Characterizations of Generalized Ordered Topological Spaces in Metrization

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Abstract: In this paper we will discuss about the triple $(Y, <_Y, T_Y)$ is called a generalized order space (abbreviated as GO space) . First we begin with a linearly order set $\{X, <\}$ and the family of all half spaces $(\leftarrow, b)\, and\, (a, \rightarrow)$ as a subspace .The resulting topology $T = T(<)$ is the open interval topology of the order $<$ and $(X, <, T)$ is linearly ordered topological spaces. we will define the linearly ordered space set as $(Y, <)$ and equip Y with any topology which contains a $T(<)$ and a base of open sets each of which is order convex. As we defined the GO space in topology spaces, further we put an effort giving characterization of those topological spaces for some compatible ordering can be constructed. Some results are so called as orderablity theorems. Characteristics of a cantor set and space of irrationals might be viewed as connected space with most non-cut points isomorphic to [0,1] and which is orderable. Let us consider the spaces which are antipodal to connected spaces, normally zero-dimension spaces , thus they are orderable which has been modified by Lynn[9] for separable spaces general case also has done by Herrlich[10].Some results has is still developed by Banaschewki [2]. We have established the some results of order on GO spaces and its resent progress and the extension of subspaces of scattered LOTS X is orderable, which has been studied by Galvin [5],and Purisch [11].

Keywords : GO spaces, topological spaces , Linearly ordered spaces, zero-dimensional spaces.

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I. INTRODUCTION

The triple product $(Y, <_Y, T_Y)$ is called a generalized order space (abbreviated as GO spaces). There is another equivalent way do define as a GO spaces as a linearly ordered spaces for a set as $(Y, <)$ and equip Y with any generalized a topology which exist with T(<) and has a base of open sets each of ordered is convex. (A set S of Y is called as order convex $x \in S$, for every point x lying between two points of S).

Here we present some of few results on GO spaces. Further we will emphasize the recent progress and also we reflects the general ideas and also it leads to get some fine technical results.

As we define above , a LOTS or a GO space is a topological space already equipped with a compatible ordering. Here some of efforts has been devoted to giving a characterization of these topological spaces for some compatible ordering can be constructed . Some of our results are also called as orderablity theorems. Some characterizations of the arc cantor set and space of irrationals might be viewed as orderablity theorems .

II. MAIN RESULTS

A. Theorem 1.0

Let (X,T) be connected and locally connected .Then there is a linear ordering $<$ of X such that (X,T,$<$) is linearly order space LOTs iff $(X \times Y) - \{(x, y) : x \in X\}$ is not connected Other orderablity theorems for connected locally connected space can be determined , we produce some results of due to Kowalsky[8].,

B. Theorem 1.1

A connected ,locally connected space is orderable iff whenever A$_1$,A$_2$, and A$_3$ are connected proper subsets of X, there exists a distinct $i, j \in \{1,2,3\}$ having set of $A_i \bigcap A_j \neq X$.

Using statements of theorems 1.0 and 1.1 there is a major stumbling block in characterization ordered spaces.

Here having a compactable dense ordering for < (ie, a<b) ,for some $c \in X$ has (a<c<b) a condition for known to be weaker than connectedness in ordered spaces.

C. Theorem 1.2

A topological space X is orderable by dense ordering iff , a uniformity u is compatible with topological space of X such that

(1). If $U \in u$ and if $U^{n+1} = U^n \circ U$ for each n>1, then $\cup \{U^n : n \geq 1\} = X \times X$:

(2). If $x, y \in X$ and satisfy for any two C sequences $x = \{x_0, x_1, x_2 ....x_n\} = y$ and $y = y_0, y_1, y_2 ....y_n = x$ in X such that for some sets of $V_1, V_2 ....V_n \in u.$ we have $V_1(x_i) \cap V_{i+1}(x_{i+1}) \neq \phi$ and $V_i(y_i) \cap V_{i+1}(y_{i+1}) \neq \phi$ for 1<i<n, then for some value of j , $V_j(x_j) \cap V_j(y_j) \neq \phi$.

Consider the spaces which are antipodal to connected spaces, normally zero dimensional spaces, then by one of the orderablity theorem which can proved .

1) *Lemma 1.1:* A topological space (X,T) admits a linear ordering < such that (X,T,<) is generalized ordered space iff (X,T) is T_1 and sub base which is union of two nests

2) *Lemma 1.2 :* A topological space (X,T) admits a linear ordering < such that (X,T,<) is a LOTS iff there is a collection of $S \subset T$ which is a T_1 space and has a subspace $S = N_1 \cup N_2$ such that each N_i is an interlocking nest.

3) *Theorem 1.3 :* A compact space(X,T) admits a linear ordering < such that (X,T, <) is a LOTS iff there is a collections of $S \subset T$ which is T_1-separates the points of X and which is union of two nests.

4) *Proof :* By using the Lemma 1.1 one can consider T'is the topology having S as a subbase .

Then (X,T') is a GO space and Hausdorff. But then the identify the map of $i : (X,T) \rightarrow (X,T')$ is continuous bijection from compact Hausdroff space, so T=T' and (X,T,<) is a compact GO space. While two next sub base theorem does solve the orderablity problem, it is not always clear how to apply the specific situations. For example we know that a square of the space of rational with half open interval is a topology, being a topological copy of the usual space of rationals. is orderable . By one of the result of Deak [3] and Purisch [11] .There is a GO space (X,T,<) when is there a liner ordering << of X having T as its open –interval topology. Further Given LOTS (Y,I_x,<) and a subset X of Y, where I_X is relative topology on X and is orderable with respect to $<_X$ and is orderable with any to any ordering spaces. By Rudin M.E [10] gave a solution for above special cases which has been studied by Purisch [11].

5) *Theorem 1.4 :* Any GO space is countably a para-compact.

6) *Proof :* A space X with S_o as fully normal if every open cover u of X has and open refinement v such that w is a countable sub collection of v with $\cap w \neq \phi$, then $\cup w$ is contained a single member of u, so that S_o is fully normal space is collection wise normal.

III. CONCLUSION

However the above results are interesting in the case of GO spaces has been obtained. we also emphasized the recent progress of a LOTS or a GO space is a topological spaces. Here some of efforts has been devoted to giving a characterization of these topological spaces for some compatible ordering can be constructed.

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