma_\theta + p \). Thus, for this shell, vonmises stress is higher than the hoop stress.

Where, \( p = \) pressure applied (N/m²)

\( r = \) radius of the dome (m)

\( t = \) thickness of the shell (m)

1) Buckling Load Factor: The buckling load factor (BLF) is an indicator of the factor of safety against buckling or the ratio of the buckling loads to the currently applied loads. When the structure is subjected to compressive stress, buckling may occur. Buckling is characterized by a sudden sideways deflection of a structural member. This may occur even though the stresses that develop in structure are well below those needed to cause failure of the material of which the structure is composed. As an applied load is increased on the member, such a column, it will ultimately become large enough to cause the member to become unstable and it is said to have buckled. Further loading will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of the member’s load carrying capacity. If the deformations that occur after buckling do not
cause the complete collapse of that member, the member will continue to support the load that caused it to buckle. If the buckled member is part of a larger assemblage of components such as building, any load applied to the buckled part of the structure beyond that which caused the member to buckle will be redistributed within the structure.

\[
\text{Buckling load factor} = \frac{\text{buckling pressure}}{\text{applied pressure}}
\]

Close form formulae are not available in literature for buckling of plates. So, the numerical formulation for buckling of plates has been considered. For the hemispherical dome,

\[
\text{Buckling pressure} = 1.21 \frac{Eh^2}{r^2} \text{ N/m}^2
\]

Where, \(E\) = modulus of elasticity (N/m²)

\(h\) = thickness of the shell (m)

\(r\) = radius of the dome (m)

### V. NUMERICAL APPROACH (FEM)

**A. Finite Element Analysis Using Ansys**

Finite element analysis (FEA) is a powerful computational technique used for solving engineering problems having complex geometries that are subjected to general boundary conditions. While the analysis is being carried out, the field variables are varied from point to point, thus possessing an infinite number of solutions in the domain. So, the problem is quite complex. To overcome this difficulty FEA is used; the system is discretized into a finite number of parts known as elements by expressing the unknown field variable in terms of the assumed approximating functions within each element. These functions are, included in terms of field variables at specific points referred to as nodes. Nodes are usually located along the element boundaries, and they connect adjacent elements. Because of its flexibility in ability to discretize the irregular domains with finite elements this method has been used as a practical analysis tool for solving problems in various engineering disciplines.

**B. Ansys Analysis**

Finite element analysis software ANSYS is a capable way to analyze a wide range of different problems. ANSYS can also solve various problems such as elasticity, fluid flow, heat transfer and electromagnetism. Beside those, it can also do non-linear and transient analysis. ANSYS analysis has the following steps for problem solving:

**C. Stiffener (Beam188)**

The following figure shows the shape of each cross section subtype:

![Cross-section subtypes](image)

Figure 5.1. Types of cross-sections in beam188

**D. Dome (Shell181)**

| Dome with Simply supported edges | 0.159*10⁻⁴ (m) | 0.74*10⁷ (N/m²) | 87.42 |

**Table 5.1. numerical values of dome without stiffener**
E. Dome (Stiffened Dome)

Poisson’s ratio = 0.3; Young’s modulus = $2 \times 10^{11}$ N/m$^2$; Pressure on area = 50,000 N/m$^2$; Density = 7850 kg/m$^3$; Thickness of the shell = 0.003 m; depth of stiffener = 75 mm; flange length is 40-50 mm; thickness is 3-4 mm; area of the stiffener is 447 mm$^2$.

VI. RESULTS AND DISCUSSIONS

Table 6.1. Comparison of numerical and theoretical results of dome

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>ANSYS VALUE</th>
<th>THEORITICAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRESS (N/m$^2$)</td>
<td>$0.74 \times 10^7$</td>
<td>$0.58 \times 10^7$</td>
</tr>
<tr>
<td>DEFORMATION (m)</td>
<td>$0.159 \times 10^{-4}$</td>
<td>$0.145 \times 10^{-4}$</td>
</tr>
<tr>
<td>BUCKLING LOAD FACTOR</td>
<td>87.42</td>
<td>87.14</td>
</tr>
</tbody>
</table>

Figure 6.1. comparison between numerical and theoretical results of dome
From the Figure 6.1, it is observed that, for the hemispherical dome without stiffener, the theoretical and analytical results are nearly equal when compared. So, the work is extended for dome with various types of stiffeners.

### Table 6.2: stress, deformation and buckling load factor values for different types of stiffeners

<table>
<thead>
<tr>
<th>S.No.</th>
<th>TYPE OF BEAM</th>
<th>MINIMUM STRESS(N/m²)</th>
<th>MAXIMUM STRESS(N/m²)</th>
<th>DEFORMATION(m)</th>
<th>BUCKLING LOAD FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solid rectangle</td>
<td>0.131*10⁷</td>
<td>0.987*10⁷</td>
<td>0.256*10⁻⁴</td>
<td>89.1486</td>
</tr>
<tr>
<td>2</td>
<td>Hollow rectangle</td>
<td>654190</td>
<td>0.103*10⁸</td>
<td>0.257*10⁻⁴</td>
<td>90.2343</td>
</tr>
<tr>
<td>3</td>
<td>I-section</td>
<td>0.130*10⁷</td>
<td>0.979*10⁷</td>
<td>0.256*10⁻⁴</td>
<td>89.6263</td>
</tr>
<tr>
<td>4</td>
<td>Z-section</td>
<td>0.124*10⁷</td>
<td>0.100*10⁸</td>
<td>0.257*10⁻⁴</td>
<td>90.0073</td>
</tr>
<tr>
<td>5</td>
<td>C-section</td>
<td>89754.1</td>
<td>0.103*10⁸</td>
<td>0.257*10⁻⁴</td>
<td>90.0575</td>
</tr>
<tr>
<td>6</td>
<td>L-section</td>
<td>659912</td>
<td>0.102*10⁸</td>
<td>0.257*10⁻⁴</td>
<td>89.8771</td>
</tr>
<tr>
<td>7</td>
<td>Hat section</td>
<td>617338</td>
<td>0.105*10⁸</td>
<td>0.257*10⁻⁴</td>
<td>90.1401</td>
</tr>
<tr>
<td>8</td>
<td>Reverse T-section</td>
<td>761217</td>
<td>0.988*10⁷</td>
<td>0.257*10⁻⁴</td>
<td>89.6107</td>
</tr>
<tr>
<td>9</td>
<td>Solid circle</td>
<td>0.129*10⁷</td>
<td>0.960*10⁷</td>
<td>0.258*10⁻⁴</td>
<td>89.7322</td>
</tr>
<tr>
<td>10</td>
<td>Hollow circle</td>
<td>5683.87</td>
<td>0.114*10⁸</td>
<td>0.255*10⁻⁴</td>
<td>91.943</td>
</tr>
</tbody>
</table>

From the above table, it is observed that the deformation is higher for hemispherical dome with stiffener when compared to hemispherical dome without stiffener. Here, in this design, the hemispherical dome acts as a membrane. So, the area of the membrane between the stiffeners is deforming more than the dome without stiffener. So, it is concluded that stiffeners are not required for a hemispherical dome to get less deformation.

The addition of stiffeners will give best results in plate-like structures to get less deformation under applied pressures and loads.

### Table 6.3: Section modulus values of plate with various types of stiffeners

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Section Type</th>
<th>Y(mm)</th>
<th>Moment of Inertia(I) (mm³)</th>
<th>Section Modulus(z) (mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I-Section</td>
<td>37.5</td>
<td>393000</td>
<td>10480</td>
</tr>
<tr>
<td>2</td>
<td>C-Section</td>
<td>37.5</td>
<td>393000</td>
<td>10480</td>
</tr>
<tr>
<td>3</td>
<td>Z-Section</td>
<td>37.5</td>
<td>393000</td>
<td>10480</td>
</tr>
<tr>
<td>4</td>
<td>Hollow Rectangle</td>
<td>37.5</td>
<td>331000</td>
<td>8826.7</td>
</tr>
<tr>
<td>5</td>
<td>Hat Section</td>
<td>37.5</td>
<td>328000</td>
<td>8746.7</td>
</tr>
<tr>
<td>6</td>
<td>Hollow circle</td>
<td>37.5</td>
<td>298000</td>
<td>7946.7</td>
</tr>
<tr>
<td>7</td>
<td>L-Section</td>
<td>37.5</td>
<td>265000</td>
<td>7067</td>
</tr>
<tr>
<td>8</td>
<td>Rev T-Section</td>
<td>37.5</td>
<td>265000</td>
<td>7067</td>
</tr>
<tr>
<td>9</td>
<td>T-Section</td>
<td>37.5</td>
<td>265000</td>
<td>7067</td>
</tr>
<tr>
<td>10</td>
<td>Solid Rectangle</td>
<td>37.5</td>
<td>210000</td>
<td>5600</td>
</tr>
<tr>
<td>11</td>
<td>Solid Circle</td>
<td>11.93133</td>
<td>15900</td>
<td>1332.62</td>
</tr>
</tbody>
</table>
FIGURE 6.2. Moment of inertia plot for dome with various types of stiffeners

From the above graph, the moment of inertia is high for I, Z, and C-sections of beam and it is lower for solid circle section of beam.

FIGURE 6.3. Section modulus plot for dome with various types of stiffeners
From the above graph, the section modulus is for I, Z, and C- sections of beam and it is lower for solid circle section of beam i.e. it is similar as that of moment of inertia of different sections.

**FIGURE 6.4.** Comparison of stress for different sections of beam

From the stress plot, the maximum stress is high for dome with hollow circle stiffener and it is low for dome with solid circle stiffener. But the stress values for dome with different sections of beam stiffener are less than the yield strength of the steel.

**FIGURE 6.5.** Comparison of deformation for different sections of beam

From the deformation plot of different cross sections of beam stiffener, the maximum deformation is for solid circle and the minimum deformation is for hollow circle. And the variation between the deformation values is comparatively very less for all the sections of beam.
From the above graph, the buckling load factor is high for hollow circle and low for solid rectangle. So, dome with hollow circle stiffener can have more buckling pressure compared to other cross sections.

VII. CONCLUSIONS AND FUTURE SCOPE

A. Conclusion

The present work covers the stress and buckling based design of hemispherical dome using finite element software ANSYS. In the first case, the stress and buckling analysis for hemispherical dome without stiffener has been analyzed. In the second case stiffener with different cross sections of beam are added to the dome in order to resist the buckling pressures and to give less deformation when compared to the dome without stiffener.

The behavior of the hemispherical dome composed of steel material subjected to static load on the outer face is investigated. In the entire work, the material is homogenous throughout and isotropic and the thickness of the shell is kept constant.

By using Euler-Bernoulli’s beam theory and Timoshenko’s shell theory we can find the beam and dome deflection values individually. So, that these theoretical results were compared with the numerical results from ANSYS software. The ANSYS results are compared with theoretical results and were obtained good agreement in case of dome without stiffener. ANSYS analysis could be used for more complicated geometry and loading where hand calculations are impossible or hard to perform. Also from the literatures, it appears that dome can be designed using experimental, analytical and numerical techniques.

VIII. FUTURE SCOPE

In the present thesis, the stress and buckling based design for hemispherical dome has been carried out in the elastic regime for homogenous and isotropic materials. It is also possible to extend the present formulation in the field of dynamic studies.

There is a scope of taking composite domes using various types of materials and also multilayered metal domes as layered structures which are widely used in diverse applications as in aircrafts, thin film deposition in semiconductor devices, heat exchangers etc.

Also this work can be extended for vibration analysis and modal analysis. The hemispherical dome with cutouts and edge ring support can also be taken.

REFERENCES
