

Pythagorean Triangle with Area/Perimeter as a 4-digit consecutive Sphenic numbers

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Abstract: We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the 4-digit consecutive Sphenic numbers namely 2013, 2014 and 2015. Also we present the number of primitive and non-primitive Pythagorean triangles. A few interesting relations among the sides are also given.

Keywords: Pythagorean triangles, 4-digit consecutive Sphenic numbers, Primitive and non-primitive Pythagorean triangles.

Notations

$Tri_n =$ Triangular number of rank n .

$Hex_n =$ Hexagonal number of rank n .

I. INTRODUCTION

Number theory is a broad and diverse part of Mathematics that developed from the study of the integers. Mathematicians all over the ages have been fascinated by Pythagorean Theorem and are solving many problems related to it thereby developing Mathematics. Pythagorean triangles which were first studied by the Pythagoreans around 400 B.C., remains one of the fascinated topics for those who just adore the numbers.

A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. These numbers have presented in [1-5]. In [6-14], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are presented.

In this communication, we search for patterns of Pythagorean triangles, such that in each of which the ratio Area/Perimeter is represented by the 4-digit consecutive Sphenic numbers 2013, 2014 and 2015.

II. BASIC DEFINITIONS

A. Definition 1

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$.

Also, in Pythagorean triangle $T(x, y, z): x^2 + y^2 + z^2$, x and y are called its legs and z its hypotenuse.

B. Definition 2

The most cited solutions of the Pythagorean equation is $x = m^2 - n^2$, $y = 2mn$, $z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

C. Definition 3

Sphenic number is a positive integer that is the product of three distinct prime numbers.

III. METHOD OF ANALYSIS

Denoting the Area and Perimeter of the triangle by A and P respectively.

The assumption

$$\frac{A}{P} = 4\text{-digit Consecutive Sphenic number.}$$

The above relation leads to the equation

$$\frac{v(u - v)}{2} = 4\text{-digit consecutive Sphenic number.} \tag{1}$$

Case 1

When
$$\frac{v(u - v)}{2} = 2013 \tag{2}$$

which implies
$$v(u - v) = 4026 \tag{3}$$

On computation, the values of the generators u, v satisfying (3) are given in the following table.

| S.No | v | $u - v$ | u | x | y | z | A | P | $\frac{A}{P}$ |
|------|------|---------|------|----------|----------|----------|--------------|----------|---------------|
| 1 | 1 | 4026 | 4027 | 8054 | 16216728 | 16216730 | 65304763660 | 32441512 | 2013 |
| 2 | 2 | 2013 | 2015 | 8060 | 4060221 | 4060229 | 16362690630 | 8128510 | 2013 |
| 3 | 3 | 1342 | 1345 | 8070 | 1809016 | 1809034 | 7299379560 | 3626120 | 2013 |
| 4 | 6 | 671 | 677 | 8124 | 458293 | 458365 | 1861586166 | 924782 | 2013 |
| 5 | 11 | 366 | 377 | 8294 | 142008 | 142250 | 588907176 | 292552 | 2013 |
| 6 | 22 | 183 | 205 | 9020 | 41541 | 42509 | 187349910 | 93070 | 2013 |
| 7 | 33 | 122 | 155 | 10230 | 22936 | 25114 | 117317640 | 58280 | 2013 |
| 8 | 61 | 66 | 127 | 15494 | 12408 | 19850 | 96124776 | 47752 | 2013 |
| 9 | 122 | 33 | 155 | 37820 | 9141 | 38909 | 172856310 | 85870 | 2013 |
| 10 | 183 | 22 | 205 | 75030 | 8536 | 75514 | 320228040 | 159080 | 2013 |
| 11 | 366 | 11 | 377 | 275964 | 8173 | 276085 | 1127726886 | 560222 | 2013 |
| 12 | 671 | 6 | 677 | 908534 | 8088 | 908570 | 3674111496 | 10002192 | 2013 |
| 13 | 1342 | 3 | 1345 | 3609980 | 8061 | 3609989 | 14550024390 | 7228030 | 2013 |
| 14 | 2013 | 2 | 2015 | 8112390 | 8056 | 8112394 | 32676706920 | 16232840 | 2013 |
| 15 | 4026 | 1 | 4027 | 32425404 | 8053 | 32425405 | 130560889200 | 64858862 | 2013 |

From the above table, it is seen that there are 15 Pythagorean triangles. Out of these, 7 triangles are primitive and remaining 8 triangles are non-primitive triangles.

Case 2:

When
$$\frac{v(u - v)}{2} = 2014 \tag{4}$$

which implies
$$v(u - v) = 4028 \tag{5}$$

On computation, the values of the generators u, v satisfying (5) are given in the following table.

| S.No | v | $u - v$ | u | x | y | z | A | P | $\frac{A}{P}$ |
|------|------|---------|------|----------|----------|----------|--------------|----------|---------------|
| 1 | 1 | 4028 | 4029 | 8058 | 16232840 | 16232842 | 65402112360 | 32473740 | 2014 |
| 2 | 2 | 2014 | 2016 | 8064 | 4064252 | 4064260 | 16387064060 | 8136576 | 2014 |
| 3 | 4 | 1007 | 1011 | 8088 | 1022105 | 1022137 | 4133392620 | 2052330 | 2014 |
| 4 | 19 | 212 | 231 | 8778 | 53000 | 53722 | 232617000 | 115500 | 2014 |
| 5 | 38 | 106 | 144 | 10944 | 19292 | 22180 | 105565824 | 52416 | 2014 |
| 6 | 53 | 76 | 129 | 13674 | 13832 | 19450 | 94569284 | 46956 | 2014 |
| 7 | 76 | 53 | 129 | 19608 | 10865 | 22417 | 106520460 | 52890 | 2014 |
| 8 | 106 | 38 | 144 | 30528 | 9500 | 31972 | 145008000 | 72000 | 2014 |
| 9 | 212 | 19 | 231 | 97944 | 8417 | 98305 | 412197324 | 204666 | 2014 |
| 10 | 1007 | 4 | 1011 | 2036154 | 8072 | 2036170 | 8217917544 | 4080396 | 2014 |
| 11 | 2014 | 2 | 2016 | 8120448 | 8060 | 8120452 | 32725405440 | 16248960 | 2014 |
| 12 | 4028 | 1 | 4029 | 32457624 | 8057 | 32457625 | 130755538300 | 64923306 | 2014 |

From the above table, it is seen that there are 12 Pythagorean triangles. Out of these, 5 triangles are primitive and remaining 7 triangles are non-primitive Pythagorean triangles.

Case 3:

When
$$\frac{v(u - v)}{2} = 2015 \tag{6}$$

which implies
$$v(u - v) = 4030 \tag{7}$$

On computation, the values of the generators u, v satisfying (7) are given in the following table.

| S.No | v | $u - v$ | u | x | y | z | A | P | $\frac{A}{P}$ |
|------|-----|---------|------|------|----------|----------|-------------|----------|---------------|
| 1 | 1 | 4030 | 4031 | 8062 | 16248960 | 16248962 | 65499557760 | 32505984 | 2015 |
| 2 | 2 | 2015 | 2017 | 8068 | 4068285 | 4068293 | 16411461690 | 8144646 | 2015 |
| 3 | 5 | 806 | 811 | 8110 | 657696 | 657746 | 2666957280 | 1323552 | 2015 |

| | | | | | | | | | |
|----|------|-----|------|----------|--------|----------|--------------|----------|------|
| 4 | 10 | 403 | 413 | 8260 | 170469 | 170669 | 1408073940 | 349398 | 2015 |
| 5 | 13 | 310 | 323 | 8398 | 104160 | 104498 | 437367840 | 217056 | 2015 |
| 6 | 26 | 155 | 181 | 9412 | 32085 | 33437 | 150992010 | 74934 | 2015 |
| 7 | 31 | 130 | 161 | 9982 | 24960 | 26882 | 124575360 | 61824 | 2015 |
| 8 | 62 | 65 | 127 | 15748 | 12285 | 19973 | 193464180 | 48006 | 2015 |
| 9 | 65 | 62 | 127 | 16510 | 11904 | 20354 | 196535040 | 48768 | 2015 |
| 10 | 130 | 31 | 161 | 41860 | 9021 | 42821 | 188809530 | 93702 | 2015 |
| 11 | 155 | 26 | 181 | 56110 | 8736 | 56786 | 245088480 | 121632 | 2015 |
| 12 | 310 | 13 | 323 | 200260 | 8229 | 200429 | 823969770 | 408918 | 2015 |
| 13 | 403 | 10 | 413 | 332878 | 8160 | 332978 | 1358142240 | 674016 | 2015 |
| 14 | 806 | 5 | 811 | 1307332 | 8085 | 1307357 | 5284889610 | 2622774 | 2015 |
| 15 | 2015 | 2 | 2017 | 8128510 | 8064 | 8128514 | 32774152320 | 16265088 | 2015 |
| 16 | 4030 | 1 | 4031 | 32489860 | 8061 | 32489861 | 130950380700 | 64987782 | 2015 |

From the above table, it is seen that there are 16 Pythagorean triangles. Out of these, 8 triangles are primitive and remaining 8 triangles are non-primitive Pythagorean triangles.

IV. OBSERVATIONS

- 1) $z - x, \frac{1}{2}(y + z)$ are perfect squares.
- 2) The expressions $6(z - x), 12(y + z)$ represents a nasty numbers.
- 3) $x \equiv 0 \pmod{2}$.
- 4) $A \equiv 0 \pmod{10}$.
- 5) $y + z + 2u = Tri_u$.
- 6) $y + z - 2u = Hex_u$.

V. CONCLUSION

One may search for the special Pythagorean triangles which are connected with the other Sphenic numbers.



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