

Mathematical Modelling of Costas Loop under Diversity Combining Technique

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Abstract: In wireless communication, carrier synchronization and symbol synchronization are essentially important. The accuracy in estimation of two quantities has a direct impact on received data error rates. For this Costas receiver is employed. It's a synchronous, applicable for the demodulation of double side-band suppressed carrier (DSB-SC) waves. Carrier recovery is very important for the operation of a coherent detector. For good reception, the transmitter and receiver ought to be fully matched in frequency and phase. To meet this purpose, Costas loop is usually employed in wireless communication and additionally, in microwave systems. It is a non-linear based Phase Locked Loop. This work, make a case for concerning the ways in which Costas loop is employed underneath the independent fading channel with three diversity combining techniques to study the application of the bit error rate problem. Due to diversity the signal reaches to the receiver with loss of power strength. The simulation result shows that Costas Loop has better performance with the maximal ratio combining than other combining techniques.

Keywords: Costas Loop, Diversity, BER, Combining techniques, I/Q imbalance.

I. INTRODUCTION

In wireless communication, there is a problem of phase loss or phase offset due to the mobility of the user or due to the Doppler effect or by fading. This lead to the poor performance of signal or the loss in signal power. To eliminate this phase loss, Costas Loop is used in the receiver. It fixes the voltage controlled oscillator (VCO) output to scale back the phase between locally generated signal and the received signal.

Costas Loop is a coherent demodulator used to synchronize the phase offset. Phase offset occurs by fading nature of the channel. Due to this, the signal received from different paths and has loss of phase information. Diversity is used to combat the fading nature of channel either at transmitter antenna or receiver antenna.

The paper presents the I/Q imbalance technique in Costas loop and diversity combining techniques is employed to analyze the bit error rate comparison of these given techniques. The work is concentrated on In-phase and Quadrature phase to recover information fast by using Fast Fourier Transform (FFT) on each arm.

II. RELATED WORK

A. Costas Loop

Costas loop or Costas receiver is used for coherent demodulation.

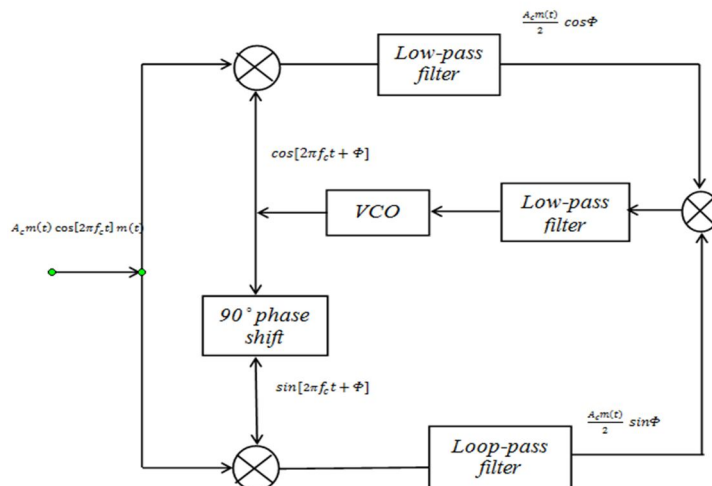


Fig.1. Costas Loop

Assuming $\frac{A_c^2 m^2(t)}{4}$ and VCO generate a local frequency with phase off-set $\cos[2\pi f_c t + \Phi]$.

The I Channel or 1st demodulator equation is given by

$$\begin{aligned} &= A_c m(t) \cos[2\pi f_c t] * \cos[2\pi f_c t + \Phi] \\ &= \frac{A_c m(t)}{2} \{\cos\Phi + \cos(4\pi f_c t + \Phi)\} \end{aligned}$$

After low pass filter

$$= \frac{A_c m(t)}{2} \cos\Phi$$

Q channel output

$$\begin{aligned} &= A_c m(t) \cos[2\pi f_c t] * \sin[2\pi f_c t + \Phi] \\ &= \frac{A_c m(t)}{2} \{\sin\Phi + \sin(4\pi f_c t + \Phi)\} \end{aligned}$$

After low pass filterz

$$= \frac{A_c m(t)}{2} \sin\Phi$$

Phase discriminator output

$$\begin{aligned} &= \frac{A_c^2 m^2(t)}{4} \sin\Phi \cos\Phi \\ &= m^2 \frac{(t)}{4} \geq 0 \end{aligned}$$

For small Φ , $-\frac{\pi}{2} \leq \Phi \leq \frac{\pi}{2}$, if $\Phi \geq 0$

Phase detector output

$$= \frac{A_c^2 m^2(t)}{4} \sin\Phi \cos\Phi \geq 0$$

Similarly, if $\Phi < 0$

Phase detector output

$$= \frac{A_c^2 m^2(t)}{4} \sin\Phi \cos\Phi < 0$$

B. I/Q Imbalance

Considering the data to be transmitted is a complex signal,

$$x = x_i + jx_q$$

The output of I-Q modulation transmission is,

$$\begin{aligned} y &= R\{x e^{j2\pi f_c t}\} \\ &= x_i \cos(2\pi f_c t) - x_q \sin(2\pi f_c t) \end{aligned}$$

At the transmitter, data x_i sent on $\cos(2\pi f_c t)$ and x_q sent on $\sin(2\pi f_c t)$. In the receiver, y signal is multiplied by the $\cos(2\pi f_c t)$ and $-\sin(2\pi f_c t)$ followed by low pass filter to extract \hat{x}_i and \hat{x}_q respectively.

According to trigonometric identity

$$\begin{aligned} \int_0^T \cos(2\pi f_c t) \cos(2\pi f_c t) &= \frac{1}{2} [1 + \cos(4\pi f_c t)] \\ \int_0^T \sin(2\pi f_c t) \sin(2\pi f_c t) &= \frac{1}{2} [1 - \cos(4\pi f_c t)] \\ \int_0^T \cos(2\pi f_c t) \sin(2\pi f_c t) &= 0 \end{aligned}$$

At receiver, the equation is as follows for I arm

$$\hat{x}_i = \int_0^T y \cos(2\pi f_c t)$$

$$= \int_0^T [x_i \cos(2\pi f_c t) - x_q \sin(2\pi f_c t)] \cos(2\pi f_c t) dt$$

$$= \frac{x_i}{2}$$

Similarly for Q arm

$$\hat{x}_q = \int_0^T y (-\sin(2\pi f_c t)) dt$$

$$= \int_0^T [x_i \cos(2\pi f_c t) - x_q \sin(2\pi f_c t)] (-\sin(2\pi f_c t)) dt$$

$$= \frac{x_q}{2}$$

By ignoring the scaling factor we can recover the both I arm x_i and Q arm x_q .

C. Diversity

Diversity is employed to overcome the fading effects or to overcome to combat fading or to overcome the effects of deep fade. Deep fade events have significantly degraded the performance of wireless communication. It can implement by using two or more receiving antennas. These techniques can be employed at both base station and mobile receivers.

D. Types of Diversity

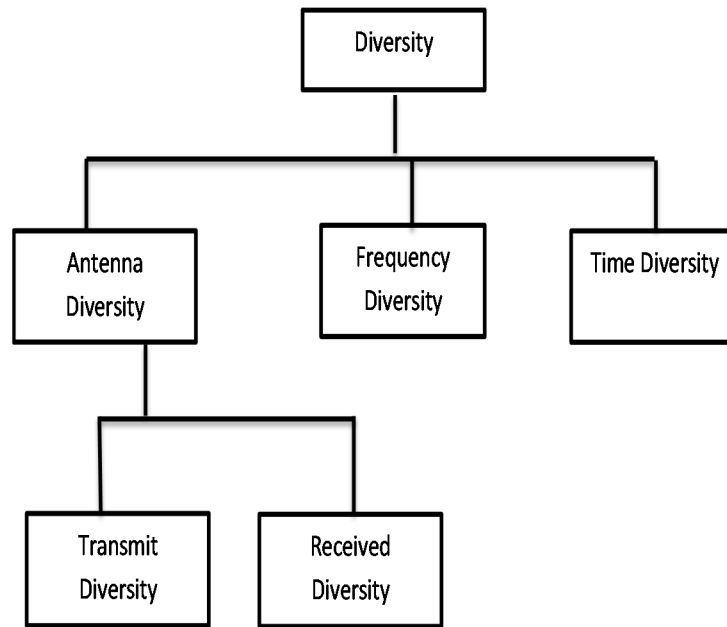


Fig. 2. Classification of Diversity

1) *Antenna Diversity*: Antenna diversity is more popular diversity techniques where we use multiple antennas either at the transmitter or at the receiver or on both sides. There are two types of antenna diversity.

2) *Transmit Diversity*: In radio communication when multiple antennas are used at the transmitter side to originate the signals from the different source.

i) Principle

More complicated than SISO.

2 or more than 2 transmitters are used and one receiver.

MISO system are lot usually referred to as transmit diversity.

The same data is distributed on each transmitting antennas but coded in such the way that the receiver will establish every transmitter.

ii) *Benefits*: Strength of the signal will increase by transmit diversity to fading and might increase performance in low Signal-to-Noise Ratio (SNR) conditions. The data rate does not increase as such but rather supports the same data rates by using less power.

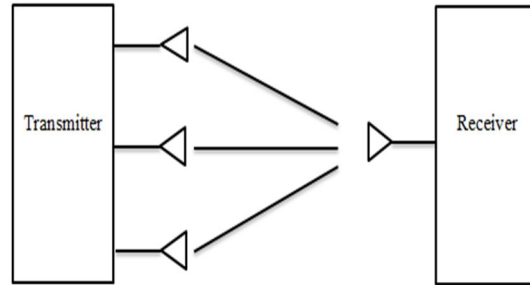


Fig. 3 Antenna Diversity

a) *Received Diversity*: In radio communication when multiple antennas are used at the receiver side to receive the signals from the different path.

i) *Principle*:

It uses one transmitter and a pair of or a lot of receivers.

It is usually mentioned as receiver diversity.

ii) *Benefits*:

It is well matched for the low SNR conditions where-ever 3 dB of gain is obtained.

It is possible when 2 receivers are used.

No amendment within the data rate since just one data stream is transmitted.

Owing to lowering of the usable SNR the cell edge is improved.

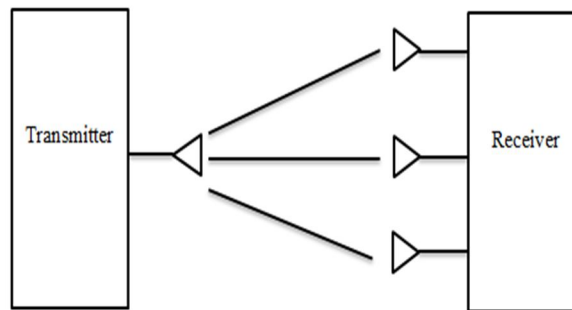


Fig. 4 Received Diversity

E. Combining Techniques

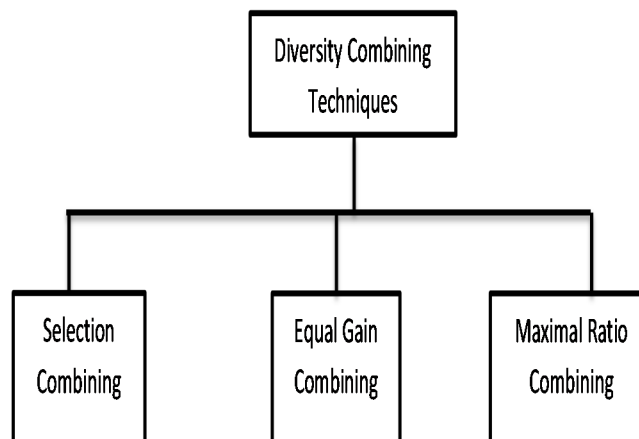


Fig 5. Classification of combining techniques

1) *Selection Combining*: Choosing the most effective signal among all the signals received from totally different branches at the receiving end. Bit error probability can be calculated from selection combining is given by:

$$P_e = \frac{1}{2} \sum_{k=0}^N (-1)^k N_k \left(1 + \frac{k}{(E_b/N_0)}\right)^{-\frac{1}{2}}$$

2) *Equal Gain Combining*: To attain the best SNR at the receiver at all times all the signals in a co-phased manner with unity weights for all signals levels this combining technique is employed.

Bit error rate can be calculated by using equal gain combining is given by:

$$P_e = \frac{1}{2} \left[1 - \sqrt{\frac{E_b/N_0 (E_b/N_0 + 2)}{E_b/N_0 + 1}} \right]$$

3) *Maximal Ratio Combining*: Combining all the signals during a co-phased and weighted manner, therefore on have the best accomplished SNR at the receiver at all times. Bit error rate calculated by maximal ratio combining is given by:

$$P_e = p^N \sum_{k=0}^N (N - 1 + k)(1 - p)^k$$

III. PROPOSED SYSTEM DESIGN

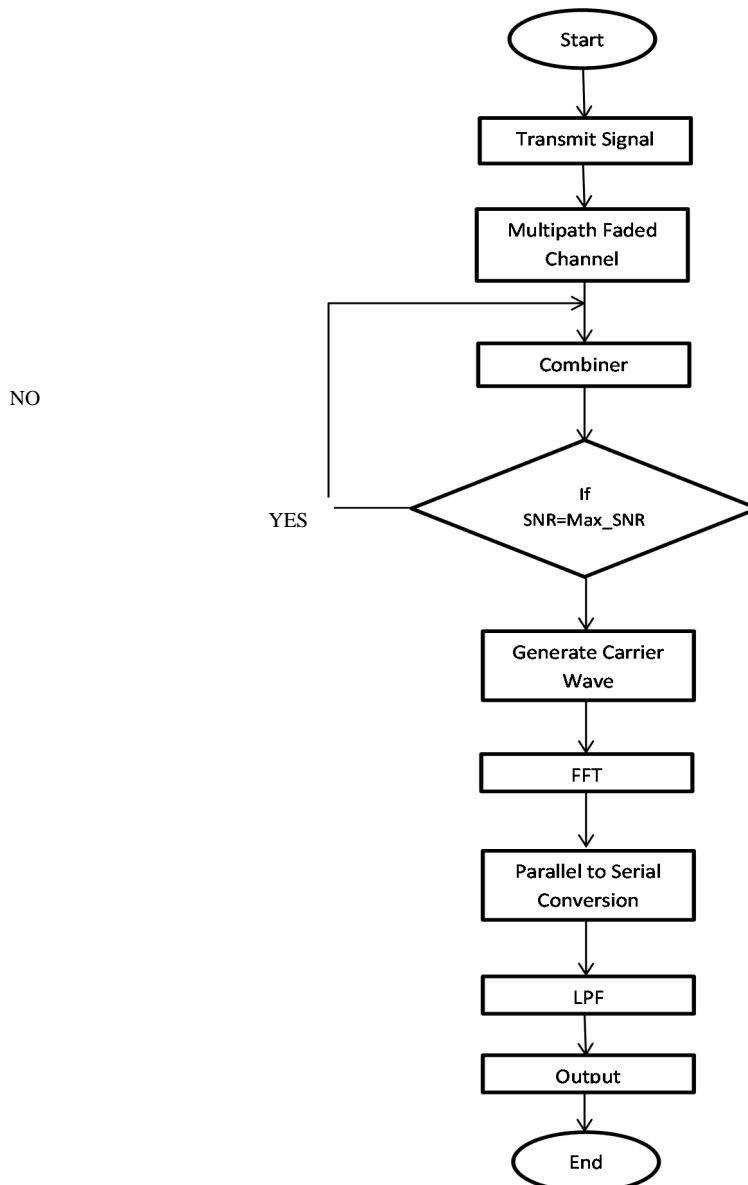


Fig. 6 System design of Combiner and Costas Loop

The following steps involved in the work for demodulation of the signal are:

Transmit the signal.

Signal passes through the multipath fading channel.

Combiner is used to select the highest value of SNR of the signal for demodulation.

After highest SNR, the VCO generates the local carrier signal.

The local carrier signal then converted to parallel to the serial bit stream.

Signal passes through low pass filter (LPF).

From I-arm the demodulated signal is obtained.

IV. RESULTS

The simulation result shows the performance comparison of bit error rate of Costas Loop with combining techniques.

Transmitted_data_bits	0 0 1 1
SNR	20
Bit_rate	4
IFilterfreq_3dB	8.0005
QFilterfreq_3dB	8.0005
Received SNR	12
Received_data_bits	0 0 1 1

Table 1. Data Sheet

Table 1 shows the data which is being transmitted and received. The maximum SNR which is used to transmit is 20 dB. But at receiver, the maximum SNR which is obtained is 12dB.

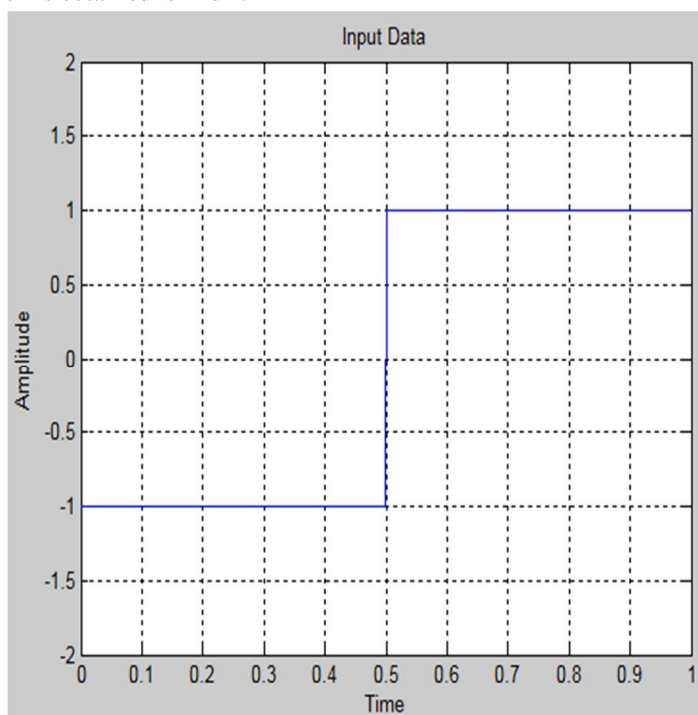


Fig. 7 Transmitted Bit

Fig. 7 represents the bit sequence which is being sent from the transmitter by using a single antenna. This signal is propagated through the multi path fading channel.

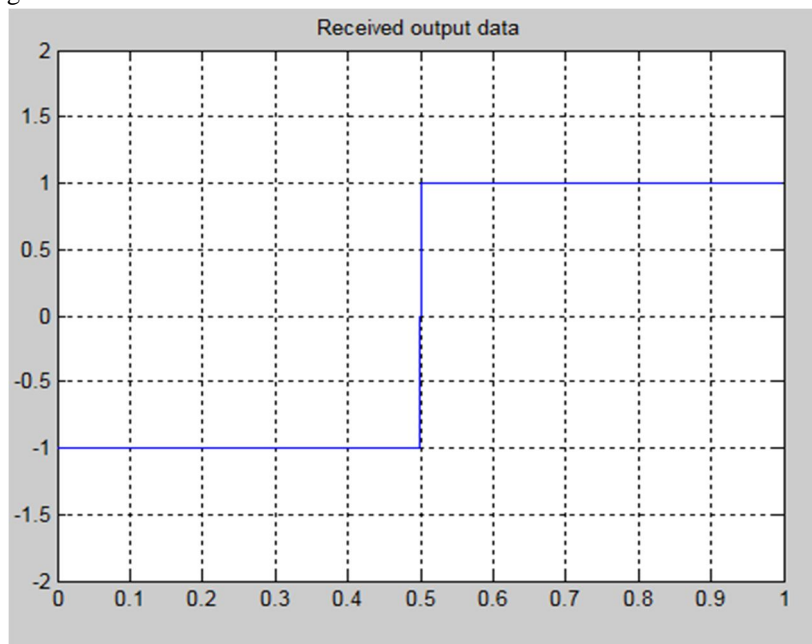


Fig. 8 Received Bit

Fig. 8 shows the recovered bit which is obtained from the multipath fading channel at the receiver having the number of multiple antennas.

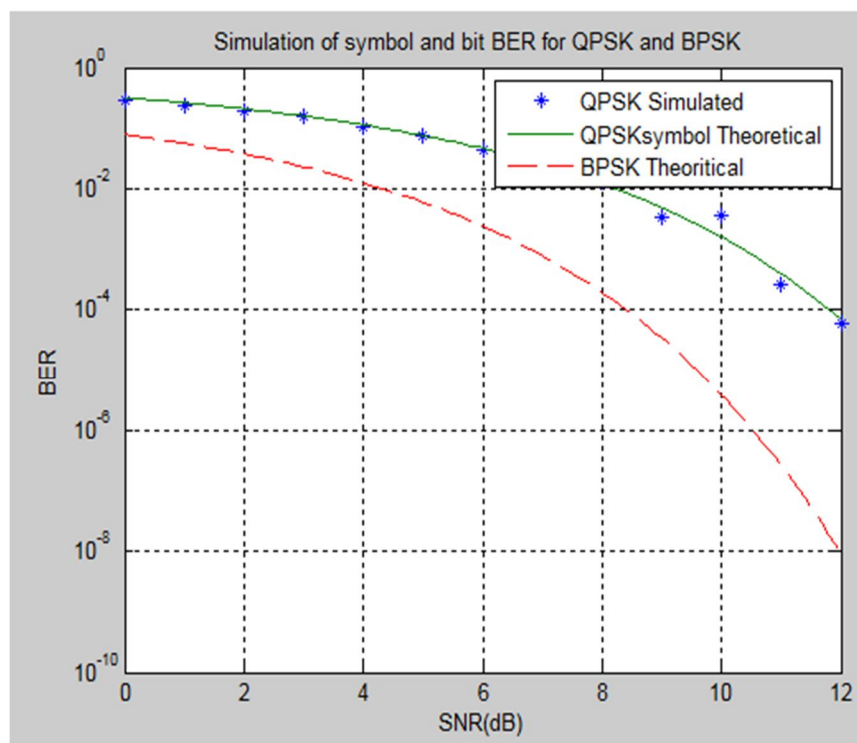


Fig. 9 BER without Diversity Combining

Fig. 9 shows the bit error rate when the signal is received from multipath and when no combining techniques are used. The receiver randomly takes the SNR value and decode it. Due to this many times, the signal power is the loss.

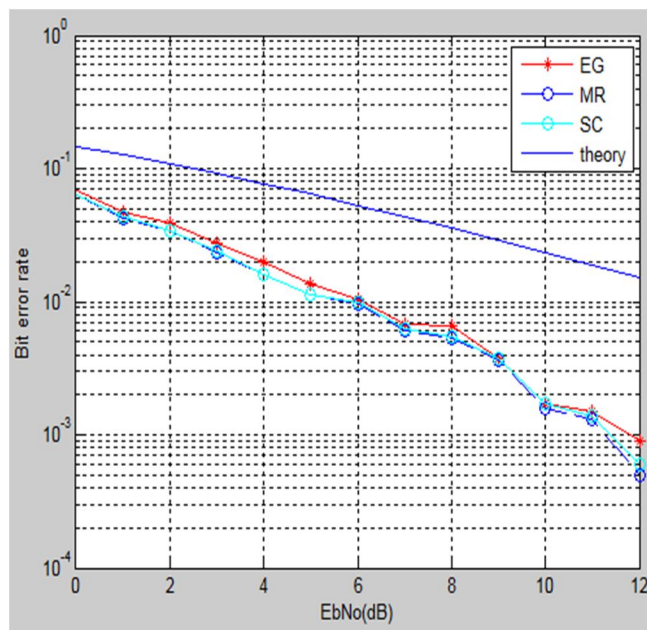


Fig. 10 BER with diversity combining technique

Fig. 10 shows when the combining techniques are used at receiver the Costas loop performance is enhanced and we get the maximum SNR of 12 dB with the low error in bits.

V. CONCLUSION

The simulation results show that the Costas loop has recovered the transmitted data which is received from multiple faded paths and an improved bit error rate performance with diversity combining techniques is observed. Fig. 7 shows the transmitted data and Fig. 8 shows the recovered received data bit. Fig. 9 shows the bit error rate when the combiner is not used and random SNR values are taken into account and Fig. 10 shows the bit error rate improvement when the combining technique is employed with Costas Loop. The comparison shows that the Maximal Ratio Combining (MRC) gives the lower bit error with maximum SNR than the other combining techniques whenever the simulation run.

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