iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: VIII Month of publication: August 2017

DOI: http://doi.org/10.22214/ijraset.2017.8268

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue VIII, August 2017- Available at www.ijraset.com

Bounds on the Metric Dimension of Graphs

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Abstract: The upper and lower bounds for the metric dimension are obtained
Keywords: Distance, Resolving set, Basis, Metric dimension.
AMS Classification: 05C12, 05C60, 05C75.

I. INTRODUCTION

The idea of resolving sets and minimum resolving sets has appeared in the literature previously in 1976 and 1979. In **[5]** and later in **[6]**, Slater introduced the concept of a resolving set for a connected graph G under the term *locating set*. He referred to a minimum resolving set as a reference set for G. He called the cardinality of a minimum resolving set (reference set) the *Location number* of G. Independently, Harary and Melter **[3]** discovered these concepts as well but used the term metric dimension, rather than location number. We adopt the terminology of Harary and Melter. Consequently, the metric dimension or, more simply, the dimension dim (G) of a connected graph G is the cardinality of a minimum resolving set. Because of the suggestiveness of this terminology to linear algebra, it is also referred a minimum resolving set a basis for G. Hence, the vertices of G have distinct representations with respect to the basis vertices.

In graph theory a Planar graph is a graph that can be embedded in a plane, that is it can be drawn on a plane in such a way that its edges intersect only at its end vertices.

It is defined that the standard distance between two vertices u and v in a connected graph as the length of the shortest path between u & v and is denoted as $d\,(u\,,v)$. By an ordered set of vertices, we mean a set $W = \{w_1, w_2, \dots, w_k\}$ on which the ordering $(w_1, w_2, \dots, w_k)$ has been imposed. For an ordered subset $W = \{w_1, w_2, \dots, w_k\}$ of V(G), we refer to the k-vector (ordered k-tuple). r(v /W)= ($d(v,w_1), d(v,w_2), \dots, d(v,w_k)$) as the (metric) representation of v with respect to W. The set W is called a resolving set for G if

r(u /W) = r(v /W) implies that $u = v$ for all $u\,,v \in V(G)$.

Hence if W is a resolving set of cardinality k for a graph G of order n, then the set $\{r(v/W) / v \in V(G)\}$ consists of n distinct k-vectors. A resolving set of minimum cardinality for a graph G is called a minimum resolving set or basis for G.

Example 1.1

Consider the following graph G

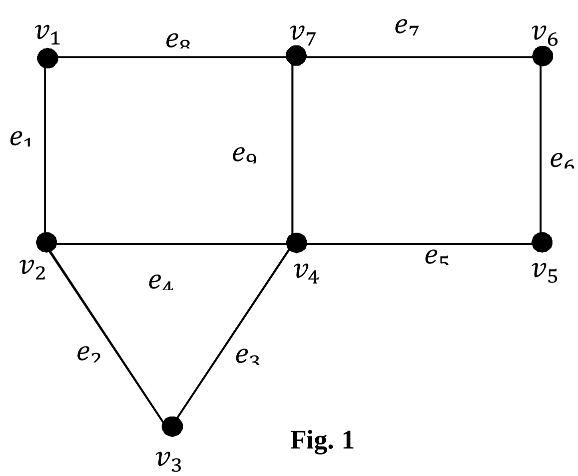


Fig. 1

Now the set $W = \{v_3, v_5\}$ is a resolving set for G, for

$r(v_1/W) = (2,3)$, $r(v_2/W) = (1,2)$, $r(v_3/W) = (0,2)$, $r(v_4/W) = (1,1)$, $r(v_5/W) = (2,0)$,

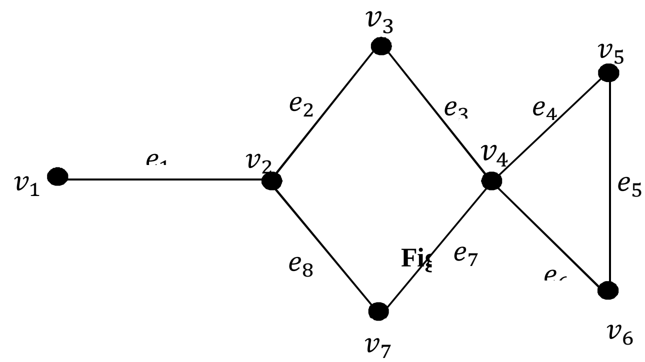
$r(v_6/W) = (3,1)$, and $r(v_7/W) = (2,2)$. All are distinct 2 vectors.

Since $W = \{v_3, v_5\}$ is a minimum resolving set it is a basis of G.

Definition 1.2

The metric dimension of G is defined to be the cardinality of a minimum resolving set for G and is denoted by $\dim(G)$.

Example 1.3



The set $W = \{v_3, v_5\}$ is a resolving set for G.

Since, $r(v_1/W) = (2,4)$, $r(v_2/W) = (1,3)$, $r(v_3/W) = (0,2)$, $r(v_4/W) = (1,1)$, $r(v_5/W) = (2,0)$, $r(v_6/W) = (2,1)$, $r(v_7/W) = (2,2)$, are all distinct two vectors.

This is a resolving set of minimum cardinality. Hence $\dim(G) = 2$.

II. BOUNDS FOR THE METRIC DIMENSION OF GRAPHS

In this section we find bounds for the metric dimension of graphs.

Here we state without proof the following theorem due to Gary Chartrand, Linda Eroh, Mark A. Johnson and Ortrud R. Oellermann.

Theorem 2.1

A connected graph G of order n has dimension 1 if and only if $G = P_n$.

Remark 2.2

From the above theorem we conclude that if G is a planar graph and is not a path then $\dim(G) \geq 2$.

Theorem 2.3

Let G be planar graph, which is not a path then the metric dimension can be made 2 deleting appropriate edges .

Proof

Let G a graph with vertex set $\{v_1, v_2, \ldots, v_n\}$. Let $W = \{w_i, w_j\}$ be an ordered set. Consider

$$d_1 = (d(v_1, w_i), d(v_1, w_j)), d_2 = (d(v_2, w_i), d(v_2, w_j)), \ldots, d_n = (d(v_n, w_i), d(v_n, w_j))$$

If all $d_1, d_2, \ldots, d_n$ are distinct then this is the minimum resolving set and $\dim(G) = 2$.

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Suppose that, $d(v_k, W) = d(v_l, W)$, then $d(v_k, v_i) = d(v_l, v_j)$, $d(v_k, v_i) = d(v_l, v_i)$. Let (v_k, W) and (v_l, W) are different paths with same length. By deleting an edge in the paths (v_k, W) or (v_l, W) will give two paths of different length. Thus we get $d(v_k, W) \neq d(v_l, W)$. Continuing this process for all equal vectors we get n distinct 2 vectors. Therefore dim(G)=2.

Hence the theorem

Example 2.4

Consider the following graph

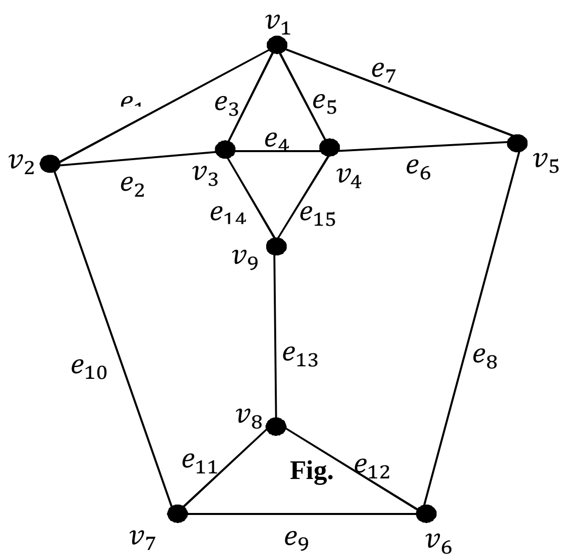

Fig.

The set $W = \{v_1, v_6\}$ is not a resolving sets for G. Since, $r(v_1/W) = (0,2)$, $r(v_2/W) = (1,2)$, $r(v_3/W) = (1,3)$, $r(v_4/W) = (1,3)$, $r(v_5/W) = (1,1)$, $r(v_6/W) = (2,0)$, $r(v_7/W) = (2,1)$, $r(v_8/W) = (3,1)$, $r(v_9/W) = (2,2)$. If $r(v_3/W) = (1, 3) = r(v_4/W)$.

Consider the following graph, delete the edge e_3

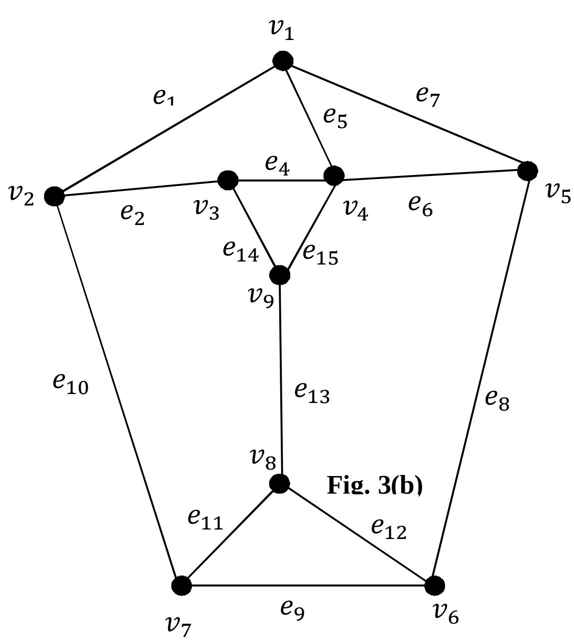

Fig. 3(b)

r(v_1/ W) = (0,2), r(v_2/ W) = (1,2), r(v_3/ W) = (2,3), r (v_4/ W) = (1,3), r (v_5/ W) = (1,1), r(v_6/ W) = (2,0), r(v_7/ W) = (2,1), r(v_8/ W) = (3,1), r(v_9/ W) = (2,2).

r(v_3/ W) $\neq$ r(v_4/ W).

Therefore the set is resolving set.

Theorem 2.5

If G is a graph and dim(G) $\leq$ 2 then G is a planar graph.

Proof

We know from theorem 2.1 that dim(G) = 1 iff G is a path and paths are planar graphs.

Let dim(G) = 2. We have to show that G is planar.

Assume that G is non planar. Therefore G cannot be embedded in a plane or sphere.

It can be represented in a three dimensional space. To represent a point in space we need three co-ordinates and it is of at least three dimensions.

Hence dim(G) $\geq$ 3.

This contradiction proves the theorem.

Example 2.6

Consider the following graph

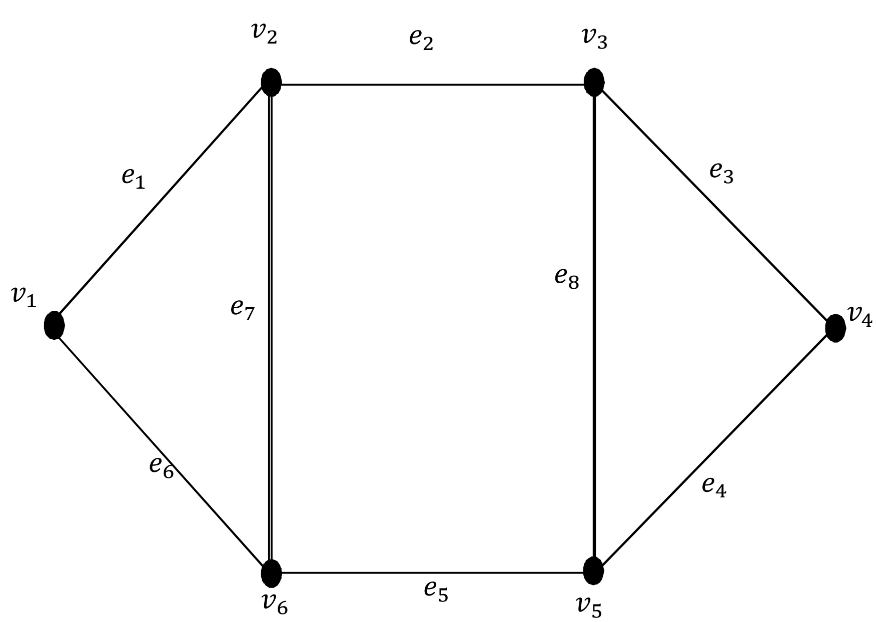

Fig. 4

The set $W = \{v_2, v_4\}$ is a resolving sets for G.
Since, r(v_1/ W) = (1,3), r(v_2/ W) = (0,2), r(v_3/ W) = (1,1), r (v_4/ W) = (2,0),
r (v_5/ W) = (2,1), r(v_6/ W) = (1,2).
Example 2.7

Consider the following graph

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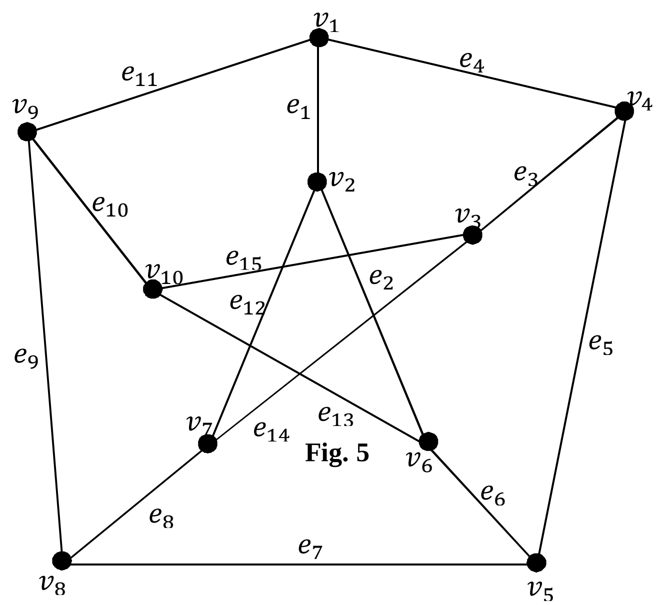

Fig. 5

The set $W = \{v_2, v_3, v_5\}$ is a resolving sets for G.

Since, r(v_1/ W) = (1,2,2), r(v_2/ W) = (0,2,2), r(v_3/ W) = (2,0,2), r (v_4/ W) = (2,1,1),

r (v_5/ W) = (2,2,0), r(v_6/ W) = 1,(2,1), r(v_7/ W) = (1,1,2), r(v_8/ W) = (2,2,1),

r(v_9/ W) = (2,1,2), r(v_{10}/ W) = (2,2,2).

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