iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: VIII Month of publication: August 2017

DOI: http://doi.org/10.22214/ijraset.2017.8308

M/M/C: N/FIFO Queueing Analysis for Patient Flow in Hospital

Rajalakshmi. R [1], Julia Rose Mary. K [2]
[1]*MPhil Research Scholar, Department of Mathematics, Nirmala College for women, Coimbatore, India*
[2]*Associate Professor, Department of Mathematics, Nirmala College for women, Coimbatore, India*

Abstract: The increasing population and health-need due to adverse environmental conditions have increased the waiting times and congestion in hospitals especially in the emergency and accident departments. In such cases, to enhance the level of admittance to care, optimal beds required in hospital is needed and this can be achieved by adequate knowledge of patient flow. In this paper, M/M/C: N/FIFO Queueing analysis of Patient flow in hospital is determined. Further their performance measures and numerical example is analysed. In this model capacity of the system is finite.
Keywords: Mean queue length, M/M/C queue, server utilization, exponential distribution, bed occupancy.

I. INTRODUCTION

Queueing theory is the mathematical study of waiting lines (or) queues. The study of queueing systems finds application in a variety of real life situations like regulating traffic flow scheduling and facility design. The theory provides models to predict the behaviour of systems that attempt to render service for randomly arising demands. Thus the Queueing theory had its origin in 1909 when E.K.Erlang published his fundamental paper relating to the study of congestion in telephone traffic.

In- Patient flow one of the vital element is in improving the delivery of health care services. From a clinical perspective, in-patient flow represents the progression of a patient's health status. As such, an understanding of patient flow can offer education and insight to health care providers, administrators, and patients about the health care needs associated with medical concerns like disease progression or recovery status. Equally important, an understanding of patient flow is also needed to support a health care facility's operational activities. From an operational perspective, patient flow can be thought of as the movement of patients through a set of locations in a health care facility. Then, effective resource allocation and capacity planning are contingent upon patient flow because patient flow, in the aggregate, is equivalent to the demand for health care services (M. J Cote,[8]). The rising population and health need due to adverse environmental conditions have led to escalating waiting times and congestion in hospital emergency departments (ED) Derlet .R.W et al [4]. It is universally acknowledged that a hospital should treat its patients, especially those in need of critical care, in a timely manner. Incidentally, this is not achieved in practice particularly in government owned health institutions because of high demand and limited resources in these hospitals.

II. LITERATURE REVIEW

Traffic management, supermarket, health care and many other fields succeed in applying queueing theory for further progress.Weiss and McCliam [10] used the M/G/∞ system to model the queue of patients needing alternative levels of care in acute care facilities whose treatment is completed and are waiting to be transferred to an extended care facility. Adeleke R. A et al [1] considered application of queuing theory to the waiting time of out- patients in a hospital. The average number of patients and the time each patient waits in the hospital were determined. Likewise in his paper Worthington [11] used queuing theory to model hospital within list. He used an M/G/C queue with state dependent arrival rate to address the long- wait list problem. Through some appropriate means he experimented with different kinds of management actions including increasing the number of beds decreasing the mean service time etc. DeBruin *et al* [3] investigated the emergency in- patient flow of cardiac patient in an hospital in order to determine the optional bed allocation so size of a hospital unit, occupancy rate and target admission rates. After analytically estimating the required number of beds in first cardiac Acid (FCA) unit, they also used numerical method to estimate the number of beds in the Critical Care Unit (CCU) and Normal care clinical ward (NC). Jonathan E. H *et al* [7] characterize an optimal admission threshold policy using control on the scheduled and expedited gate way for a new Markov Decision process model. In their work, they presented a practical policy base on insight from the analytical model that yield reduced emergency blockages, cancelations and off- units through simulation based on historical hospital data. Recently, Adeleke R. A et al [9] considered queueing Analysis of patient flow in Hospital.

Application of queueing theory to model health care is growing more popular as hospital management teams are becoming aware of the advantages of these techniques. In this research we will use both analytical techniques and simulation to study a simple queuing network composed of only two service stations placed in tandem. In this paper, we studied all admissions into the Emergency and accident Department (EAD) of a tertiary hospital. We will show that admissions into this system has a Poisson distribution, hence it has exponential inter-arrival rate. We also examine the average length of stay, the occupancy rate and we determine the optimal bed count in the Intensive Care Wards (ICW) and the Medical and Surgical Wards (MSW). Since the ICW and MSW have multiple beds we will consider the M/M/c queue.

III. RELATED WORK

A. Finite Capacity

In an M/M/c/N queue only N customers can queue at any one time (including those in service). Any further arrivals to the queue are considered "lost". We assume that N≥ c. The model has transition rate matrix

$$Q=\begin{pmatrix} -\lambda & \lambda & & & & & & & \\ \mu & -(\mu+\lambda) & \lambda & & & & & & \\ & 2\mu & -(2\mu+\lambda) & \lambda & & & & & \\ & & 3\mu & -(3\mu+\lambda) & \lambda & & & & \\ & & & \ddots & & & & & \\ & & & C\mu & -(C\mu+\lambda) & \lambda & & & \\ & & & & \ddots & & & & \\ & & & & C\mu & -(C\mu+\lambda) & \lambda & & \\ & & & & & & \ddots & & \\ & & & & & & c\mu & -c\mu \end{pmatrix}$$

On the state space {0, 1, 2, ...,*c*, ..., *N*}. In the case where *c* =*N*, the M/M/*c*/N queue is also known as the Erlang–B model. This model is same as M/M/C: (∞/FIFO) except that the maximum number in the system is limited to N. where N ≥ c. therefore utilizing the steady state Probabilities of

$$\lambda_n = \lambda \quad if \quad 0 \le n < N \quad and$$
$$0 \quad otherwise$$

$$\mu_n = n\mu \quad if \quad 0 \le n < c$$
$$c\mu \quad if \quad c \le n \le N$$

We get,

$$p_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} p_0 & 0 \le n < c \\ \dfrac{(\lambda/\mu)^n}{c!\, c^{n-c}} p_0 & c \le n \le N \end{cases}$$

Now for $\lambda/c\mu < 1$, *the normalization condition* $\sum_{n=0}^{\infty} p_n = 1$ *gives*

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!} \left(1 - \left(\frac{\lambda}{c\mu} \right)^{N-c+1} \left(\frac{c\mu}{c\mu-\lambda} \right) \right) \right]^{-1}$$

By Noting that the probability an arrival unit has to wait on arrival is given by the probability,

$$p(N \ge c) = \sum_{n=c}^{N} p_n = \frac{(\lambda/\mu)^c}{c!} (1 - (\frac{\lambda}{c\mu})^{N-C+1})(\frac{c\mu}{c\mu-\lambda}) p_0 = \frac{p_c}{(1-\rho)} (1-\rho^{N-C+1})$$

We now proceed to compute some performance measures.

B. Performance Measures

The expected queue length L can be computed as,

$$L=\sum_{n=c}^{N}(n-c)p_n$$

$$=\frac{(\lambda/\mu)^{c+1}p_0}{c\,c!}\left[\frac{1-\rho^{N-C+1}-(N-c+1)(1-\rho)\rho^{N-c}}{(1-\rho)^2}\right]$$

Where $\rho=\frac{\lambda}{c\mu}<1$ is referred to as the server utilization.

C. Expected number of busy and idle servers:

The expected number of busy servers E(B) is given by

$$E(B)=\sum_{n=0}^{c-1}np_n+\sum_{n=c}^{N}cp_n$$

$$=\frac{\lambda}{\mu}\left[\sum_{n=0}^{c-1}\frac{(\lambda/\mu)^{n-1}}{(n-1)!}p_0+\sum_{n=0}^{N}\frac{(\lambda/\mu)^{n-1}}{(c-1)!\,c^{n-c}}p_0\right]$$

$$=\frac{\lambda}{\mu}\left[\sum_{m=0}^{c-2}\frac{(\lambda/\mu)^{m}}{m!}+\frac{(1-\rho+\rho)}{(c-1)!}(\frac{\lambda}{\mu})^{c-1}\left[1-(\frac{\lambda}{c\mu})^{N-C+1}\right]\frac{1}{(1-\rho)}\right]p_0$$

$$=\frac{\lambda}{\mu}\left[\sum_{m=0}^{c-2}\frac{(\lambda/\mu)^{m}}{m!}+\frac{(\frac{\lambda}{\mu})^{c-1}}{(c-1)!}\left[1-(\frac{\lambda}{c\mu})^{N-C+1}\right]+\frac{(\frac{\lambda}{\mu})^{c}}{c!}\left[1-(\frac{\lambda}{c\mu})^{N-C+1}\right]\frac{1}{(1-\rho)}\right]p_0$$

$$=\frac{\lambda}{\mu}\left[\sum_{m=0}^{c-1}\frac{(\lambda/\mu)^{m}}{m!}++\frac{(\frac{\lambda}{\mu})^{c}}{c!}\left[1-(\frac{\lambda}{c\mu})^{N-C+1}\right]\frac{1}{(1-\rho)}\right]p_0$$

$$=\frac{\lambda}{\mu}=c\rho \qquad \text{since} \quad \rho<1 \quad so \quad \rho^{N-c+1}\to 0$$

Hence the expected number of idle servers E(I) is given by

E(I)=E(c-B)=E(c)-E(B) =c-c $\rho=c(1-\rho)$

Applying little's formula we also obtain expected waiting time in the queue.

$$w=\frac{L_q}{\lambda}=\left[\frac{p_c p_0\rho\left[1-\rho^{N-C+1}-(N-c+1)(1-\rho)\rho^{N-c}\right]}{\lambda(1-\rho)^2}\right]$$

IV. LENGTH OF STAY DISTRIBUTION

The number of days in hospital for a patient is described by the term length of stay (LOS). LOS is defined as the time of discharge minus time of admission. Following, the average length of stay is abbreviated as ALOS.

The average length of stay(in days)= $\frac{total\ discharge\ days}{total\ discharges}$ or

The average length of stay(in days)= $\frac{total\ inpatient\ days\ of\ care}{total\ admission}$

Below are the definitions for each of the four data items included in the above calculations.

TOTAL DISCHARGE DAYS-the sum of the number of days spent in the hospital for each inpatient who was discharged during the time period examined regardless of when the patient was admitted.

TOTAL DISCHARGES- the number of inpatients released from the hospital during the time period examined. This figure includes deaths. Births are excluded unless the infant was transferred to the hospital's neonatal intensive care unit prior to discharge.
TOTAL INPATIENT DAYS OF CARE- sum of each daily inpatient census for the time period examined.
TOTAL ADMISSIONS- the total number of individuals formally accepted into inpatient units of the hospital during the time period examined. Births are excluded from this figure unless the infant was admitted to the hospital's neonatal intensive care unit.

V. BED OCCUPANCY

It is common practice in health services to estimate the required number of beds as the average number of daily admissions times average length of stay in days and divided by average bed occupancy rate(average number of occupied beds during a day) Huang X (1995)

Bed requirement= $\frac{average\ no.of\ daily\ admissions}{average\ bed\ occupancy\ rate} \times average\ length\ of\ stay$

Hospital bed capacity decisions have been made based on Target occupancy rate (TOR)- the average percentage of occupied beds and the most commonly used occupancy target has been 85% Linda V. Green [5]. Another metric often cited in the literature is the target access rate (TAR), which measures the percentage of the time that a census count will show that the hospital contains at least one empty bed, Kumar and john[2].

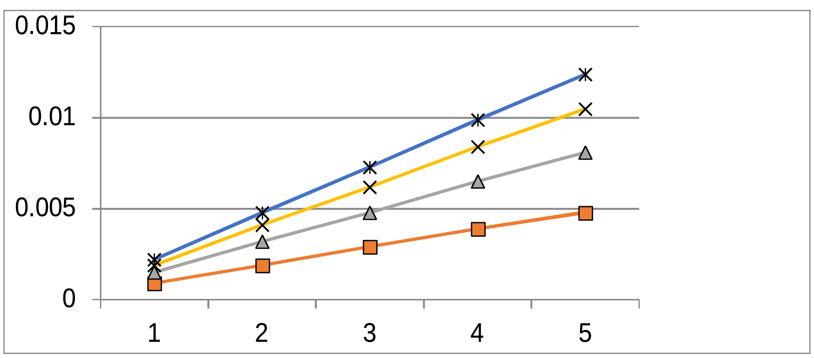
VI. NUMERICAL SOLUTION

The occupancy rate (ρ) is related to the real demand (λ) and LOS (μ) and can be defined as follows,

$$\rho = \frac{average\ number\ of\ beds\ occupied}{number\ of\ beds\ available} = \frac{(1-p_c)\lambda\mu}{c}$$

Table .1 Server utilization for M/M/C:N/FIFO Model

λ	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**
C=1	0.0019	0.0036	0.0052	0.0067	0.0081
C=2	0.0009	0.0019	0.0029	0.0039	0.0048
C=3	0.0006	0.0013	0.0019	0.0026	0.0033
C=4	0.0004	0.0009	0.0014	0.0019	0.0024
C=5	0.0003	0.0007	0.0011	0.0015	0.0019



Graph(1) $\rho\ versus(\lambda, c)$

From the table and graph, we conclude that, the server utilization factor(ρ) increases when the arrival rate increases. Also, we find that the server utilization factor(ρ) decreases when the number of beds increases.

The term $(1-p_c)\lambda$ can be entitled as the effective demand as the refused admissions are subtracted from the real demand. Furthermore, the product $\lambda\mu$ which is the expected number of patients in the system is also known as the workload of the system. Many hospitals use the same target occupancy rate for all hospital units, no matter the size of the unit. The target occupancy rate is typically set at 85% and has developed into a golden standard (green [5]). The conclusion is clear and important. Larger hospital units can operate at higher occupancy rates than smaller ones while attaining the same percentage of refused admissions. Therefore, one target occupancy rate for all hospital units is not realistic.

Total admission into ICW=50, ALOS in ICW=4.44 days, percentage of patients reneged, k=3.4%. we also have the following set of data for the MSW. Total admission into MSW=100, ALOS= 6 days.

From the parameter values specified, we estimate the arrival rate to each station as,

$$\lambda_{ICW} = \frac{N_{ICW}}{30days} = 1.67days^{-1} \quad \lambda_{MSW} = \frac{N_{MSW}}{30days} = 3.33days^{-1}$$

But the queue leading to the MSW is composed of new arrivals and blocked patients from the ICW. Also we have only a fraction 1-k=96.6% of patients arrived into ICW without reneging during service. So that the effective arrival into the ICW is

$$\lambda^e_{ICW} = \lambda_{ICW}(1-k) = 1.613days^{-1}$$

$$\lambda^e_{MSW} = \lambda^e_{ICW} + \lambda_{ICW} = 3.283days^{-1}$$

Table .2 Performance measure for ICW

C1	**% server utilization**	**Mean waiting time in the queue**	**Mean waiting time in the system**
10	83.5	0.0002	4.4402
11	75.9	0.0001	4.4401
12	69.58	0.0000	4.4400
13	64.23	0.0000	4.4400
14	59.64	0.0000	4.4400
15	55.66	0.0000	4.4400
16	52.18	0.0000	4.4400
17	49.11	0.0000	4.4400
18	46.38	0.0000	4.4400
19	43.94	0.0000	4.4400
20	41.75	0.0000	4.4400

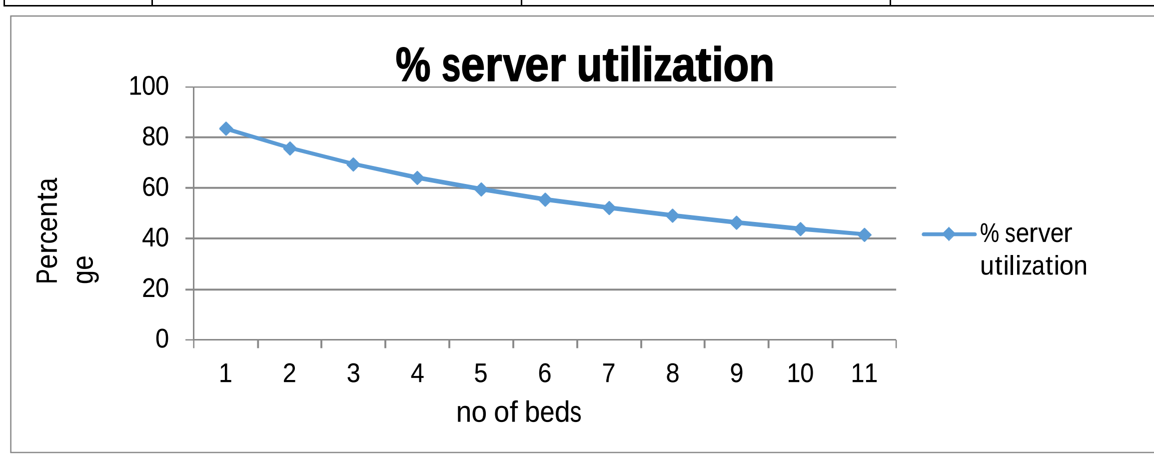


Table .3 Performance measure for MSW

C2	% server utilization	Mean waiting time in the queue	Mean waiting time in the system
21	95.14	0.0235	0.1835
22	90.81	0.0225	0.1825
23	86.86	0.0215	0.1815
24	83.25	0.0209	0.1809
25	79.92	0.0202	0.1802
26	76.84	0.0196	0.1796
27	74	0.0191	0.1791
28	71.35	0.0185	0.1785
29	68.89	0.018	0.178
30	66.66	0.0175	0.1775

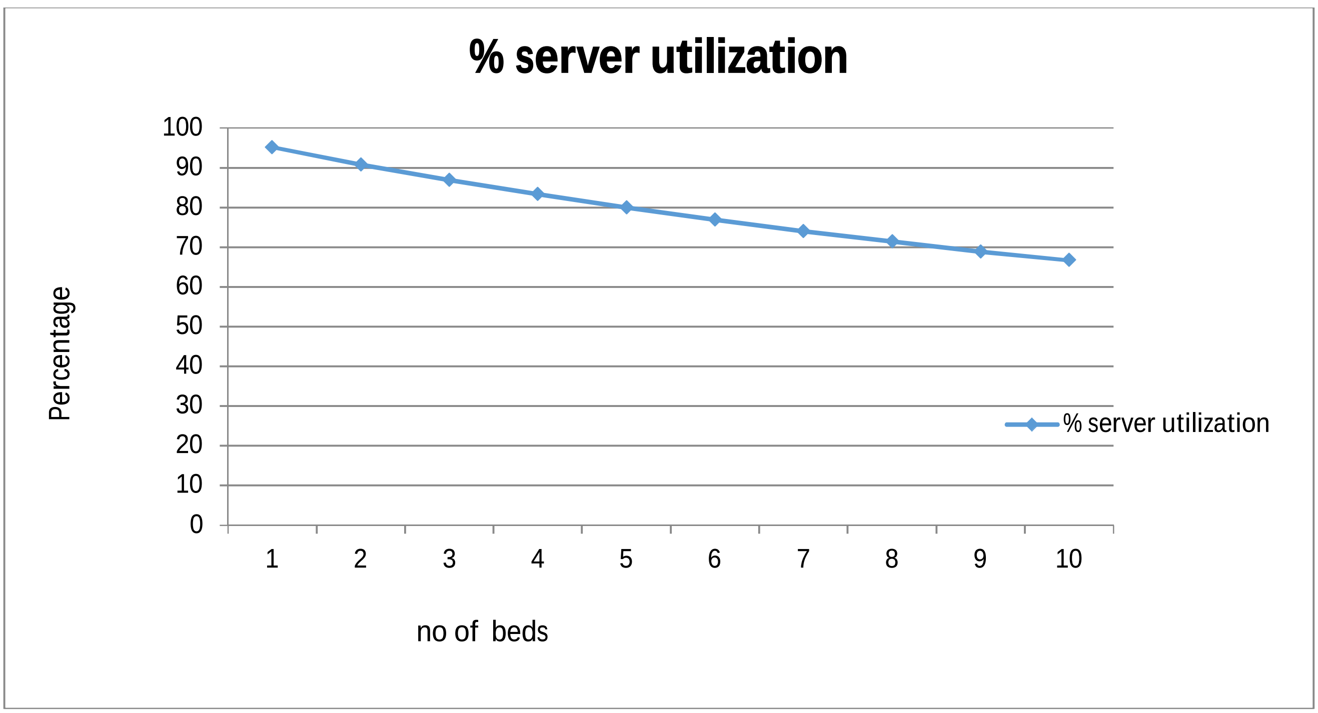


Table 2 and 3 shows the result for various values of C_1 and C_2 . From the tables we can see that C_1=12 guarantees that there is no waiting at the EAW, since urgent Patient needing urgent care are brought in through it. In the MSW, C_2=28, will guarantee an approximate of 71.35% server utilization and a minimum waiting time in queue.

VII. CONCLUSION

In this work, we analysed a queueing network model with reneging to study how waiting time in the Emergency and Accident department (EAD) of an Hospital is influenced by the number of beds in the ICW and MSW. The system was decomposed into two independent multi-server queues so as to obtain estimates for the required number of beds in the wards. We found that the required number of beds to ensure that emergent patients are promptly attended and there is easy flow is approximately 12 in the ICW and 28 in the MSW for the test hospital under consideration.

REFERENCES

[1] Adeleke. R. A, Ogunwale O.D, Halid O. Y(2009) Application of queueing theory to waiting time of out patients in hospitals, Pacific Journal of science and technology Vol.10(2) 270-274.
[2] Arun Kumar and John Mo (2010) Models for bed occupancy Management of a hospital in Singapore. Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management Dhaka, Bangladesh.
[3] Debruin A.M, A.C Van Rossun MC Visser, GM Koole (2006) Modeling the Emergency Cardiac In-Patient flow : An application of queueing theory. Health care Management Science Vol. 10, No 2, pp125-137.
[4] Derlet R W, John R Richard, Richard L.kravitz(2006) Frequent overcrowding in U.S Emergency Departments, Academic Emergency Medicine, vol 8,issue 2,pp 151-155.
[5] Green L.V (2002) How many beds Decision, Risk and Operation. Working Paper series
[6] Green L.V, soares J, Giglio JF, Green R A(2006) using queueing theory to increase the effectiveness of emergency department provider staffing. Academic Emergency Medicine 13(1):61-68.
[7] Janathan. E H,Shervin Ahmed Beygi, Mark P. Van Oyen(2009), design and Analysis of Hospital of Hospital Admission Control for Operational Effectiveness Technical Report 09-05. University of Michigan. Michigan
[8] Murray J. Cote (2000) understanding patient flow. Production/Operations Management Decision Line pp8-10
[9] Olorunsola S. A, Adeleke R.A and ogunlade T.O (2014) Queueing analysis of patient flow in Hospital, IOSR Journal of Mathematics Vol.10 ISSN:2278-5728.
[10] WeissE.N, J.O Mcclain (1986) Administrative days in acute care Facilities: a queueing analytic approach Operations ResearchVol.35 no 1 pp 35-44.
[11] Worthington D.J (1987) queueing models for Hospital waiting list. The journal of the Operational Research Society Vol.38, No 5,pp413-422.