# Product Cordiality of Barycentric Subdivision of $\mathbf{C}_{\mathbf{n}}$ 

Nikhil Parmar ${ }^{1}$, Dr. Dharamvirsinh Parmar ${ }^{2}$<br>${ }^{1}$ M.Phil. Scholar, C.U. Shah University and Assistant Professor in Mathematics, Sardar Patel College of Engineering, Bakrol, ${ }^{2}$ Assistant Professor in Mathematics, C. U. Shah University, Wadhwan

Abstract: Graph labelling is an important area of research in Graph theory. There are many kinds of graph labelling. We investigate some new results in product cordial labelling of $C_{n}\left(C_{n}\right)$. We investigate that even number of copies of $C_{n}\left(C_{n}\right)$, each consecutive copies joined by same length of path is product cordial graph.<br>Keywords: Cordial graph, Product cordial graph, Path union, Barycentric subdivision

## I. INTRODUCTION

Graph theory plays a important role in various fields. Graph labelling is one of the important area of graph theory. One of the productive and famous labelling of graph theory is cordial labelling. The concept of cordial graph was introduced by Cahit [1]. Motivated through the concept of cordial labelling the product cordial labelling was introduced by Sundaram [5].
In present work we consider finite, connected and undirected graph without multiple edges and loops. We consider graph $G=(V(G), E(G))$ having set of vertices $V$ and set of edges $E$ respectively. Our work mainly roam around $C_{\mathrm{n}}$ means cycle and $P_{n}$ means path of length $n-1$. We refer Gross and Yellen [3] for all kind of definitions and notations. And base for this work is taken from Vaidya and Kanani [6]. We will discuss some basic terminology and definitions which is useful to present work.

## A. Definitions 1.1

If the vertices of graph are assigned by some values or numbers subject to certain rules is known as graph labelling.

## B. Definition 1.2

Labelling of vertices of graph by binary numbers, 0 's and 1 's under certain condition is called binary vertex labelling. Notations:
$v_{f}(0)=$ Number of vertices with label 0 .
$v_{f}(1)=$ Number of vertices with label 1 .
$\varepsilon_{f}(0)=$ Number of edges with label 0 .
$\varepsilon_{f}(1)=$ Number of edges with label 1.

## C. Definition 1.3

A binary vertex labelling of graph $G$ with induced edge labelling $f^{*}: E \rightarrow\{0,1\}$ defined by $f^{*}(e=u v)=|f(w)-f(v)|$ is called a cordial labelling if $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\mid \theta_{f}(0)-\varepsilon_{f}(1) \| \leq 1$. A graph which admits cordial labelling is called cordial graph.

## D. Definition 1.4

A binary vertex labelling of graph $G$ with induced labelling $f^{*}: E \rightarrow[0,1]$ defined by $f^{*}(e=w)=f(w) f(v)$ is called a product cordial labelling if $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-\varepsilon_{f}(1)\right| \leq 1$. A graph which admits product cordial labelling is called product cordial graph.

## E. Definition 1.5

(Shee and $\mathrm{Ho}[4]$ ) Let $G_{1}, G_{2}, \ldots, G_{n}, n \geq 2$ be $n$ copies of a fixed graph $G$. The graph obtained by adding an edge between $G_{\mathbb{I}}$ and $G_{i+1}$ for $\bar{i}=1,2, \ldots, n-1$ is called the path union of $G$.

## F. Definition 1.6

Barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of the original graph.

## G. Observation

Present work is motivated by some results of Vaidya and Kanani [6] in product cordial labelling which are given below:

1) The path union $k$ copies of $C_{n}\left(C_{\mathrm{P}}\right)$ is a product cordial graph except for odd $k$.
2) The graph obtained by joining two copies of $C_{n}\left(C_{n}\right)$ by a path of arbitrary length is a product cordial graph.

The present work is done to investigate product cordial labelling for the path union of arbitrary length of copies of $C_{n}\left(C_{n}\right)$.

## II. RESULTS

Theorem 2.1 k copies of $C_{\mathrm{n}}\left(C_{\mathrm{n}}\right)$ joined by a path where each consecutive copies joined by a path of same length is product cordial except for odd $k$.
Proof: Let $G_{1}, G_{2}, G_{3}, \ldots, G_{k}$ be $k$ copies of cycle $C_{N}$ and $G_{1}^{R}, G_{2}^{f}, \ldots, G_{k}^{R}$ be $k$ copies of cycle $C_{n}$ which are obtained by joining each newly inserted vertices of adjacent edges by an edge.
So $G_{1}$ and $G_{1}^{r}$ together makes a graph which is denoted by $H_{1} ; G_{2}$ and $G_{2}^{r}$ together makes a graph which is denoted by $H_{2}$.
Same way we get graphs $H_{3}, H_{4}, \ldots, H_{R}$.
Now join $H_{1}$ and $H_{2}$ by a path $P_{l}$ of length $l-1$, join $H_{2}$ and $H_{2}$ by a path $P_{l}$ of length $l-1$, and same way each consecutive $H_{i}$ and $H_{i+1}$ joined by a path $P_{i}$ of same length $l-1$.
The graph obtained by joining each consecutive $H_{i}$ and $H_{i+1}$ is denoted by $G$.
Let $S_{i 1}, S_{i 2}, \ldots, S_{i n}$ be the vertices of $G_{i}$ and $S_{i 1}^{\mathrm{F}}, S_{i 2}^{\mathrm{r}}, \ldots, S_{i n}^{\mathrm{F}}$ be the corresponding vertices of the $G_{i}^{\mathrm{F}}$, for $i=1,2, \ldots, k$.
Let $w_{j 1}, w_{j 2}, \ldots, w_{j l}$ be the vertices of path $P_{i}$ joining graph $H_{j}$ and $H_{j+1}$ for $j=1,2, \ldots, k-1$.
It can be easily see that $S_{i 1}=w_{j 1}$ for $i=j$ and
$i=1,2, \ldots, k-1$
Also, $w_{j 1}=S_{[j+1] 1}$ for $j=1,2, \ldots, k-1$,
$w_{j l}=w_{(j+1) 1}$ for $j=1,2, \ldots, k-1$.
Note that $\|V(G)\|=2 n k+(k-1)(l-2)$
$\|E(G)\|=3 n k+(k-1)(l-1)$.
To define binary vertex labelling $f: V(G) \rightarrow\{0,1\}$ we consider following cases.
(i) $l$ is even
(ii) $I$ is odd

As we know that $k$ is even.
For $1 \leq i \leq k / 2$
Define $\left.\begin{array}{c}f\left(s_{i j}^{t}\right)=0 \\ f\left(s_{i j}\right)=0\end{array}\right\} 1 \leq j \leq n$
also for $k / 2 \leq i \leq k$
Define $\left.\begin{array}{l}f\left(s_{i j}\right)=1 \\ f\left(s_{i j}\right)=1\end{array}\right\} 1 \leq j \leq n$.
A. Case $1: l$ is even.

For $i=1,2, \ldots, \frac{k}{2}-1$
$f\left(w_{u}\right)=0$, where $1 \leq j \leq l$.
For $i=\left(\frac{k}{2}+1\right), 2, \ldots, k-1$.
$f\left(w_{i j}\right)=1$, where $1 \leq j \leq l$
For $i=k / 2$

$$
f\left(w_{v}\right)=0, \quad 2 \leq j \leq / / 2
$$

$$
f\left(w_{0}\right)=1, \quad l_{2}<j<1
$$

$v_{f}(0)=v_{f}(1)$ and $\theta_{f}(0)=\theta_{f}(1)+1$
Hence $\left\|v_{f}(0)-v_{f}(1)\right\| \leq 1$ and $\left\|_{f}(0)-e_{f}(1)\right\| \leq 1$.
Graph under consideration is a product cordial graph.
B. Case 2: lis odd.

For $i=1,2, \ldots, \frac{k}{2}-1$
$f\left(w_{i j}\right)=0$, where $\mathbf{1} \leq j \leq l$.
For $i=\left(\frac{k}{2}+1\right), 2, \ldots, k-1$.

$$
f\left(w_{i j}\right)=1, \text { where } 1 \leq j \leq l
$$

For $i=k / 2$

$$
\begin{aligned}
& f\left(w_{i j}\right)=0, \quad 2 \leq j \leq \frac{l-1}{2} \\
& f\left(w_{i j}\right)=1, \quad \frac{i-1}{2}<j<l
\end{aligned}
$$

$v_{f}(0)+1=v_{f}(1)$ and $e_{f}(0)=e_{f}(1)$
Hence $\left\|v_{f}(0)-v_{f}(1)\right\| \leq 1$ and $\mid \theta_{f}(0)-e_{f}(1) \| \leq 1$.
Graph under consideration is a product cordial graph.
Example 2.2 In the figure 1 the product cordial labelling of 4 copies of $C_{8}\left(C_{\mathrm{B}}\right)$ is demonstrated. Where each consecutive copies are joined by path $P_{3}$ of length 2 .


Figure 1: 4 copies of $C_{\mathbb{Z}}\left(C_{\mathbb{8}}\right)$ with path $P_{\mathbf{8}}$
Example 2.3 In the figure 2, product cordial labelling of 4 copies of $C_{7}\left(C_{3}\right)$ is demonstrated. Where each consecutive copies are joined by path $P_{4}$ of length 3 .


Figure 2: 4 copies of $C_{3}\left(C_{3}\right)$ with path $P_{4}$

## III. CONCLUSIONS

Vaidya and Kanani [6] investigated product cordial labelling for path union of various graph while we have extended one of their results. Whether similar type of result can be true for even number of copies of $C_{n}$ as well as Petersen graph is an open research problem.

## REFERENCES

[3] Gross, J. and Yellen, J. (1999), "Graph Theory and its applications", CRC Press
[4] Narsingh Deo, " Graph Theory with Applications to Engineering and Computer Science", Paperback - 1979
[5] Shee, S. C. and Ho, Y. S., (1996), " The cordiality of the path union of n copies of a graph", Discrete Math., 151, 221-229.
[6] Sundaram, Ponraj, M. and Somsundaram, R., (2004), "Product cordial labelling of graphs", Bull. Pure and Applied Sciences (Mathematics and Statistics), 23E, 155-163.
[7] Vaidya, S. K. and Kanani, K. K., (2011), " Some new product cordial graphs", Mathematics Today Vol.27,64-70
[8] Vaidya, S. K. and Kanani, K. K., (2010), "Some New Results on Cordial Labelling in the Context of Arbitrary Super subdivision of Graph", Applied Mathematical Sciences, Vol. 4, no. 47, 2323-2329

do
cross ${ }^{\text {ref }}$
10.22214/IJRASET


IMPACT FACTOR: 7.129

TOGETHER WE REACH THE GOAL.

IMPACT FACTOR:
7.429

## INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE \& ENGINEERING TECHNOLOGY
Call : 08813907089 @ (24*7 Support on Whatsapp)

